MARK SCHEME

Maximum Mark: **75**

Q	Answer / Working	Marks	Guidance
1(a)	Forms $x^2 - 4 = 3x$ or $x^2 - 4 = -3x$	**M1**	Or squares both sides (up to quartic).
	$x^2 - 3x - 4 = 0 \implies (x-4)(x+1) = 0 \implies x = 4, x = -1$ $x^2 + 3x - 4 = 0 \implies (x+4)(x-1) = 0 \implies x = 1, x = -4$	**M1**	Solves to find potential roots.
	Valid roots are $x = 4$ and $x = 1$.	**A1**	Rejects -1 and -4 since $3x \geqslant 0$.
		[3]	
1(b)	Recognises translation: replaces x with $x-1$.	**M1**	
	$x - 1 = 4 \implies x = 5$ and $x - 1 = 1 \implies x = 2$.	**A1**	Both correct without re-solving.
		[2]	
2(a)	Tangent to x-axis implies $P(2) = 0$ and $P'(2) = 0$.	**B1**	Can be implied by working.
	$P(2) = 0 \implies 16 + 4a + 2b + 12 = 0 \implies 2a + b = -14$	**M1**	Correct linear equation 1.
	$P'(x) = 6x^2 + 2ax + b \implies P'(2) = 24 + 4a + b = 0 \implies 4a + b = -24$	**M1**	Correct linear equation 2.

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	Solves simultaneously: $2a = -10 \implies a = -5$. $b = -4$.	A1	Both values correct.
		[4]	
2(b)	Tangent at $x = 2$ implies $(x-2)^2$ is a factor. $2x^3 - 5x^2 - 4x + 12 = (x-2)^2(cx+d) = (x^2 - 4x + 4)(2x+3)$	M1	Equates coefficients or uses polynomial division.
	Other root is $x = -1.5$.	A1	
		[2]	
3(a)	$R = \sqrt{4^2 + 3^2} = 5$	B1	Exact value.
	$\tan\alpha = \frac{3}{4}$	M1	
	$\alpha = 36.87°$	A1	Correct to 2 d.p.
		[3]	
3(b)	The curve is $y = 5\sin(x + 36.87°) + k$. Minimum value of the un-translated curve is -5.	M1	Uses their R.
	To touch the x-axis, minimum must be $0 \implies -5 + k = 0 \implies k = 5$.	A1	
		[2]	
4(a)	$(4+2x)^{-1/2} = 4^{-1/2}\left(1+\frac{x}{2}\right)^{-1/2} = \frac{1}{2}\left(1+\frac{x}{2}\right)^{-1/2}$	B1	Correct extraction of $\frac{1}{2}$.
	Expands $(1+X)^{-1/2}$: $1 + (-\frac{1}{2})(\frac{x}{2}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(\frac{x}{2})^2$	M1	Unsimplified correct structure.

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	$= \frac{1}{2} - \frac{1}{8}x + \frac{3}{64}x^2$	A1	All signs and coefficients correct.
		[3]	
4(b)	$f(x) = (3-x)\left(\frac{1}{2} - \frac{1}{8}x + \frac{3}{64}x^2\right)$	M1	Multiplies by $(3-x)$.
	$= \frac{3}{2} - \frac{3}{8}x + \frac{9}{64}x^2 - \frac{1}{2}x + \frac{1}{8}x^2 = \frac{3}{2} - \frac{7}{8}x + \frac{17}{64}x^2$	A1	
		[2]	
4(c)	$\lvert\frac{x}{2}\rvert < 1 \implies \lvert x\rvert < 2 \quad$ (or $-2 < x < 2$)	B1	
		[1]	
5	$3x^2 + 3y^2\frac{dy}{dx} - \left(3y + 3x\frac{dy}{dx}\right) = 0$	M1	Correct implicit differentiation.
	Factorises to make $\frac{dy}{dx}$ the subject: $\frac{dy}{dx}(y^2 - x) = y - x^2$	A1	
	Tangent parallel to x-axis implies $\frac{dy}{dx} = 0 \implies y = x^2$	M1	Equates numerator to 0.
	Substitutes $y = x^2$ into original: $x^3 + (x^2)^3 - 3x(x^2) = 3$ $x^6 - 2x^3 - 3 = 0$	M1	Forms equation in one variable.
	$(x^3-3)(x^3+1) = 0 \implies x^3 = 3$ or $x^3 = -1$	M1	Solves hidden quadratic for x^3.
	$x = \sqrt[3]{3} \implies y = 3^{2/3}$ and $x = -1 \implies y = 1$ Coordinates are $(-1, 1)$ and $(\sqrt[3]{3}, 3^{2/3})$.	A1	Exact coordinates required.
		[6]	

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6(a)	Let $u = x + \mathrm{i}y \implies u^* = x - \mathrm{i}y \implies uu^* = x^2 + y^2$.	B1	
	$x^2 + y^2 + 4\mathrm{i}(x + \mathrm{i}y) - 5 + 12\mathrm{i} = 0 \implies (x^2 + y^2 - 4y - 5) + \mathrm{i}(4x + 12) = 0$	M1	Groups real and imaginary parts.
	Imaginary part: $4x + 12 = 0 \implies x = -3$.	A1	
	Real part: $(-3)^2 + y^2 - 4y - 5 = 0 \implies y^2 - 4y + 4 = 0 \implies (y-2)^2 = 0 \implies y = 2$. Hence $u = -3 + 2\mathrm{i}$.	A1	
		[4]	
6(b)	Circle centre $(-3, 2)$ drawn with radius 2.	B1	
	Horizontal line drawn at $y = 2$.	B1	
	Top half of the circle (boundary included) shaded.	B1	
		[3]	
6(c)	$z = w + 1$ represents translating the region right by 1. Centre of semi-circle for z is $(-2, 2)$, radius 2.	M1	Recognises geometric transformation.
	Max $\lvert z \rvert$ is the distance from origin through the centre plus radius. Max $\lvert z \rvert = \sqrt{(-2)^2 + 2^2} + 2 = \sqrt{8} + 2 = 2\sqrt{2} + 2$.	A1	Valid geometric deduction.
		[2]	
7(a)	$\frac{4a^2}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a} \implies A(x+a) + B(x-a) = 4a^2$	M1	Setup.
	$A = 2a$ and $B = -2a$. Hence $\frac{2a}{x-a} - \frac{2a}{x+a}$.	A1	
		[2]	

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7(b)	$\int \left(\frac{2a}{x-a} - \frac{2a}{x+a}\right) \mathrm{d}x = 2a\ln\lvert x-a\rvert - 2a\ln\lvert x+a\rvert = 2a\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	M1	Proper integration yielding ln.
	Evaluates limits: $2a\left[\ln\left(\frac{2a}{4a}\right) - \ln\left(\frac{a}{3a}\right)\right] = 2a\left[\ln\left(\frac{1}{2}\right) - \ln\left(\frac{1}{3}\right)\right]$	M1	Substitutes $3a$ and $2a$ correctly.
	$= 2a\ln\left(\frac{3}{2}\right) = a\ln\left(\frac{9}{4}\right)$	A1	Must be in requested form.
		[3]	
7(c)	$\int_{2a}^{N} f(x)\mathrm{d}x = 2a\ln\left(\frac{N-a}{N+a}\right) - 2a\ln\left(\frac{1}{3}\right) = 2a\ln\left(\frac{3(N-a)}{N+a}\right)$	M1	Correct upper limit substitution.
	As $N \to \infty$, $\frac{N-a}{N+a} \to 1$.	M1	Limit logic explicitly shown.
	Integral converges to $2a\ln 3$ (or $a\ln 9$).	A1	
		[3]	

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8(a)	Let N be foot of perpendicular from P to l. $\vec{PN} = (1+2\lambda)\mathbf{i} + (2-\lambda-3)\mathbf{j} + (1+\lambda-1)\mathbf{k} = \binom{1+2\lambda}{-1-\lambda}\lambda$	M1	Vector connecting P to generic point on l.
	$\vec{PN}\cdot\binom{2}{-1}1 = 0 \implies 2(1+2\lambda)-(-1-\lambda)+\lambda = 0$	M1	Dot product with direction vector.
	$2+4\lambda+1+2\lambda = 0 \implies 6\lambda = -3 \implies \lambda = -0.5$. So $N = (0, 2.5, 0.5)$ and $\vec{PN} = \binom{0}{-0.5}-0.5$.	A1	Finds λ and N or $\vec{PN}$.
	$P' = P + 2\vec{PN} = \binom{0}{3}1 + \binom{0}{-1}-1 = \binom{0}{2}0 = 2\mathbf{j}$.	A1	Correct reflection vector.
		[4]	
8(b)	Area $= \frac{1}{2}\times\|PP'\|\times\|QN\| = \sqrt{3}$. $\|PP'\| = \sqrt{0^2+(-1)^2+(-1)^2} = \sqrt{2}$.	M1	Uses geometry of triangle.
	$\frac{1}{2}\sqrt{2}\|QN\| = \sqrt{3} \implies \|QN\| = \sqrt{6}$.	M1	Length from N to Q.
	Q has parameter μ. $\vec{NQ} = (\mu-(-0.5))\binom{2}{-1}1$. $\|\vec{NQ}\|^2 = (\mu+0.5)^2(2^2+(-1)^2+1^2) = 6(\mu+0.5)^2$. $6(\mu+0.5)^2 = 6 \implies \mu+0.5 = \pm 1 \implies \mu = 0.5$ or $\mu = -1.5$.	M1	Translates length to parameter difference.
	$\mu = 0.5 \implies Q_1 = 2\mathbf{i}+1.5\mathbf{j}+1.5\mathbf{k}$. $\mu = -1.5 \implies Q_2 = -2\mathbf{i}+3.5\mathbf{j}-0.5\mathbf{k}$.	A1	Both position vectors correct.
		[4]	
9(a)	$\frac{dx}{dt} = -kx^2\sqrt{t}$	M1	Initial translation to math model.
	$x = 2, t = 1 \implies -0.5 = -k(4)(1) \implies k = 0.125 \implies \frac{dx}{dt} = -0.125x^2\sqrt{t}$	A1	Fully shown.

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		[2]	
9(b)	Separate variables: $\int x^{-2}\mathrm{d}x = \int -\frac{1}{8}t^{1/2}\mathrm{d}t$	**M1**	
	$-\frac{1}{x} = -\frac{1}{8}\left(\frac{2}{3}t^{3/2}\right) + C = -\frac{1}{12}t^{3/2} + C$	**M1**	Correct integration.
	Use condition $x = 2, t = 1 \implies -0.5 = -1/12 + C \implies C = -5/12$	**M1**	Finds constant of integration.
	$-\frac{1}{x} = -\frac{1}{12}t^{3/2} - \frac{5}{12} \implies \frac{1}{x} = \frac{t^{3/2}+5}{12}$	**M1**	Rearrangement steps.
	$x = \frac{12}{t^{3/2}+5}$	**A1**	Exact correct form.
		[5]	
9(c)	As $t \to \infty$, $x \to 0$. (The substance completely dissolves).	**B1**	
		[1]	
10(a)	$u = x^2 \implies \mathrm{d}u = 2x\mathrm{d}x$. Integral becomes $\frac{1}{2}\int \mathrm{e}^u \mathrm{d}u$	**M1**	Correct application of substitution.
	$= \frac{1}{2}\mathrm{e}^{x^2} + C$	**A1**	
		[2]	
10(b)	Splits $x^3\mathrm{e}^{x^2}$ into $x^2 \cdot (x\mathrm{e}^{x^2})$. Let $u = x^2 \implies \mathrm{d}u = 2x\mathrm{d}x$, and $\mathrm{d}v = x\mathrm{e}^{x^2}\mathrm{d}x \implies v = \frac{1}{2}\mathrm{e}^{x^2}$.	**M1**	Chooses correct parts.
	$uv - \int v\mathrm{d}u = \left[\frac{1}{2}x^2\mathrm{e}^{x^2}\right]_0^1 - \int_0^1 x\mathrm{e}^{x^2}\mathrm{d}x$	**M1**	Applies integration by parts formula.

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	Evaluates first bracket: $\frac{1}{2}\mathrm{e} - 0$.	M1	
	Evaluates second integral: $\left[\frac{1}{2}\mathrm{e}^{x^2}\right]_0^1 = \frac{1}{2}\mathrm{e} - \frac{1}{2}$		Integrates the $v\mathrm{d}u$ term using part (a).
	Total $= \frac{1}{2}\mathrm{e} - \left(\frac{1}{2}\mathrm{e} - \frac{1}{2}\right) = \frac{1}{2}$.	A1	Exact correct answer.
		[4]	
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\cos 2x - 2x^2 \sin 2x$	M1	Product rule correctly applied.
	Equates to 0: $2x(\cos 2x - x\sin 2x) = 0$. Since $x \neq 0$ in interval, $\cos 2x = x\sin 2x$	M1	Isolates the required terms.
	$\tan 2x = \frac{1}{x} \implies 2x = \tan^{-1}\left(\frac{1}{x}\right) \implies x = \frac{1}{2}\tan^{-1}\left(\frac{1}{x}\right)$	A1	AG. Shown clearly.
		[3]	
11(b)	Let $f(x) = x - 0.5\tan^{-1}(1/x)$. $f(0.5) = 0.5 - 0.5\tan^{-1}(2) = -0.0535$	M1	Must be in radians.
	$f(0.6) = 0.6 - 0.5\tan^{-1}(1/0.6) = +0.0848$. Change of sign indicates root lies between 0.5 and 0.6.	A1	Statement required.
		[2]	
11(c)	$x_1 = 0.5$, $x_2 = 0.55357$, $x_3 = 0.53176$	M1	First few iterations correct.
	$x_4 = 0.54088, x_5 = 0.53704, x_6 = 0.53866, x_7 = 0.53798,$ $x_8 = 0.53826, x_9 = 0.53815, \ldots$	M1	Shows sufficient iterations to 5 d.p.
	Root converges to 0.538 (correct to 3 d.p.)	A1	

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		[3]	
			Total: 75 Marks