MARK SCHEME

Maximum Mark: **75**

Q	Answer / Working	Marks	Guidance
1(a)	Term in x^2 is $\binom{6}{2}(2)^4(-kx)^2$	**B1**	Can be implied by correct unsimplified expression.
	Coefficient $= 15 \times 16 \times k^2 = 240k^2$	**B1**	
		[2]	
1(b)	Expansion of $(2-kx)^6$: $64 - 192kx + 240k^2x^2 + \dots$	**M1**	Identifies necessary terms.
	Term in x^2 in product: $1(240k^2) + 3(-192k) - 1(64) = 0$	**M1**	Sums relevant combinations to 0.
	$240k^2 - 576k - 64 = 0 \implies 15k^2 - 36k - 4 = 0$ $k = \frac{36 \pm \sqrt{1296 - 4(15)(-4)}}{30} \implies k = \frac{18 \pm 8\sqrt{6}}{15}$	**A1**	Accept $k \approx 2.51$ or $k \approx -0.106$.
		[3]	
2	Equate line and curve: $mx - 2 = 2x^2 - 3x + 6$	**M1**	
	$2x^2 - (3+m)x + 8 = 0$	**M1**	Forms correct quadratic.
	No real roots $\implies b^2 - 4ac < 0$: $(-(3+m))^2 - 4(2)(8) < 0 \implies (3+m)^2 < 64$	**M1**	Uses discriminant correctly.

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	$-8 < 3+m < 8 \implies -11 < m < 5$	A1	Strict inequalities required.
		[4]	
3(a)	$\text{LHS} = \frac{\cos\theta(1+\sin\theta)-\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$	M1	Common denominator.
	$= \frac{\cos\theta+\cos\theta\sin\theta-\cos\theta+\cos\theta\sin\theta}{1-\sin^2\theta}$	M1	Simplifies numerator or denominator.
	$= \frac{2\sin\theta\cos\theta}{\cos^2\theta} = \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$	A1	AG. Legitimate steps shown.
		[3]	
3(b)	$2\tan\theta = 3\cos\theta \implies \frac{2\sin\theta}{\cos\theta} = 3\cos\theta \implies 2\sin\theta = 3\cos^2\theta$	M1	
	$2\sin\theta = 3(1-\sin^2\theta) \implies 3\sin^2\theta + 2\sin\theta - 3 = 0$	M1	Uses identity correctly.
	$\sin\theta = \frac{-2\pm\sqrt{4-4(3)(-3)}}{6} = \frac{-1\pm\sqrt{10}}{3}$	A1	Rejects the negative root (≈ -1.387).
	$\sin\theta = 0.7207\cdots \implies \theta = 0.805, 2.34$	A1	Both values correct in radians.
		[4]	
4(a)	$u_1 = a, u_3 = a+2d = ar, u_4 = a+3d = ar^2$	M1	Relates AP terms to GP terms.

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	$2d = ar - a$ and $3d = ar^2 - a$ $\implies 3(ar-a) = 2(ar^2-a)$	M1	Eliminates d.
	$3ar - 3a = 2ar^2 - 2a \implies 2ar^2 - 3ar + a = 0$ Divide by a (since $a \neq 0$): $2r^2 - 3r + 1 = 0$	A1	AG. Shown clearly.
		[3]	
4(b)	Condition for convergence: $\lvert r \rvert < 1$	B1	
	$(2r-1)(r-1) = 0 \implies r = 0.5$ or $r = 1$. Since $r = 1$ doesn't satisfy $\lvert r \rvert < 1$, valid $r = 0.5$.	B1	Independent marks.
		[2]	
4(c)	$S_\infty = \frac{a}{1-r} = \frac{a}{1-0.5}$	M1	Uses correct formula with their r.
	$S_\infty = 2a$	A1	
		[2]	
5(a)	Translation right by $q \implies x_{new} = x_{old} + q \implies 1 = -2 + q \implies q = 3$	M1	Setup for translation.
	Stretch vertical by $p \implies y_{new} = p \cdot y_{old} \implies 15 = p(5) \implies p = 3$	A1	Both constants correct.
	New minimum $x = 4 + 3 = 7$, $y = 3(-1) = -3 \implies (7, -3)$	B1	Coordinates of new min.
		[3]	
5(b)	1: Reflection in the y-axis (or $f(-x)$)	B1	Reverses x-coordinates so $x_{max} = 2 > x_{min} = -4$.

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	2: Reflection in the x-axis (or $-f(x)$)	**B1**	Flips max/min so $x_{max} = 4 > x_{min} = -2$.
		[2]	

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6(a)	Length of Arc AB (center O) $= 12 \times \frac{\pi}{3} = 4\pi$	**M1**	
	Length of Arc OB (center A): Triangle OAB is equilateral so angle $OAB = \frac{\pi}{3}$. Arc $= 12 \times \frac{\pi}{3} = 4\pi$	**M1**	Recognises equilateral properties.
	Perimeter $= OA + \text{Arc } AB + \text{Arc } OB = 12 + 4\pi + 4\pi = 12 + 8\pi$	**A1**	AG. Shown clearly.
		[3]	
6(b)	Area of Sector OAB (center O) $= \frac{1}{2}(12)^2(\frac{\pi}{3}) = 24\pi$	**M1**	
	Area of $\Delta OAB = \frac{1}{2}(12)^2 \sin(\frac{\pi}{3}) = 36\sqrt{3}$	**M1**	
	Area of segment (chord OB	**arc OB center A)** $=$ **Sector $AOB -$** $\Delta OAB = 24\pi - 36\sqrt{3}$	M1
Proper strategy for area of overlap.			
	Shaded Area $=$ Sector $OAB -$ Segment $= 24\pi - (24\pi - 36\sqrt{3}) = 36\sqrt{3} \quad (c = 0, d = 36)$	**A1**	Exact form correctly derived. Note $c = 0$ is correct.

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		[4]	
7(a)	$f(x) = (x-2)^2 + 3$. Minimum at $x = 2$, so smallest $k = 2$.	**B1**	
		[1]	
7(b)	Domain of gf is the domain of f: $x \geqslant 2$	**B1**	
	Range of f: $f(x) \geqslant 3$	**B1**	
	Range of gf: $g(f(x)) = 2(f(x)) + 1 \geqslant 2(3) + 1 \implies gf(x) \geqslant 7$	**B1**	
		[3]	
7(c)	Let $y = (x-2)^2 + 3 \implies (x-2)^2 = y - 3$	**M1**	Rearranges to make x subject.
	$x = 2 + \sqrt{y-3}$ (taking positive root since $x \geqslant 2$)	**M1**	Must choose positive root.
	$f^{-1}(x) = 2 + \sqrt{x-3}$, Domain: $x \geqslant 3$	**A1**	Both expression and domain.
		[3]	
8(a)	Stat point at $x = 4 \implies \frac{k}{\sqrt{4}} - 2 = 0 \implies \frac{k}{2} = 2 \implies k = 4$	**M1**	Equates derivative to 0.
	$k = 4$	**A1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2x^{-3/2}$. At $x = 4$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{4} < 0$, hence Maximum.	**A1**	Shows second derivative is negative.
		[3]	

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8(b)	$y = \int \left(4x^{-1/2} - 2\right) \mathrm{d}x = 8x^{1/2} - 2x + C$	M1	Integration attempt (powers +1).
	Passes through $(9,5) \implies 5 = 8(3) - 18 + C \implies C = -1$	M1	Substitutes coordinates.
	$y = 8\sqrt{x} - 2x - 1$	A1	
		[3]	
8(c)	Uses chain rule $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
	At $x = 9$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{3} - 2 = -\frac{2}{3}$. $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{3} \times 0.6 = -0.4$ units/sec	A1	
		[2]	

Q	Answer / Working	Marks	Guidance
9(a)	Completing square: $(x-5)^2 - 25 + (y+2)^2 - 4 + 4 = 0$	B1	Or uses $x = -g, y = -f$.
	Centre $(5, -2)$, Radius $= \sqrt{25} = 5$	B1	Both correct.
		[2]	
9(b)	Perpendicular distance from $(5, -2)$ to $3x - 4y + k = 0$ is 5: $\frac{\lvert 3(5)-4(-2)+k \rvert}{\sqrt{3^2+(-4)^2}} = 5$	M1	Setup distance formula or substitutes line into circle eq.
	$\frac{\lvert 15+8+k \rvert}{5} = 5 \implies \lvert 23+k \rvert = 25$	M1	Correctly evaluates distance expression.
	$23 + k = 25 \implies k = 2$ $23 + k = -25 \implies k = -48$	A1	Both values found.
		[3]	
9(c)	Line is $3x - 4y + 2 = 0$. Perpendicular line through $(5, -2)$ has gradient $-\frac{4}{3} \implies 4x + 3y = 14$	M1	Valid geometric or algebraic substitution method.
	Solves simultaneously: $x = 2, y = 2 \implies P(2, 2)$	A1	
		[2]	
9(d)	P is the midpoint of C_1 and C_2 centres. Let C_2 centre be (x_2, y_2). $\frac{5+x_2}{2} = 2 \implies x_2 = -1$. $\frac{-2+y_2}{2} = 2 \implies y_2 = 6$.	M1	Midpoint logic to find new centre.
	Eq of C_2: $(x+1)^2 + (y-6)^2 = 25$	M1	Uses centre and radius 5.

Q	Answer / Working	Marks	Guidance
	$x^2 + y^2 + 2x - 12y + 12 = 0$	A1	Correct expanded form.
		[3]	
10(a)	Area $= \int_0^1 x^3\,dx +$ Area of Triangle	M1	Strategy splitting region or integrating line minus curve.
	Area $= \left[\frac{x^4}{4}\right]_0^1 + \frac{1}{2}(1)(1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$	A1	
		[2]	
10(b)	$V = \pi\int_0^1 (x^3)^2\,dx + \pi\int_1^2 (2-x)^2\,dx$	M1	Setup two volume integrals (or one integral + cone volume).
	$V_1 = \pi\left[\frac{x^7}{7}\right]_0^1 = \frac{\pi}{7}$	A1	Curve portion evaluated correctly.
	$\int (2-x)^2 dx = \left[-\frac{(2-x)^3}{3}\right]_1^2$	M1	Proper integration technique for line.
	$V_2 = \pi\left(0 - \left(-\frac{1}{3}\right)\right) = \frac{\pi}{3}$	M1	(Or volume of cone $= \frac{1}{3}\pi r^2 h = \frac{\pi}{3}$).
	Total Volume $= \frac{\pi}{7} + \frac{\pi}{3} = \frac{10\pi}{21}$	A1	Exact answer.
		[5]	
11(a)	$\frac{dy}{dx} = 4x - 5$	M1	Differentiation attempt.

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	At $x = 2$, gradient $m = 4(2) - 5 = 3$.	M1	
	Tangent: $y - 2 = 3(x - 2) \implies y = 3x - 4$	A1	
		[3]	
11(b)	Translated curve: $y = 2(x - p)^2 - 5(x - p) + 4$	M1	
	Gradient of tangent is $3 \implies 4(x - p) - 5 = 3 \implies x - p = 2 \implies x = p + 2$.	M1	Discriminant method: equating $y_{curve} = 3x - 10$ is also valid.
	Tangent touches line $y = 3x - 10$. Sub $(p + 2, 2)$ into line: $2 = 3(p + 2) - 10 \implies 2 = 3p - 4 \implies p = 2$	A1	
		[3]	
11(c)	Substitute $p = 2$ into $x = p + 2 \implies x = 4$.	M1	
	y-coordinate is $3(4) - 10 = 2 \implies (4, 2)$.	A1	
		[2]	

Total: 75 Marks