CAMBRIDGE INTERNATIONAL AS & A LEVEL

FURTHER MATHEMATICS 9231/11

Paper 1 Further Pure Mathematics 1 May/June 2026

MARK SCHEME

Mark Scheme Notes:

- **M1**: Method mark, awarded for a valid method applied to the problem.
- **A1**: Accuracy mark, awarded for a correct answer or intermediate step following a correct method.
- **B1**: Independent mark, awarded for a correct result or statement without needing a method.
- **AG**: Answer Given.

Q	Answer	Marks	Guidance
1(a)	States $\sum \alpha = -a = 0 \Rightarrow a = 0$	B1	
	States $\alpha\beta\gamma\delta = d/1 = 1 \Rightarrow d = 1$	B1	
	Uses $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta \Rightarrow 2 = 0 - 2b \Rightarrow b = -1$	B1	Shows clear working to **AG**.
		[3]	
1(b)	States $\sum \frac{1}{\alpha} = \frac{\sum \alpha\beta\gamma}{\alpha\beta\gamma\delta} = \frac{-c}{1} = -c$	B1	Obtains given result **AG**.
	Divides by x and sums: $\sum \alpha^3 + a\sum \alpha^2 + b\sum \alpha + 4c + d\sum \frac{1}{\alpha} = 0$	M1	Or finds roots of $x^4 + cx^3 + bx^2 + ax + d = 0$.
	Substitutes known values: $3 + 0 + 0 + 4c - c = 0 \Rightarrow c = -1$	A1	Correct value of c.
		[3]	
1(c)	Uses $\sum \frac{1}{\alpha^2} = (\sum \frac{1}{\alpha})^2 - 2\sum \frac{1}{\alpha\beta}$ OR divides original eq by x^2 and sums	M1	Valid method for sum of inverse squares.
	Finds $\sum \frac{1}{\alpha\beta} = \frac{b}{d} = -1$	A1	Intermediate calculation.
	$\sum \frac{1}{\alpha^2} = (-(-1))^2 - 2(-1) = 1 + 2 = 3$	A1	Correct final exact value.
		[3]	

Q	Answer	Marks	Guidance
2(a)	$\begin{pmatrix} a & -2 \\ 1 & b \end{pmatrix}\begin{pmatrix} x \\ x \end{pmatrix} = \begin{pmatrix} ax-2x \\ x+bx \end{pmatrix}$	M1	Applies matrix to $(x,x)^T$.
	Equates coordinates since line is invariant $y=x$: $(a-2)x=(1+b)x \Rightarrow a-2=1+b \Rightarrow a=b+3$	A1	Concludes correctly **AG**.
		[2]	
2(b)	Area scale factor is $\lvert\det \mathbf{M}\rvert = 6 \Rightarrow \lvert ab+2\rvert = 6$	M1	Sets up determinant equation.
	Substitutes $b=a-3$: $a(a-3)+2=\pm 6$ $\Rightarrow a^2-3a-4=0$ (rejects $+8=0$ as no real roots)	M1	Forms quadratic in a.
	Solves to get $a=4$ (since $a>0$), and $b=1$	A1	Correct pair of values.
		[3]	
2(c)	Matrix is $\mathbf{M} = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$. Let invariant line be $y=mx$. $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \lambda\begin{pmatrix} x \\ mx \end{pmatrix}$ OR $\frac{x+mx}{4x-2mx} = m$	M1	Sets up equation for invariant line through origin.
	$m(4-2m) = 1+m \Rightarrow 2m^2-3m+1=0$	M1	Forms quadratic in m.
	Solves to get $m=1/2$. Equation is $y=\frac{1}{2}x$	A1	Must state the equation, not just m.
		[3]	
3(a)	$f(r)-f(r+1) = \frac{r}{(r+1)!} - \frac{r+1}{(r+2)!} = \frac{r(r+2)-(r+1)}{(r+2)!}$	M1	Common denominator $(r+2)!$.
	$= \frac{r^2+2r-r-1}{(r+2)!} = \frac{r^2+r-1}{(r+2)!}$	A1	Simplifies correctly **AG**.
		[2]	
3(b)	$\sum_{r=1}^{n} \frac{r^2+r-1}{(r+2)!} = \sum_{r=1}^{n} (f(r)-f(r+1))$	M1	Recognises Method of Differences.
	$= (f(1)-f(2)) + (f(2)-f(3)) + \cdots + (f(n)-f(n+1))$	M1	Shows partial terms to identify cancellation.
	$= f(1)-f(n+1)$	A1	Intermediate structure.
	$= \frac{1}{2} - \frac{n+1}{(n+2)!}$	A1	Correct final expression in n.
		[4]	

Q	Answer	Marks	Guidance
3(c)	States $\lim_{n\to\infty}\frac{n+1}{(n+2)!}=0$	M1	Considers limit.
	Sum to infinity is $\frac{1}{2}$	A1	Exact value.
		[2]	
4(a)	Base case: When $n=1$, $u_1=5>4$. True for $n=1$.	B1	
	Assume true for $n=k$: $u_k>4$. Consider $u_{k+1}=5-\frac{4}{u_k}$.	M1	Inductive hypothesis and substitution.
	Since $u_k>4\Rightarrow 0<\frac{4}{u_k}<1\Rightarrow u_{k+1}=5-\frac{4}{u_k}>5-1=4$.	A1	Shows $u_{k+1}>4$.
	Conclusion: True for $n=1$, if true for $n=k$ then true for $n=k+1$. Hence true for all $n\geqslant 1$ by mathematical induction.	A1	Valid concluding statement.
		[4]	
4(b)	$u_{n+1}-u_n=\frac{5u_n-4-u_n^2}{u_n}$	M1	Common denominator.
	$=\frac{-(u_n^2-5u_n+4)}{u_n}=\frac{-(u_n-1)(u_n-4)}{u_n}$	A1	Correct factorisation **AG**.
	Since $u_n>4$, $u_n-1>0$ and $u_n-4>0$. Thus $u_{n+1}-u_n<0$, strictly decreasing.	A1	Proper reasoning using part (a).
		[3]	
5(a)	Direction vector $\mathbf{d}_1=\mathbf{n}_1\times\mathbf{n}_2=\begin{pmatrix}1\\1\\-1\end{pmatrix}\times\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Cross product of normal vectors.
	$\mathbf{d}_1=2\mathbf{i}-5\mathbf{j}-3\mathbf{k}$	A1	Correct direction vector.
	Finds a point on l_1: let $z=0\Rightarrow x+y=2, 2x-y=1$	M1	Solving system for arbitrary coord.
	Point is $(1,1,0)$	A1	Correct point.
	Eq: $\mathbf{r}=\mathbf{i}+\mathbf{j}+\lambda(2\mathbf{i}-5\mathbf{j}-3\mathbf{k})$	A1	Any valid vector equation.
		[5]	
5(b)	$\cos\theta=\frac{\lvert\mathbf{d}_1\cdot\mathbf{d}_2\rvert}{\lvert\mathbf{d}_1\rvert\lvert\mathbf{d}_2\rvert}=\frac{\lvert(2)(1)+(-5)(1)+(-3)(-2)\rvert}{\sqrt{2^2+(-5)^2+(-3)^2}\sqrt{1^2+1^2+(-2)^2}}$	M1	Correct dot product formula.
	$\cos\theta=\frac{3}{\sqrt{38}\sqrt{6}}=\frac{3}{\sqrt{228}}$	M1	Correct values.
	$\theta=78.5^\circ$ or 1.37 radians	A1	
		[3]	

Q	Answer	Marks	Guidance
5(c)	Finds vector $\vec{AB}$ between lines: $\mathbf{a}_1 - \mathbf{a}_2 = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$	**B1**	Using $(1,1,0)$ and $(2,0,1)$.
	Common perpendicular $\mathbf{n} = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$	**M1**	
	$\mathbf{n} = \begin{pmatrix} 13 \\ 1 \\ 7 \end{pmatrix}$	**A1**	
	Distance $d = \frac{\lvert \vec{AB} \cdot \mathbf{n} \rvert}{\lvert \mathbf{n} \rvert}$	**M1**	Correct distance formula.
	Numerator: $\lvert -1(13) + 1(1) - 1(7) \rvert = 19$. Denominator: $\sqrt{219}$	**A1**	Calculates components correctly.
	Distance $= \frac{19}{\sqrt{219}}$ (or $\frac{19\sqrt{219}}{219}$)	**A1**	Exact exact shortest distance.
		[6]	
6(a)	Correct sketch showing closed single loop in upper half-plane.	**B1**	Starts/ends at origin, peaks roughly right of center.
	Pole coordinates: $(0,0)$ or just $r = 0$	**B1**	
		[2]	
6(b)	Area $= \frac{1}{2}\int_0^{\pi} r^2\, d\theta = \frac{1}{2}\int_0^{\pi} \theta^2 \sin\theta\, d\theta$	**M1**	Correct setup.
	Integrates by parts once: $-\theta^2\cos\theta + \int 2\theta\cos\theta\, d\theta$	**M1A1**	Method and correct first step.
	Integrates by parts twice: $\cdots + 2\theta\sin\theta - \int 2\sin\theta\, d\theta$	**M1**	
	Final integral: $[-\theta^2\cos\theta + 2\theta\sin\theta + 2\cos\theta]_0^{\pi}$	**A1**	Fully correct anti-derivative.
	Evaluates limits to get $\frac{1}{2}(\pi^2 - 4) = \frac{\pi^2}{2} - 2$	**A1**	Correct exact final area.
		[6]	
6(c)	$y = r\sin\theta = \theta(\sin\theta)^{3/2}$	**M1**	Uses $y = r\sin\theta$.

Q	Answer	Marks	Guidance
	$\frac{dy}{d\theta} = (\sin\theta)^{3/2} + \theta \cdot \frac{3}{2}(\sin\theta)^{1/2}\cos\theta$ Sets to 0. Divides by $(\sin\theta)^{1/2}$: $\sin\theta + \frac{3}{2}\theta\cos\theta = 0$ $\Rightarrow \tan\theta = -\frac{3}{2}\theta$	**M1A1** **A1** **[4]**	Product rule and chain rule applied correctly. Obtains given result **AG**.
6(d)	Let $f(\theta) = \tan\theta + 1.5\theta$. $f(1.9) = -0.077\ldots$ and $f(2.0) = +0.815\ldots$ Sign change (and continuity in interval) indicates a root between 1.9 and 2.0.	**M1** **A1** **[2]**	Evaluates function at endpoints. Mentions sign change.
7(a)	Vertical asymptote $x = 1 \Rightarrow c = -1$ $f(x) = \frac{x^2+ax+b}{x-1} = x + (a+1) + \frac{b+a+1}{x-1}$ Equates quotient to oblique asymptote: $x + (a+1) = x - 2 \Rightarrow a = -3$	**B1** **M1** **A1** **[3]**	 Polynomial division. Correct value of a.
7(b)	Substitutes $(2,-1)$ into $f(x) = \frac{x^2-3x+b}{x-1}$: $-1 = \frac{4-6+b}{1} \Rightarrow -1 = -2 + b \Rightarrow b = 1$	**B1** **[1]**	Shows steps clearly **AG**.
7(c)	$f'(x) = \frac{(2x-3)(x-1)-(x^2-3x+1)}{(x-1)^2}$ $f'(x) = \frac{x^2-2x+2}{(x-1)^2}$ $x^2 - 2x + 2 = (x-1)^2 + 1 > 0$ for all x. Since $f'(x) \neq 0$, no stationary points.	**M1** **A1** **A1** **[3]**	Uses quotient rule. Simplifies numerator. Complete algebraic justification.
7(d)	Draws asymptotes $x = 1$ and $y = x - 2$ clearly. Two correct branches in opposite quadrants formed by asymptotes, always increasing. Identifies correct axes intersections: $(0,-1)$ and roughly $(0.38,0), (2.62,0)$.	**B1** **B1** **B1** **[3]**	 Exact roots $\frac{3\pm\sqrt{5}}{2}$ not required on graph if coordinates indicated.
7(e)	Splits into $\frac{x^2-3x+1}{x-1} > 2$ and $\frac{x^2-3x+1}{x-1} < -2$	**M1**	Correct approach for absolute value.

Q	Answer	Marks	Guidance
	Case 1: $\frac{x^2-5x+3}{x-1} > 0$. Roots of num are $\frac{5\pm\sqrt{13}}{2}$	**A1**	
	Case 2: $\frac{x^2-x-1}{x-1} < 0$. Roots of num are $\frac{1\pm\sqrt{5}}{2}$	**A1**	
	Determines correct intervals using sign analysis or graph.	**M1**	
	Exact set: $x < \frac{1-\sqrt{5}}{2}$ **or** $\frac{5-\sqrt{13}}{2} < x < 1$ **or** $1 < x < \frac{1+\sqrt{5}}{2}$ **or** $x > \frac{5+\sqrt{13}}{2}$	**A1**	Must exclude $x = 1$ and have strict inequalities.
		[5]	