MARK SCHEME

Maximum Mark: **50**

Q	Answer / Working	Marks	Guidance
1	Initial Elastic Potential Energy (EPE) $= \frac{3mg(a)^2}{2a} = \frac{3}{2}mga$	**B1**	
	Frictional force $F = \mu R = \frac{3}{4}mg$	**B1**	
	Work done against friction $= \frac{3}{4}mgd$ (where d is total distance)	**M1**	
	Applies conservation of energy (Final KE and EPE = 0): $\frac{3}{2}mga - \frac{3}{4}mgd = 0$	**M1**	Accept two-part method (Taut phase: $d = a$, Slack phase: $d = a$).
	$d = 2a$	**A1**	Since $d = 2a$, it stops exactly at O (does not overshoot).
		[5]	
2(a)	Uses $mv\frac{dv}{dx} = -mcv^3 \implies \frac{dv}{dx} = -cv^2$	**M1**	Correct setup of DE.
	$\int v^{-2}\mathrm{d}v = \int -c\,\mathrm{d}x \implies -\frac{1}{v} = -cx + A$	**M1**	Integrates.
	Uses $v = U$ when $x = 0 \implies A = -1/U$, leading to $v = \frac{U}{1+cUx}$	**A1**	
		[3]	
2(b)	Uses $\frac{dv}{dt} = -cv^3$	**M1**	DE with respect to t.
	$\int v^{-3}\mathrm{d}v = \int -c\,\mathrm{d}t \implies -\frac{1}{2v^2} = -ct + B$	**M1**	Integrates.

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	$v = U$ at $t = 0 \implies B = -\frac{1}{2U^2}$. Substitute $v = U/2$ to find T: $\frac{1}{2(U/2)^2} = cT + \frac{1}{2U^2} \implies \frac{4}{2U^2} - \frac{1}{2U^2} = cT \implies T = \frac{3}{2cU^2}$	A1	
		[3]	
3(a)	Velocity of A perpendicular to line of centres is $u\sin\alpha$	B1	Component unchanged.
	Conservation of momentum: $u\cos\alpha = v_A + kv_B$ Newton's law of restitution: $v_B - v_A = eu\cos\alpha$	M1	Both equations correct.
	Solves for v_A along line of centres: $v_A = \frac{1-ke}{k+1}u\cos\alpha$	M1	
	Uses perpendicularity condition (e.g. Dot Product = 0): $\mathbf{u}\cdot\mathbf{v}_{\text{final}} = (u\cos\alpha)(v_A) + (u\sin\alpha)(u\sin\alpha) = 0$ $\implies v_A = -u\frac{\sin^2\alpha}{\cos\alpha} = -u\sin\alpha\tan\alpha$ Equating: $\frac{1-ke}{k+1}u\cos\alpha = -u\frac{\sin^2\alpha}{\cos\alpha} \implies \tan^2\alpha = \frac{ke-1}{k+1}$	A1	Dot product is the cleanest method, but accept valid trigonometry.
		[4]	
3(b)	$\frac{1}{2}m(v_A^2 + u^2\sin^2\alpha) = \frac{1}{2}kmv_B^2$ Sub v_A and v_B: $u^2\tan^2\alpha = k\left(\frac{1+e}{k+1}u\cos\alpha\right)^2$	M1	Equates KEs. Must include perpendicular component for A.
	$\frac{ke-1}{k+1}\cdot\frac{k(1+e)}{k+1} = k\frac{(1+e)^2}{(k+1)^2} \implies ke-1 = 1+e \implies k = 5$	A1	Uses exact $e = 0.5$.
	$\tan^2\alpha = \frac{0.5(5)-1}{5+1} = 0.25 \implies \alpha = \tan^{-1}(0.5)$	A1	Or $\alpha = 26.6°$ (accept exact or 1 d.p).
		[3]	
4(a)	$t_{P(\text{max height})} = \frac{u\sin\alpha}{g} \implies T_Q = T_P - t_P = \frac{u\sin\alpha}{g}$	B1	

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	$T_Q = \frac{2v\sin\beta}{g} \implies 2v\sin\beta = u\sin\alpha$	M1	Relates vertical motions.
	Equal horizontal ranges: $u\cos\alpha \times \frac{2u\sin\alpha}{g} = v\cos\beta \times \frac{u\sin\alpha}{g} \implies 2u\cos\alpha = v\cos\beta$	M1	Relates horizontal motions.
	Dividing the equations: $\frac{2v\sin\beta}{v\cos\beta} = \frac{u\sin\alpha}{2u\cos\alpha} \implies \tan\beta = \frac{1}{4}\tan\alpha$	A1	Shown.
		[4]	
4(b)	$v^2\sin^2\beta + v^2\cos^2\beta = \frac{1}{4}u^2\sin^2\alpha + 4u^2\cos^2\alpha$	M1	Squaring and adding.
	$\frac{v^2}{u^2} = \frac{1}{4}\sin^2\alpha + 4\cos^2\alpha$	A1	
		[2]	
4(c)	$v^2 = u^2 \implies \frac{1}{4}\sin^2\alpha + 4\cos^2\alpha = 1$	M1	Sets ratio to 1.
	Divide by $\cos^2\alpha$: $\frac{1}{4}\tan^2\alpha + 4 = \sec^2\alpha = 1 + \tan^2\alpha \implies \tan\alpha = 2$	A1	
		[2]	
5(a)	Velocity of P at B: $u = \sqrt{2ga}$	B1	
	Collision: $v_Q = \frac{1+e}{2}u = \frac{1+\sqrt{2}/2}{2}\sqrt{2ga} = \frac{\sqrt{2}+1}{2}\sqrt{ga}$	M1	Conservation of mom & restitution.
	Height: $h = \frac{v_Q^2}{2g} = \frac{3+2\sqrt{2}}{8}a$	A1	
	$3+2\sqrt{2} \approx 5.828 \implies h \approx 0.728a < a$, so does not reach L.	A1	Must explicitly deduce $< a$.

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		[4]	
5(b)	Energy for P: $u = \sqrt{2ga(1+\cos\theta)}$	B1	
	Energy for Q to exactly reach L: $v_Q = \sqrt{2ga}$	B1	
	Collision gives $v_Q = \frac{1+e}{2}u = \frac{3}{4}u$	M1	Using $e = 1/2$.
	$\sqrt{2ga} = \frac{3}{4}\sqrt{2ga(1+\cos\theta)} \implies 1 = \frac{9}{16}(1+\cos\theta) \implies \cos\theta = \frac{7}{9}$	A1	Exact value only.
		[4]	

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6(a)	Distance $AC = \frac{a}{\sin\theta}$	B1	
	Moments about A: $W \cdot a\cos\theta = R_C \cdot \frac{a}{\sin\theta}$	M1	
	$R_C = W\sin\theta\cos\theta$	A1	Shown.
		[3]	
6(b)	Resolve horizontally: $F = R_C\sin\theta = W\sin^2\theta\cos\theta$	B1	Friction acts to the right.
	Resolve vertically: $R_A = W - R_C\cos\theta = W(1-\sin\theta\cos^2\theta)$	B1	
	$\mu = \frac{F}{R_A} = \frac{\sin^2\theta\cos\theta}{1-\sin\theta\cos^2\theta}$	A1	
		[3]	
6(c)	Moments about A: $R_C \cdot a\sqrt{2} = W \cdot a\frac{\sqrt{2}}{2} + kW \cdot 2a\frac{\sqrt{2}}{2} \implies R_C = W(k+0.5)$	B1	Using $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$.
	$\mu = 1 \implies F = R_A \implies R_C\frac{1}{\sqrt{2}} = W(1+k) - R_C\frac{1}{\sqrt{2}}$	M1	Resolves forces horiz & vert.
	$R_C\sqrt{2} = W(1+k) \implies W(k+0.5)\sqrt{2} = W(1+k)$ $k(\sqrt{2}-1) = 1-0.5\sqrt{2} \implies k = \frac{\sqrt{2}}{2}$	A1	Accurate exact simplification.
		[3]	
7(a)	Volume of cone = $\frac{1}{3}\pi r^3 k$, Hemisphere = $\frac{2}{3}\pi r^3$, Total = $\frac{1}{3}\pi r^3(k+2)$	B1	
	Moments about flat face ($\bar{y}$ towards vertex): $\frac{1}{3}\pi r^3(k+2)\bar{y} = \frac{1}{3}\pi r^3 k\left(\frac{1}{4}kr\right) + \frac{2}{3}\pi r^3\left(-\frac{3}{8}r\right)$	M1	Both COM displacements correct ($kr/4$ and $-3r/8$).

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	$(k+2)\bar{y} = \frac{1}{4}k^2 r - \frac{3}{4}r \implies \bar{y} = \frac{r(k^2-3)}{4(k+2)}$	**A1**	Shown.
		[3]	
7(b)	Toppling condition: vertical passes through base rim $\implies \tan\theta = \frac{r}{\bar{y}}$	**M1**	
	$\frac{10}{3} = \frac{4(k+2)}{k^2-3} \implies 10k^2 - 30 = 12k + 24 \implies 5k^2 - 6k - 27 = 0$ $(5k+9)(k-3) = 0 \implies k = 3$ (since $k > 0$)	**A1**	
		[2]	
7(c)	For equilibrium on any point, COM must be exactly at the centre O ($\bar{y} = 0$)	**M1**	
	$k^2 - 3 = 0 \implies k = \sqrt{3}$	**A1**	
		[2]	

Total: 50 Marks