新灯塔国际教育

AL 数学逆袭宝典

Answers　答案手册

Edexcel IAL P1 / P2 • Basis

目录

1 Number & Sequences

Basic Drill

1 $\frac{5}{6}$; 2 $\frac{3}{8}$; 3 $\frac{2}{3}$; 4 $\frac{3}{16}$; 5 $\frac{2}{3}$; 6 4; 7 -3; 8 24; 9 -6; 10 $-\frac{1}{10}$; 11 $\frac{2}{7}$; 12 $\frac{4}{9}$; 13 $-\frac{27}{64}$; 14 9; 15 $\frac{1}{5}$; 16 $\frac{5}{8}$; 17 3; 18 3; 19 10; 20 3; 21 18; 22 47; 23 -8; 24 $d = 7$, $u_5 = 30$; 25 66; 26 185; 27 104; 28 $a = 5$, $d = 3$; 29 $d = 3$; 30 $d = 2$; 31 96; 32 162; 33 3; 34 155; 35 64; 36 40; 37 3; 38 $r = 3$, $a = 2$; 39 AP; 40 GP; 41 $\frac{5}{7}$; 42 $\frac{3}{2}$; 43 $3\frac{3}{4}$; 44 $\frac{17}{5}$; 45 21; 46 54; 47 210; 48 $u_1 = 2$, $u_{10} = 29$; 49 50; 50 $\frac{1}{4}$.

Practice

1 $-\dfrac{26}{3}$; 2 $a = 8$, $d = 3$, $u_{15} = 50$, $S_{15} = 435$; 3 $a = 5$, $u_6 = 160$; 4 8, 19, 20; middle mean $14 = \dfrac{2+26}{2}$; 5 $n = 11$; 6 $u_n = 4n - 1$, $a = 3$, $d = 4$; 7 $\dfrac{1}{5}$; 8 $r = 2$, $a = 6$, $S_4 = 90$; 9 $\dfrac{2}{5}$; 10 $d = 7$, $r = 2$; 11 $x = \dfrac{29}{8}$; 12 $\dfrac{380}{9}$ m; 13 $u_n = 6n - 4$; $u_5 + u_6 = 58 = S_6 - S_4$; 14 $a = 2$, $r = 3$; 15 $n = 22$; 16 8; 17 5, 7, 9; 18 $r = \dfrac{1}{3}$, $S_4 = \dfrac{160}{9}$; 19 shown; 20 $n = 12$; 21 185; 22 $-\dfrac{5}{8}$; 23 $n = 11$; 24 $a = \dfrac{25}{6}$, $r = 2$; 25 $d = 6$.

Advanced Practice

1 shown; 2 $a = 5$, $r = 2$; 3 $r = \dfrac{a+4d}{a+d}$; 4 $n = 8$, 49, …; 5 $r = 2$, $a = 2$ or $r = -2$, $a = -6$; 6 shown; 7 $S_n = 2 - 2^{1-n}$, limit 2; 8 2, 6, 18; 9 $\dfrac{3n+1}{n+1}$; 10 $a = 2$, $r = \dfrac{1}{2}$.

2 Indices

Basic Drill

1 729; 2 625; 3 64; 4 x^9; 5 a^5; 6 y^{12}; 7 16; 8 1; 9 1; 10 $\frac{1}{4}$; 11 $\frac{1}{9}$; 12 $\frac{1}{1000}$; 13 8; 14 x^{-4}; 15 $2x^{-1}$; 16 2^{-3}; 17 3^2; 18 7; 19 3; 20 4; 21 2; 22 9; 23 8; 24 $\frac{1}{3}$; 25 $2x$; 26 $5y^2$; 27 x^2; 28 x^2; 29 x^2; 30 1000; 31 $8a^6b^3$; 32 $3x^3$; 33 $4a^5$; 34 $12x^3$; 35 $\frac{1}{4}x^{-6}$; 36 $5x^{-2}$; 37 $7x^{-1}$; 38 $4x^{1/2}$; 39 $x^{-1/2}$; 40 a^5; 41 a; 42 4; 43 27; 44 $\frac{x}{y^2}$; 45 16; 46 1; 47 $\frac{1}{25}$; 48 10^{-2}; 49 x^4y^4; 50 $4x^2$.

Practice

1 $3x^{-2}y^4$; 2 $y = 2x^{-2} + x^{3/2} - 5x^{-1}$; $y \cdot x^2 = 2 + x^{7/2} - 5x$; 3 4; 4 $x - 4 + 4x^{-1}$; 5 $x = -4$; 6 $\dfrac{9}{4a^6b^4}$; 7 $5x^{-2/3}$; $x = 8$ gives $\dfrac{5}{4}$; 8 $3a^2 + \dfrac{a}{3}$; 9 x^2; 10 $x = 2$; 11 1; 12 $\dfrac{15}{4}$; 13 $x^{3/2} + 3x^{-1/2}$; $x = 4$ gives $\dfrac{19}{2}$; 14 shown; 15 $x^{5/2}y^{-3}$; 16 $n = 2$; 17 2^{3x}; 18 $\dfrac{1}{8}$; 19 $4x^4y^{-6}$; 20 4; 21 $a^{-1/2}$; 22 $x = 1$; 23 $\dfrac{2x^2}{y}$; 24 3; 25 6.

Advanced Practice

1 $x = 2$; 2 1 ($a \neq b$); 3 $x^2 - 2 = 3^{2t} + 3^{-2t}$; 4 $x = 1$ or $x = 2$; 5 $\dfrac{2x^2}{3y}$; 6 shown; 7 shown; 8 $x^{(a+b)\left(2b-(a-b)^2\right)}$; 9 $x^{3/2} + x^{-1/2}$; 10 $x = 1$ or $x = 2$.

3 Surds

Basic Drill

1 $2\sqrt{3}$; 2 $3\sqrt{2}$; 3 $4\sqrt{2}$; 4 $5\sqrt{3}$; 5 $2\sqrt{2}$; 6 $3\sqrt{3}$; 7 $3\sqrt{5}$; 8 $7\sqrt{2}$; 9 $10\sqrt{2}$; 10 $6\sqrt{2}$; 11 $7\sqrt{3}$; 12 $4\sqrt{2}$; 13 $3\sqrt{3}$; 14 $3\sqrt{2}$; 15 $7\sqrt{2}$; 16 $4\sqrt{2}$; 17 6; 18 6; 19 4; 20 6; 21 3; 22 5; 23 $\frac{\sqrt{2}}{2}$; 24 $\frac{\sqrt{5}}{5}$; 25 $\sqrt{3}$; 26 $\frac{\sqrt{10}}{2}$; 27 $\frac{\sqrt{2}}{2}$; 28 1; 29 -1; 30 3; 31 $3+2\sqrt{2}$; 32 $11-6\sqrt{2}$; 33 $5+2\sqrt{6}$; 34 $\frac{3}{4}$; 35 $\frac{2}{5}$; 36 3; 37 0.7; 38 $4\sqrt{7}$; 39 a $(a \geq 0)$; 40 $2x$ $(x \geq 0)$; 41 $3x^2$ $(x \geq 0)$; 42 $\frac{\sqrt{3}}{6}$; 43 $5\sqrt{7}$; 44 $16\sqrt{3}$; 45 7; 46 $\sqrt{3}$; 47 $3\sqrt{2}$; 48 No; 49 $4\sqrt{3}$; 50 $\sqrt{48}$.

Practice

1 $8\sqrt{3}$; 2 $\sqrt{7}+\sqrt{2}$; 3 $13-4\sqrt{3}$; $\dfrac{2\sqrt{3}+1}{11}$; 4 4; 5 1; 6 3; 7 $96\sqrt{2}$; 8 $x=1$; 9 shown; $\sqrt{10}-1$; 10 $\dfrac{5\sqrt{3}}{18}$; 11 $a=8$, $b=15$ (or swap); 12 $4x\sqrt{2}$; 13 $7+4\sqrt{3}$; 14 $\sqrt{50}-\sqrt{32}=\sqrt{2}<2\sqrt{2}$; 15 $\dfrac{4\sqrt{3}}{3}$; 16 3; 17 $\dfrac{7\sqrt{2}}{10}$; 18 $3\sqrt{5}+3$; 19 $x=\dfrac{7}{3}$; 20 $k=5$; $\sqrt{26}$; 21 $2\sqrt{3}-\sqrt{6}$; 22 $3\sqrt{3}$; 23 $a=5$; 24 $2+\sqrt{3}$; 25 area 4; perimeter $4\sqrt{5}$.

Advanced Practice

1 $\dfrac{a+b-2\sqrt{ab}}{a-b}$; 2 shown; 3 shown; 4 $m=6$, $n=5$ (or swap); 5 shown; 6 $x=4$; 7 $k=6$; 8 $-7+\sqrt{5}+2\sqrt{3}$; 9 $p=3$, $q=-2$; 10 $|a+b|$; equals $a+b$ when $a+b \geq 0$.

4 Expanding

Basic Drill

1 $4x+12$; 2 $10x-5$; 3 $-3x+12$; 4 x^2+5x; 5 $6x^2-14x$; 6 x^2+5x+6; 7 x^2+6x+5; 8 x^2+x-12; 9 $x^2-8x+12$; 10 x^2-49; 11 x^2+6x+9; 12 $x^2-10x+25$; 13 x^2+2x+1; 14 x^2-2x+1; 15 $2x^2+7x+3$; 16 $2x^2-x-10$; 17 $3x^2-10x-8$; 18 $6x^2+13x-5$; 19 $4x^2+12x+9$; 20 $9x^2-12x+4$; 21 $x^2+3xy+2y^2$; 22 a^2-b^2; 23 $4a^2-9b^2$; 24 x^3+2x^2+x; 25 $3x^2-12x+15$; 26 x^3+2x^2+3x+6; 27 x^3-1; 28 x^3+1; 29 $-6x+10$; 30 $25-x^2$; 31 $y^2+8y+16$; 32 x^2+5x+6; 33 $7+5x-2x^2$; 34 $16x^2-1$; 35 $x^2+\frac{5}{2}x+1$; 36 $6x-2$; 37 $2x^2+9x+9$; 38 $5x^2-17x+6$; 39 $x^2+4xy+4y^2$; 40 $9x^2-6xy+y^2$; 41 x^3+x^2-4x-4; 42 $2x^2+8x+6$; 43 $-x^2+3x+4$; 44 x^4-4; 45 $6x^2+17x+5$; 46 $5x-4$; 47 x^3+3x^2+3x+1; 48 $1-2x+x^2$; 49 $4-12x+9x^2$; 50 $2x^2+2x-8$.

Practice

1 $x^2+4x-10$; 2 $x^3+6x^2+12x+8$; at $x=-1$ gives 1; 3 $8x^3-12x^2+6x-1$; 4 $(x+y)^2-1$; 5 $3x^2-13xy-10y^2$; coeff. of xy is -13; 6 x^3+5x^2+5x-2; coeff. of x^2 is 5; 7 $2x^3-9x^2+7x-12$; 8 $2n+1$; always odd; 9 $6-3x^{-2}$; 10 $1+8x^3$; 11 $12x$; $12x=12(x)$; 12 $4ab$; 13 $3x^2+14x+8$; 14 1; 15 -36; verified at $x=0$; 16 $x^3+9x^2+26x+24$; sum of coefficients 60; 17 $1-x^4$; 18 $2x^3+9x^2-5x$; 19 x^3-5x^2+3x+9; 20 $4x-13$; $x=\dfrac{13}{4}$; 21 $4x^2-4x-3$; 22 $4x^2+2x+\dfrac{1}{4}$; 23 $(x^2+1)^2=x^4+2x^2+1$; $(x^2-1)^2=x^4-2x^2+1$; differ by $4x^2$; 24 x^4-2x^2+1; 25 x^4-16.

Advanced Practice

1 $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$; 2 shown; 3 $p=13$; 4 $1+4x+6x^2+4x^3+x^4$; 5 shown; 6 $x^2+2+x^{-2}-1=\left(x+x^{-1}\right)^2-1$; 7 720; 8 $(x^2-4)^2$; 9 $k=3$; 10 5.

5 Factorising

Basic Drill

1 $4(x+2)$; 2 $3(3x-4)$; 3 $x(x+3)$; 4 $5x(x-2)$; 5 $3x(2x+3)$; 6 $(x+3)(x+4)$; 7 $(x+4)(x+5)$; 8 $(x+3)(x-2)$; 9 $(x-4)(x+3)$; 10 $(x-2)(x-3)$; 11 $(x-4)^2$; 12 $(x+3)^2$; 13 $(x-4)(x+4)$; 14 $(x-7)(x+7)$; 15 $(3x-2)(3x+2)$; 16 $(5x-1)(5x+1)$; 17 $(2x+1)(x+2)$; 18 $(3x+1)(x+3)$; 19 $(2x-3)(x+1)$; 20 $(3x+1)(x-2)$; 21 $x(x-2)$; 22 $7(2x-3)$; 23 $(x-1)(x+1)$; 24 $(x+2)^2$; 25 $(x-5)^2$; 26 $(x+3)(x+8)$; 27 $(x-2)(x-5)$; 28 $(x+5)(x-3)$; 29 $(x-7)(x+4)$; 30 $(2x-3)(2x+3)$; 31 $(4x-5)(4x+5)$; 32 $2x(x+4)$; 33 $3x(x-4)$; 34 $(x+3)(x+5)$; 35 $(x-3)(x-6)$; 36 $5x(x+1)$; 37 $(x-6)(x+6)$; 38 $(2x+1)(x+1)$; 39 $(2x+1)^2$; 40 $(3x+2)(2x-1)$; 41 $x(a+b)$; 42 $x^2(x+1)$; 43 $xy(x+y)$; 44 $(x-2)(x+1)$; 45 $(x+7)(x-2)$; 46 $2(x-2)(x+2)$; 47 $3(x-3)(x+3)$; 48 $(x+6)^2$; 49 $(x-7)^2$; 50 $5x(2x-3)$.

Practice

1 $2x^2(x-3)(x+3)$; $x=0,\pm3$; 2 $4ab$; 3 $3y(2x^2-5x+3)=3y(2x-3)(x-1)$; 4 $(x-y)(x-1)$; always linear factors in x; 5 $(x+3)(y+2)$; 6 $(x^2+1)(x+2)$; 7 $(x-2)(x^2+2x+4)$; $x=2$; 8 $(x+3)(x^2-3x+9)$; $x=-3$; 9 $(3x-2)(4x+1)$; $x=\frac{2}{3},-\frac{1}{4}$; 10 $(4x+3)(2x+5)$; 11 $(x-2)(x+2)(x^2+4)$; $x=\pm2$; 12 $(x^2+1)(x^2+3)$; no real roots; 13 $2x(x-5)(x+5)$; $x=0,\pm5$; 14 $(x+y-3)(x+y+3)$; 15 $(x-2y)(x+y)$, $(2x-y)(x+y)$; 16 $(5x-2)(3x+4)$; 17 $x(x-1)(x+1)$; roots $0,\pm1$; 18 $(3x-5-2y)(3x-5+2y)$; 19 $(2x-3y)^2$; $2x=3y$; 20 $(3x+4)(2x-3)$; 21 $m=3$, $n=-8$; 22 $(x-1)(x+1)(x-2)(x+2)$; 23 $(x+2-y)(x+2+y)$; 24 $k=0$ or $k=4$ (e.g. $k=4$ gives $(x+2)^2$); 25 $(a-b-c)(a-b+c)$.

Advanced Practice

1 $(x^2+2x+2)(x^2-2x+2)$; 2 shown; if $a+b+c=0$ then $a^3+b^3+c^3=3abc$; 3 $k\in\{-13,-8,-7,7,8,13\}$; 4 $(x-1)(x+1)(x^2-x+1)(x^2+x+1)$; 5 $n(n-1)(n+1)$ divisible by 2 and 3; 6 $(x+2-y)(x+2+y)$; 7 $(2x+a)(x+2)$; 8 $(x-1)(x+1)(x+3)$; 9 $(x-1)(x-2)(x-3)$; 10 always (since $n^3-1=(n-1)(n^2+n+1)$).

6 Algebraic Fractions

Basic Drill

1 $\frac{2x}{3}$; 2 $\frac{x}{2}$; 3 x; 4 $\frac{x+1}{2}$; 5 $x+2$; 6 $x+1$ $(x \neq 1)$; 7 $x-2$ $(x \neq -2)$; 8 $x+5$ $(x \neq 0)$; 9 $x+2$ $(x \neq 0)$; 10 $x+2$ $(x \neq -1)$; 11 $x+4$ $(x \neq -3)$; 12 $\frac{x+3}{x+2}$ $(x \neq -3, -2)$; 13 $\frac{2}{x}$; 14 $\frac{5}{x}$; 15 $\frac{2+x}{2x}$; 16 $\frac{2x+1}{x(x+1)}$; 17 $\frac{2}{x}$; 18 $\frac{3x+10}{x(x+2)}$; 19 $\frac{2}{x}$; 20 1; 21 $2x$; 22 $2x$; 23 $x+1$ $(x \neq -1)$; 24 $\frac{x+4}{x-4}$ $(x \neq 4)$; 25 $\frac{2}{x}$ $(x \neq 0, -2)$; 26 $2x$; 27 x; 28 $\frac{3}{4x}$; 29 $\frac{5}{4x}$; 30 $\frac{x}{x+1}$ $(x \neq \pm 1)$; 31 $\frac{x+1}{x+2}$; 32 $\frac{1}{x}$; 33 3; 34 5; 35 $\frac{x}{x+2}$; 36 1; 37 2; 38 $\frac{3x}{4}$; 39 x; 40 x; 41 $x+1$; 42 $x-3$; 43 $\frac{4}{3}$; 44 2; 45 $x-2$; 46 $x \neq 5$; 47 $x \neq \pm 3$; 48 $\frac{5}{3x}$; 49 $-(x+5)$ $(x \neq 5)$; 50 2.

Practice

1 1 $(x \neq 2, \pm 3)$; 2 $\dfrac{x+2}{x-2}$; 3 $\dfrac{5-x}{(x-1)(x+1)}$; 4 $\dfrac{x^2+x+6}{(x+2)(x-2)}$; 5 $\dfrac{x+2}{(x-1)(x+1)}$; 6 $\dfrac{2x+3}{x+1}$; 7 $\dfrac{2-x}{(x-2)(x+2)}$; 8 $\dfrac{x^2+2x+4}{x+2}$; 9 $\dfrac{(3x+4)(x-2)}{x(x+4)}$; 10 $\dfrac{ab}{a+b}$; 11 $\dfrac{-h}{x(x+h)}$; 12 $\dfrac{-x}{x^2-1}$; 13 $\dfrac{(3x-2)(2x+1)}{(3x-1)(x+2)}$; 14 $\dfrac{3x+4}{x(x+3)}$; 15 $\dfrac{2x}{x^2-1}$ $(x \neq \pm 1)$; 16 $\dfrac{x}{(x+1)^2}$; 17 $\dfrac{5-3x}{x(x+1)}$; 18 $x = 1 \pm \sqrt{2}$ $(x \neq \pm 1)$; 19 $\dfrac{4x-1}{(2x-1)(2x+1)}$; 20 $\dfrac{3x^2-2x+1}{x^3}$; 21 $\dfrac{x+y}{x^2+xy+y^2}$; 22 $\dfrac{x+4}{(x+2)(x+3)}$; 23 $\dfrac{6(x+1)}{(x-3)(x+3)}$; 24 $\dfrac{x}{2}$ $(x \neq \pm 1)$; 25 $\dfrac{2x}{x^2-4}$.

Advanced Practice

1 $\dfrac{n}{n+1}$; 2 $\dfrac{x+a}{x+c}$; 3 $\dfrac{2(b-a)x+a^2-b^2}{(x-a)(x-b)}$; 4 $A=-2$, $B=7$; 5 $\dfrac{x+y}{y-x}$; 6 $A=\dfrac{8}{9}$, $B=-\dfrac{8}{9}$, $C=\dfrac{1}{3}$; 7 1 $(x \neq \pm 1)$; 8 $x > 2$; 9 $\dfrac{3-x}{x(x-1)(x-2)}$; 10 x.

7 Equations & Inequalities

Basic Drill

1 $x = 7$; 2 $x = 9$; 3 $x = 3$; 4 $x = 3$; 5 $x = 4$; 6 $x = 20$; 7 $x = 9$; 8 $x = 4$; 9 $x = 8$; 10 $x = 6$; 11 $x = \pm 4$; 12 $x = \pm 3$; 13 $x = 0$ or 5; 14 $x = -1$ or -3; 15 $x = 2$ or 5; 16 $x = -4$ or 3; 17 $x = -\frac{1}{2}$ or 3; 18 $x = 1 \pm \sqrt{2}$; 19 $x = -3$; 20 $x = 4$ or -1; 21 $x = 6$, $y = 4$; 22 $x = 3$, $y = 1$; 23 $x = 2$, $y = 3$; 24 $x = 3$, $y = 6$; 25 $x = 1$, $y = 2$; 26 $x > 3$; 27 $x \leq -1$; 28 $x < 4$; 29 $x \leq 1$; 30 $x < 4$; 31 $x < -3$; 32 $x \leq -2$ or $x \geq 2$; 33 $1 < x < 4$; 34 $x < -3$ or $x > 3$; 35 $x = \pm 5$; 36 $x = -1$ or 5; 37 $x = 7$; 38 $x = 14$; 39 $x = \pm 2$; 40 $x = 0$ or 5; 41 $t = \dfrac{v-u}{a}$; 42 $r = \sqrt{\dfrac{A}{\pi}}$; 43 $x = \dfrac{y-c}{m}$; 44 $u = \dfrac{2s}{t} - v$; 45 $x = -2$ or 3; 46 $x = 2$, $y = 3$; 47 $x > 6$; 48 $x \geq -3$; 49 $x = -1$ or -4; 50 $x = -1$ or $\dfrac{1}{2}$.

Practice

1 $x = 4$; 2 $x = 1$; 3 $x = 2 \pm \sqrt{5}$; 4 $x = \dfrac{-3 \pm \sqrt{41}}{4}$; 5 $x = -3 \pm \sqrt{11}$; 6 $x = 2$, $y = 3$; 7 $(x, y) = (3, 8)$ or $(-1, 0)$; 8 $x < 2$ or $x > 3$; 9 $x = -7$; 10 $x = -2$ or 3; 11 $-5 < x < -1$; 12 $x = 2$ or $x = -\dfrac{3}{5}$; 13 $k = \pm 6$; 14 $k < 1$; 15 $x = -1$ or 3; 16 $x = \dfrac{7 \pm \sqrt{33}}{4}$ $(x \neq 0, 2)$; 17 $x = \dfrac{5 \pm \sqrt{17}}{2}$ $(x \neq \pm 2)$; 18 $x \leq -3$ or $x \geq \dfrac{1}{2}$; 19 $(x, y) = (3, 4)$ or $(4, 3)$; 20 $k = \dfrac{13 \pm 3\sqrt{17}}{4}$; 21 $x = \dfrac{3}{2}$; 22 $x = 3$; 23 $[-\sqrt{14}, -2] \cup [2, \sqrt{14}]$; 24 $2 - \sqrt{10} < x < 2 + \sqrt{10}$; 25 $-2 \leq x \leq 2$.

Advanced Practice

1 $x = \pm 1, \pm 2$; 2 $m \leq 0$ or $m \geq 4$; 3 $x = 1$, $y = 2$, $z = 3$; 4 $x > 2$; 5 shown; 6 $x = -3$ or $x = 2$; 7 $x = 0$ or $x = 4$; 8 $x = \dfrac{k+2}{k-1}$ $(k \neq 1)$; no solution if $k = 1$; 9 $k > 1$; 10 $(x, y) = \pm(3, 4),\ \pm(4, 3)$.

8 Quadratic Functions

Basic Drill

1 up; 2 down; 3 7; 4 $x = 2, 3$; 5 $x = 3$; 6 $x = -2$; 7 $(3, 2)$; 8 $(-1, -5)$; 9 $(x+3)^2 - 4$; 10 $(x-2)^2 - 3$; 11 $(x+1)^2 - 8$; 12 5; 13 9; 14 $[3, \infty)$; 15 $(-\infty, 8]$; 16 $(-1, 0), (2, 0), (0, -2)$; 17 $(1, -9)$; 18 $(x+5)^2 - 4$; 19 $(1, 5)$, min 5; 20 $(-2, 7)$, max 7; 21 $c = 3$; 22 No; 23 none; 24 one ($x = 2$); 25 two; 26 $x = \frac{5}{2}$; 27 $(0, 0)$; 28 $(-4, 0)$; 29 $y = x^2 - 4x + 7$; 30 $y = 2x^2 + 4x - 3$; 31 2; 32 -2; 33 $2(x+2)^2 - 5$; 34 $-(x-1)^2 + 6$; 35 min; 36 max; 37 $(-3, 0), (1, 0), (0, -3)$; 38 $(2, 0), (6, 0), (0, -12)$; 39 $x = 2$; 40 4; 41 4; 42 $[1, \infty)$; 43 $(-\infty, 3]$; 44 $y = (x-2)^2 - 3$; 45 $a = 1$; 46 $y = 3x^2$ is narrower; 47 $k = 9$; 48 $\left(-\frac{1}{2}, \frac{3}{4}\right)$; 49 $(2, 7)$; 50 $\left(x - \frac{1}{2}\right)^2 - 1$.

Practice

1 $3(x-2)^2 - 5$; $(2, -5)$; $[-5, \infty)$; 2 $(-1, 0), (5, 0), (0, -5)$; vertex $(2, -9)$; 3 16 at $x = 3$; 4 $[-6, 3]$; 5 $b = -6$, $c = 5$; 6 $k = 1$; 7 $x = 2 \pm \sqrt{6}$; parabola and line meet at two points; 8 $a = -2$, $b = 5$; 9 shown; 10 $x = \dfrac{5}{6}$; vertex $\left(\dfrac{5}{6}, -\dfrac{13}{12}\right)$; 11 $-2(x-2)^2 + 5$; max 5; 12 $y = 2x^2 - 5x + 3$; 13 $a = 1$, $b = -4$, $c = 5$; 14 $[-1, 3]$; 15 $y = (x-4)^2 - 1$; 16 $-3(x-2)^2 + 7$; max 7 on $[0, 4]$; 17 $|m| > 2$; 18 $4\left(x - \dfrac{1}{2}\right)^2 + 6$; 19 $(1 \pm \sqrt{2},\ 4 \pm 2\sqrt{2})$; 20 shown; 21 10 at $t = 3$; 22 $y = -2(x+1)(x-3)$; 23 $p = 0$ or $p = 6$; 24 vertex $(-3, 5)$; axis $x = -3$; max 5; 25 $(-\frac{3}{2}, 0), (2, 0), (0, -6)$; axis $x = \frac{1}{4}$; range $\left[-\frac{49}{8}, 0\right]$ on $[-1, 2]$.

Advanced Practice

1 $\dfrac{\sqrt{11}}{2}$; 2 shown; 3 $0 < k < 4$; 4 $8b = a^2 + 8$; 5 $[2, \infty)$; 6 $(x+2)^2 + (y-3)^2 - 13$; 7 $b = \pm 4$; 8 shown; 9 vertex $(c,\ c - c^2)$; locus $y = x - x^2$; 10 $k = 6$.

9 Coordinate Geometry

Basic Drill

1 $\frac{1}{2}$; 2 2; 3 0; 4 undefined; 5 $-\frac{3}{2}$; 6 $(3,2)$; 7 $(5,3)$; 8 $(1,1)$; 9 5; 10 5; 11 5; 12 $y=3x+2$; 13 $y=-2x-1$; 14 $y=x$; 15 $y=-\frac{1}{2}x+4$; 16 $c=4$; 17 $m=4$, $c=-1$; 18 $y=-2x+5$; 19 $m=\frac{3}{2}$; 20 e.g. $y=5x$; 21 $m=-\frac{1}{2}$; 22 $m=3$; 23 Yes; 24 Yes; 25 $y=3x$; 26 $y=3x-1$; 27 $y=-x+4$; 28 -1; 29 $\left(\frac{a+c}{2},\frac{b+d}{2}\right)$; 30 $\sqrt{a^2+b^2}$; 31 $x=4$; 32 9; 33 $y+1=2(x-4)$; 34 $x-2y+6=0$; 35 No; 36 Yes; 37 $x=3$; 38 $y=7$; 39 -2; 40 $k=4$; 41 $k=1$; 42 0; 43 5; 44 $y=-5$; 45 $x=2$; 46 0; 47 $y=3$; 48 $m=-4$; 49 $y=3x-4$; 50 $y=-\frac{1}{3}x+\frac{8}{3}$.

Practice

1 $x-2y-7=0$; 2 No ($m_{AB}=\frac{3}{2}$, $m_{AC}=-4$); 3 $M(5,1)$; $y=x-4$; 4 $y=-4x+12$; 5 All sides 6 (equilateral); 6 $p=3$; 7 $(2,3)$; distance $\sqrt{13}$; 8 $(2,3)$; area 6; 9 6; 10 $y=x-1$; 11 No; 12 $5x+10y=11$ (or $y=-\frac{1}{2}x+\frac{11}{10}$); 13 $2x+y-3=0$; 14 $k=9$; 15 $y=\sqrt{3}x+1$; 16 $(3,4)$; 17 $t=5$; 18 $k=\pm5$; 19 $\dfrac{x}{3}+\dfrac{y}{6}=1$; $2x+y-6=0$; 20 $(1,1)$; 21 $D(2,-3)$; 22 $P(4,0)$, $Q(0,6)$; 23 15°; 24 $x=2$; 25 $\frac{21}{2}$.

Advanced Practice

1 Midpoints of diagonals both $(\frac{a+c}{2},\frac{b+d}{2})$; 2 $P\left(\frac{11}{3},0\right)$; 3 $y=3x-2$; 4 $c=5$; 5 Collinear $\iff$ gradients equal (or area zero); 6 $y=x\pm2\sqrt{2}$; 7 Opposite sides of midpoint quadrilateral are equal and parallel; 8 Minimum $a+b=3+2\sqrt{2}$ at $a=1+\sqrt{2}$, $b=2+\sqrt{2}$; 9 $2\sqrt{2}$; 10 Maximum 6 at $P(2,0)$ or $P(0,2)$.

10 Functions & Graphs

Basic Drill

1 7; 2 5; 3 $\frac{1}{5}$; 4 $x = \frac{5}{2}$; 5 $x \neq 4$; 6 $x \geq -3$; 7 $x > 0$; 8 $\mathbb{R}$; 9 $[0, \infty)$; 10 $[0, \infty)$; 11 5; 12 $x^2 + 3$; 13 $(x-2)^2$; 14 up 4; 15 left 5; 16 right 1, down 2; 17 reflect in x-axis; 18 reflect in y-axis; 19 steeper line through origin (stretch in y); 20 steeper line (compress in x); 21 $3a + 3$; 22 2; 23 $x \neq \pm 1$; 24 $x \leq 5$; 25 $\{5\}$; 26 No; 27 $2, 4, 6$; 28 $f(x) = 3x + 1$; 29 right 2; 30 down 2; 31 right 4; 32 up 3; 33 stretch in y by 3; 34 compress in x by $\frac{1}{2}$; 35 $f(f(2)) = f(-1) = 2$; 36 No (vertical line $x = a$); 37 domain $[0, 4]$, range $[0, 4]$; 38 27; 39 $|x|$; 40 one output per input; 41 $x = 5$; 42 $x = \pm 3$; 43 No; 44 $y = \dfrac{1}{x-1}$; 45 $y = \dfrac{1}{x} + 2$; 46 $(0, -1)$, $(\pm 1, 0)$; 47 reflect in x-axis then up 1; 48 Yes (all reals); 49 $(0, 1]$; 50 $g(3) = 7$.

Practice

1 domain $[-3, 3]$, range $[0, 3]$; 2 domain $x \neq 3$, range $y \neq 2$; 3 right 1, stretch in y by 2, up 3; 4 $y = -x^2 + 4$; 5 $2x - 1$; 6 $x \geq -2$, $x \neq 1$; 7 $[2, \infty)$; 8 $(-\infty, 1]$; 9 each x would give two y values; 10 $(1, 5)$; 11 $\left(\frac{1}{2}, 2\right)$; 12 $(2, 2)$; 13 $f(0) = 2$, $f(2) = 2$, $f(-3) = 4$; 14 $x < \frac{3}{2}$; 15 $x = 2$, $y = 1$; 16 $f^{-1}(x) = \sqrt{x}$; domain $[0, \infty)$, range $[0, \infty)$; 17 $x = \dfrac{y+3}{2}$; 18 left 2, reflect in x-axis, up 5; 19 $a = 4$; 20 $x = 0, \pm\sqrt{2}$; 21 $[2, \infty)$; 22 W-shape; $(\pm 1, 0)$, $(0, 1)$; minimum 0 at $x = \pm 1$; 23 No; 24 $f(g(2)) = 9$, $g(f(2)) = 5$; 25 domain is $x \leq 0$; not obtained from $y = \sqrt{x}$ by stretch/reflection alone.

Advanced Practice

1 $[-5, 5]$; 2 $x = -1$; 3 $f(f(f(x))) = x$ for $x \neq 1$; 4 reflect in y-axis; right 1; stretch in y by 3 and reflect in x-axis; up 2; 5 domain $[0, 1]$, range $[1, \sqrt{2}]$; 6 $m = 0$ (any c) or $m = 1$, $c = 0$; 7 not continuous at 0; range $[0, 4]$ on $[-2, 2]$; 8 $m = -1$ (any c) or $m = 1$, $c = 0$; 9 $y = x$ (since $y = x + \dfrac{1}{x}$ for $|x| \to \infty$); 10 $\left[\frac{2}{3}, 2\right]$.

11 Trigonometry

Basic Drill

1 $\frac{1}{2}$; 2 $\frac{\sqrt{3}}{2}$; 3 $\frac{1}{\sqrt{3}}$; 4 $\frac{\sqrt{2}}{2}$; 5 $\frac{\sqrt{2}}{2}$; 6 1; 7 $\frac{\sqrt{3}}{2}$; 8 $\frac{1}{2}$; 9 $\sqrt{3}$; 10 0, 1; 11 1, 0; 12 $\frac{\pi}{2}$; 13 π; 14 $\frac{\pi}{4}$; 15 $\frac{\pi}{6}$; 16 90°; 17 30°; 18 120°; 19 $\frac{1}{2}$; 20 $\frac{1}{2}$; 21 1; 22 1; 23 1; 24 $\frac{4}{5}$; 25 $\frac{5}{12}$; 26 positive; 27 negative; 28 negative; 29 $\frac{1}{2}$; 30 $-\frac{\sqrt{3}}{2}$; 31 0; 32 -1; 33 $0°, 180°$; 34 90°; 35 45°; 36 $60°, 120°$; 37 60°; 38 Yes; 39 $30°, 150°$; 40 $\sqrt{3}$; 41 1; 42 $\frac{3}{5}$; 43 $\frac{4}{5}$; 44 $\frac{3}{4}$; 45 $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$; 46 $\frac{3\pi}{2}$; 47 $\frac{\sqrt{2}}{2}$; 48 $\frac{\sqrt{3}}{2}$; 49 $\sin 60°$; 50 $\cos\theta$.

Practice

1 $60°, 120°$; 2 $120°, 240°$; 3 $120°, 300°$; 4 45°; 5 $\sin 225° = -\frac{\sqrt{2}}{2}$, $\cos 225° = -\frac{\sqrt{2}}{2}$; 6 0; 7 $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$; 8 $30°, 60°$; 9 $15°, 75°$; 10 $c = 4$, area $4\sqrt{3}$; 11 $40°, 320°$; 12 $\frac{5}{4}$; 13 $a = 6$; 14 $\frac{\pi}{3}, \frac{2\pi}{3}$; 15 $\frac{\pi}{4}, \frac{5\pi}{4}$; 16 $\sin\theta = \frac{2}{\sqrt{5}}$; 17 $60°, 120°$; 18 70°; 19 -1; 20 $4\sqrt{3}$ m; 21 $30°, 150°$; 22 $20°, 160°$; 23 $\frac{\sqrt{6}-\sqrt{2}}{4}$; 24 $\sin 70° = \cos 20°$; 25 arc length 2π, area 6π.

Advanced Practice

1 $45°, 225°$; 2 $0°, 30°, 150°, 180°, 360°$; 3 $30°, 150°$; 4 $\sin(90° + \theta) = \cos\theta$; 5 $\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$, $\cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$; 6 $5\sin(\theta + \alpha)$ with $R = 5$; maximum 5; 7 $0°, 180°$; 8 $\sin\theta\cos\theta = 0$, $\sin 2\theta = 0$; 9 $90°, 330°$; 10 identity holds for all p, q.

12 Exponentials & Logarithms

Basic Drill

1 2; 2 3; 3 4; 4 0; 5 1; 6 3; 7 $\frac{1}{2}$; 8 $\frac{1}{2}$; 9 0; 10 $2^x = 8$; 11 $x = 3$; 12 $x = 3$; 13 $x = 0$; 14 $x = 4$; 15 5; 16 1; 17 2; 18 7; 19 3; 20 5; 21 $x > 0$; 22 $x > 1$; 23 -3; 24 -2; 25 $x = -2$; 26 $x = -2$; 27 $a^c = b$; 28 4; 29 $\frac{1}{2}$; 30 5; 31 3; 32 $\log_{10} x + \log_{10} y$; 33 $\log_{10} x - \log_{10} y$; 34 $\log_2(xy)$; 35 $\log_2(x^3)$; 36 $x = 32$; 37 $x = 100$; 38 $x = \frac{1}{9}$; 39 $\frac{3}{2}$; 40 $\frac{3}{2}$; 41 No; 42 0; 43 $x = 3$; 44 $x = 2$; 45 3; 46 k; 47 3; 48 0.903 (3 s.f.); 49 $x = 1$; 50 $x = 5$.

Practice

1 $x = \dfrac{\ln 2}{\ln 3 - \ln 2}$; 2 $x = 3$; 3 $x = 5$; 4 $x = 3$ $(x > 1)$; 5 $\log_a b \cdot \log_b a = 1$; 6 $x = 2$; 7 $x = 3$; 8 $x = 7$ $(x > 1)$; 9 $x = \frac{1}{2}\ln 7$; 10 $x = -1$; 11 $2a + b$; 12 $x = \log_{10} 3$; 13 $x < 2$ or $x > 3$; 14 $x = 5$; 15 $2\log_{10} 2 + \log_{10} 3$; 16 $1 + 2a$; 17 $x = 1$ or $x = 2$; 18 $x = 1$; 19 0.778 (3 s.f.); 20 $x = -3$; 21 $f^{-1}(x) = \log_3 x$; 22 $x = 3$; 23 $x = \log_2 24$ (or $3 + \log_2 3$); 24 1.54 (3 s.f.); 25 $x = \dfrac{\ln 3}{\ln 3 - \ln 2}$.

Advanced Practice

1 $x = 4$ or $x = \sqrt{2}$; 2 $\log_{a^n} b^m = \frac{m}{n}\log_a b$; 3 $x = 1$ or $x = 2$; 4 $x = 0$ or $x = 3$; 5 $2^x + 2^{-x} \geq 2$ with equality at $x = 0$; 6 minimum 2 at $x = 0$; 7 $(x, y) = (2, 4)$ or $(4, 2)$; 8 product equals 1; 9 $x = 100$ or $x = 0.1$; 10 $x = 8$.