Pearson

Mark Scheme (Results)

October 2025

International Advanced Level in Pure Mathematics P4

WMA14/01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread

however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to x = ...

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	Assume there are positive numbers *a* and *b* such that $\frac{9a}{b}+\frac{4b}{a}<12$	B1
	Multiplies by $ab \Rightarrow 9a^2+4b^2<12ab$	M1
	$\Rightarrow 9a^2-12ab+4b^2<0 \Rightarrow (3a-2b)^2<0$	A1
	Square numbers cannot be negative, so we have a contradiction and hence, if *a* and *b* are positive real numbers, then $\frac{9a}{b}+\frac{4b}{a}\geq 12$	A1*
		(4 marks)

Condone $\leq$ for < for the first 3 marks in this question

B1: Sets up the argument. It requires the following three aspects

- Words such as ‘assume’ or ‘let’
- ‘positive (real) numbers *a* and *b*’ or equivalent such as $a>0$ and $b>0$
- $\frac{9a}{b}+\frac{4b}{a}<12$ but condone $\frac{9a}{b}+\frac{4b}{a}\leq 12$

M1: Multiplies $\frac{9a}{b}+\frac{4b}{a}\ldots 12$ by *ab* to reach $9a^2+4b^2\ldots 12ab$ or $9a^2-12ab+4b^2\ldots 0$ where …

can be = or any inequality.

Alternatives exist so $\frac{9a^2-12ab+4b^2}{ab}\ldots 0$, $\frac{9a^2+4b^2}{ab}\ldots 12$ and $9\left(\frac{a}{b}\right)^2-12\left(\frac{a}{b}\right)+4\ldots 0$ are all valid methods

A1: Reaches a point at which the contradiction can be proven. Examples of suitable alternatives include

$9a^2-12ab+4b^2<0 \Rightarrow (3a-2b)^2<0$, $\frac{9a^2-12ab+4b^2}{ab}<0 \Rightarrow \frac{(3a-2b)^2}{ab}<0$ or

$9\left(\frac{a}{b}\right)^2-12\left(\frac{a}{b}\right)+4<0 \Rightarrow \left(\frac{3a}{b}-2\right)^2<0$ but condone attempts with ≤ instead of <

A1*: CSO. Fully correct proof with reason and conclusion.

Look for all of the following

- A correct set up
- correct algebra leading to $(3a-2b)^2<0$ o.e. NOT $(3a-2b)^2\leq 0$

- a correct statement or reason in the correct place that indicates a contradiction has been reached. For example, ‘numbers are ≥ 0 when squared’ or ‘but $(3a-2b)^2 \geq 0$’ o.e.

  If the intermediate form $\dfrac{(3a-2b)^2}{ab}$ is reached there must be some statement about

  ab being >0 and $(3a-2b)^2 \geq 0$ (Note that stating > 0 would be incorrect)

- a correct conclusion such as ‘hence $\dfrac{9a}{b}+\dfrac{4b}{a} \geq 12$’ ‘hence contradiction, so proven’ if the inequality is restated you can condone the omission of *a* and *b* being positive

Simply stating $(3a-2b)^2 \geq 0$ without any words like ‘but’ is insufficient. Stating ‘proven’ without stating a reason such as there is a ‘contradiction’ is also insufficient

A minimal acceptable proof would be;

Assume there are positive numbers *a* and *b* such that $\dfrac{9a}{b}+\dfrac{4b}{a} < 12$

$9a^2 + 4b^2 < 12ab \Rightarrow 9a^2 - 12ab + 4b^2 < 0 \Rightarrow (3a-2b)^2 < 0$

As $(3a-2b)^2 \geq 0$, we have a contradiction. Hence proven

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\mathrm{d}x = \int x^{-2}\ln x\,\mathrm{d}x = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$	M1A1
	$= \frac{x^{-1}}{-1}\ln x + \int x^{-2}\,\mathrm{d}x$	
	$= \frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}(+c)$	dM1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{\mathrm{e}} = \left(\frac{-1}{\mathrm{e}}\ln \mathrm{e} - \frac{1}{\mathrm{e}}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	ddM1
	$= 1 - \frac{2}{\mathrm{e}}$	A1
		(6)

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)

Accept as evidence $\int x^{-2}\ln x\,\mathrm{d}x = \pm Ax^{-1}\ln x \pm B\int x^{-1}\times\frac{1}{x}\,\mathrm{d}x$ o.e.

A1: Accept a correct un-simplified application $= \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$ o.e

dM1: Dependent upon having scored the previous M mark.
It is for reaching a form $\pm Ax^{-1}\ln x \pm Bx^{-1}$ o.e

A1: A completely correct integral.
For students who substitute in limits look for $= \left(\frac{\mathrm{e}^{-1}}{-1}\ln \mathrm{e}\right) - \left(\frac{1^{-1}}{-1}\ln 1\right) + \left[\frac{x^{-1}}{-1}\right]_1^{\mathrm{e}}$

ddM1: Dependent upon both previous M's.
It is awarded for

- achieving an integral of the form $\pm Ax^{-1}\ln x \pm Bx^{-1}$
- substituting in limits and subtracting either way around
- using both $\ln \mathrm{e} = 1$ & ln1 = 0 and achieving an expression of the form $a + \frac{b}{\mathrm{e}}$ which may be un-simplified. Note that an intermediate expression of $-\frac{1}{\mathrm{e}} + 1 + \frac{1}{\mathrm{e}}$ (which may be implied) leading to 1 is fine for this mark

A1 : cso and cao $= 1 - \frac{2}{\mathrm{e}}$ or $-\frac{2}{\mathrm{e}} + 1$ Condone $1 + -\frac{2}{\mathrm{e}}$ but $1 - \frac{2}{\mathrm{e}} + c$ is A0

Note that you may see attempts at the integration via D & I methods. These are essentially the same as via the main method. So still expect to see variations on $-\frac{1}{x}\ln x + \int \frac{1}{x^2}\,\mathrm{d}x$ for the M1, A1.

All stages of working must be seen (bold sentences), so there is an expectation that the intermediate step is seen.

If $\int \frac{\ln x}{x^2}\mathrm{d}x == \frac{-1}{x}\ln x - \frac{1}{x}(+c)$ appears without any justification score this 1,0,1, 0 followed by potentially 1,0

Question Number	Scheme	Marks
3	Differentiates wrt *x* $\quad 3^x \ln 3 + \underline{\underline{6\frac{dy}{dx}}} = \underline{\tfrac{3}{2}y^2 + 3xy\frac{dy}{dx}}$ Substitutes (2, 3) **AND** rearranges to get $\frac{dy}{dx}$ or vice versa $\Rightarrow 9\ln 3 + 6\frac{dy}{dx} = \frac{27}{2} + 18\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9\ln 3 - \frac{27}{2}}{12} = \frac{6\ln 3 - 9}{8} = \frac{-9 + \ln 729}{8}$	B1 $\underline{\underline{\text{B1}}}, \underline{\text{M1}}, \text{A1}$ M1 A1, A1 **(7)** **(7 marks)**

B1: Differentiates $3^x \to 3^x \ln 3$ or equivalent such as $e^{x\ln 3} \to e^{x\ln 3}\ln 3$

B1: Differentiates $6y \to 6\frac{dy}{dx}$

M1: Uses the product rule to differentiate $\frac{3}{2}xy^2$.

Look for sight of $\underline{py^2 + qxy\frac{dy}{dx}}$ $\quad p, q > 0$

A1: A completely correct differential. It need not be simplified.

Variations on this include $3^x \ln 3dx + \underline{\underline{6dy}} = \underline{\tfrac{3}{2}y^2dx + 3xydy}$

Note that $\frac{dy}{dx} = 3^x \ln 3 + \underline{\underline{6\frac{dy}{dx}}} = \underline{\tfrac{3}{2}y^2 + 3xy\frac{dy}{dx}}$ is A0 unless there is later recovery to the correct answer

M1: Substitutes $x = 2,\ y = 3$ into their expression to find a 'numerical' value for $\frac{dy}{dx}$

The expression must contain only two $\frac{dy}{dx}$ terms, one from differentiating $6y$, the other from differentiating $\frac{3}{2}xy^2$

Alternatively rearranges to find $\frac{dy}{dx}$ and substitutes in $x = 2,\ y = 3$ FYI a correct $\frac{dy}{dx}$ is

$$\frac{3^x \ln 3 - 1.5y^2}{3xy - 6}$$

Condone slips (e.g. sign and coefficient slips) when rearranging or simplifying under this method.

A1: Any correct numerical answer e.g. $\frac{dy}{dx} = \frac{9\ln 3 - \frac{27}{2}}{12}$ which must be partially simplified.

So, the denominator for instance must not be left as (18 – 6)

A1: Exact answer only and in the required form.

The order of the terms on the numerator is not important so accept $\dfrac{\ln 729 - 9}{8}$

ISW after a correct answer

Question Number	Scheme	Marks
4 (a)	$\dfrac{1}{\sqrt{1-2x}}=(1-2x)^{-\frac{1}{2}}=1+\left(-\dfrac{1}{2}\right)(-2x)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x)^2$	M1A1
	$\dfrac{2+3x}{\sqrt{1-2x}}=(2+3x)(1+x+\dfrac{3}{2}x^2+\ldots$	dM1
	$=2+5x+6x^2$ *	A1*
		(4)
(b)	Sub $x=\dfrac{1}{20}$ into both sides of $\dfrac{2+3x}{\sqrt{1-2x}}=2+5x+6x^2$	M1
	$\dfrac{\frac{43}{20}}{\sqrt{\frac{9}{10}}}=\dfrac{453}{200}\Rightarrow\sqrt{10}=..$	dM1
	$\sqrt{10}=\dfrac{1359}{430}$	A1
		(3) **(7 marks)**

(a)

M1: Uses the binomial expansion with $n=-\frac{1}{2}$ to achieve correct binomial coefficients for terms 2 and 3 combined with the correct powers of *x*

Look for $1+\left(-\dfrac{1}{2}\right)(*x)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}(*x)^2\ldots$

or $1+\left(-\dfrac{1}{2}\right)(\pm 2x)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(\pm 2x)^2$ condoning missing/invisible brackets.

If no intermediate form is seen award for $1\pm x\pm\frac{3}{2}x^2$

A1: Correct simplified or unsimplified expression for $(1-2x)^{-\frac{1}{2}}$.

The correct simplified form $1+x+\frac{3}{2}x^2$ scores M1, A1. Condone additional terms

dM1: Multiplies their binomial expansion by $(2+3x)$. It is dependent upon the previous M.
Note that this is a proof with a given answer so individual terms must be seen, and not just the simplified expansion

Look for 4 correct terms of the 5 for their $(2+3x)(1+ax+bx^2+)=2+2ax+2bx^2+3x+3ax^2+....$

A1*: $2+5x+6x^2$ Correct answer only and it must follow all M1, A1, dM1 .
There must be no extra terms

(b)

M1: Sub $x=\frac{1}{20}$ into **both sides** of the given expression AND sets equal. Condone missing brackets.

So allow $\dfrac{2+3\times\frac{1}{20}}{\sqrt{1-2\times\frac{1}{20}}}=2+5\times\frac{1}{20}+6\times\frac{1}{20}^{2}$

It is acceptable to have partially simplified or indeed simplified expressions.

E.g. $\dfrac{43/20}{\sqrt{9/10}}=\dfrac{453}{200}$ or $\dfrac{43\sqrt{10}}{60}=\dfrac{453}{200}$

dM1: For an allowable method of proceeding to an answer for $\sqrt{10}$ as a fraction
Examples of allowable methods are

- simplifying the lhs to $\dfrac{43\sqrt{10}}{60}$ and then multiplying their $\left(2+5\times\frac{1}{20}+6\times\frac{1}{20}^{2}\right)$ by $\frac{60}{43}$ o.e
- simplifying the lhs to $\dfrac{43}{6\sqrt{10}}$ o.e. and then dividing $\frac{43}{6}$ o.e by their $\left(2+5\times\frac{1}{20}+6\times\frac{1}{20}^{2}\right)$

A1: Accept $\sqrt{10}=\dfrac{1359}{430}$ or $\sqrt{10}=\dfrac{4300}{1359}$

Question Number	Scheme	Marks
5.	One of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$	B1
	Two of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$	B1
	All three of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$	B1
Main		
	Full attempt to find $\frac{dV}{dt}$ E.g. $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dA}\times\frac{dA}{dt}=18\pi x^2\times\frac{1}{2\pi x}\times\frac{\pi}{20}$	M1
	Finds $\frac{dV}{dt}$ at $x=2\Rightarrow$ $\frac{dV}{dt}=18\pi 2^2\times\frac{1}{80}=\frac{9}{10}\pi$	dM1 A1
		(6 marks)
	Note that the M1 may be found in two steps: E.g.I Finds $\frac{dx}{dt}$ from $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ followed by $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ E.g II Finds $\frac{dV}{dA}$ from $\frac{dV}{dx}=\frac{dV}{dA}\times\frac{dA}{dx}$ followed by $\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt}$	
Alt	$\frac{dA}{dt}=\frac{\pi}{20}$	B1
	$V=6\pi x^3, A=\pi x^2\Rightarrow V=6\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}$ $\frac{dV}{dA}=9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$	B1, B1
	$\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt}\Rightarrow\frac{dV}{dt}=9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}\times\frac{\pi}{20}$	M1
	$x=2\Rightarrow A=4\pi$ $\Rightarrow\left.\frac{dV}{dt}\right\|_{A=4\pi}=\frac{9}{10}\pi$	dM1A1
		(6 marks)

Main method:

Condone the use of other letters such as *S* for *A* or *r* for *x*

Ignore any reference to units

The marks have now changed in this question to B1, B1, B1, M1, dM1, A1

B1: States or uses one of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$

B1: States or uses two of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$

B1: States or uses all three of $\frac{dA}{dt}=\frac{\pi}{20}$, $\frac{dA}{dx}=2\pi x$, $\frac{dV}{dx}=18\pi x^2$

Note that the circumference of the circle is $2\pi x$. You should not be awarding B1 for sight of $2\pi x$ unless you can see it set equal to, or implied equal to $\frac{dA}{dx}$

M1: For a full attempt at finding $\frac{\mathrm{d}V}{\mathrm{d}t}$. There are many ways to do this including

- In one step: Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}$ with $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{20}$, $\frac{\mathrm{d}A}{\mathrm{d}x}=px$ and $\frac{\mathrm{d}V}{\mathrm{d}x}=qx^2$
- In two steps: Finds $\frac{\mathrm{d}x}{\mathrm{d}t}$ from $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ with $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{20}$ and $\frac{\mathrm{d}A}{\mathrm{d}x}=px$

  followed by finding $\frac{\mathrm{d}V}{\mathrm{d}t}$ from $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ with $\frac{\mathrm{d}V}{\mathrm{d}x}=qx^2$ and their $\frac{\mathrm{d}x}{\mathrm{d}t}$

Condone slips when rearranging the chain rules but they must at least be correct to start with.

Condone slips on the coefficients of x and x^2 of $\frac{\mathrm{d}A}{\mathrm{d}x}=px$ and $\frac{\mathrm{d}V}{\mathrm{d}x}=qx^2$ respectively

If they state the chain rule $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ then make an algebraic error they may be awarded this method mark

dM1: For a full attempt at finding the value of $\frac{\mathrm{d}V}{\mathrm{d}t}$ at $x=2$ It is dependent upon the previous M

A1: $\frac{9}{10}\pi$ or 0.9π or $\frac{18}{20}\pi$ ……or k =0.9 etc

Alt method: Solution obtained directly via $\frac{\mathrm{d}V}{\mathrm{d}A}$

B1: States or uses $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{20}$

B1: Differentiates their $V=\alpha A^{\frac{3}{2}}\rightarrow\frac{\mathrm{d}V}{\mathrm{d}A}=rA^{\frac{1}{2}}$

B1: A correct $\frac{\mathrm{d}V}{\mathrm{d}A}=9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$ o.e such as $\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{9}{\sqrt{\pi}}A^{\frac{1}{2}}$

M1: For a full attempt at finding $\frac{\mathrm{d}V}{\mathrm{d}t}$ using $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}$.

To award this, candidates must be using $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{20}$ and $\frac{\mathrm{d}V}{\mathrm{d}A}=rA^{\frac{1}{2}}$

dM1: Finds the value of $\frac{\mathrm{d}V}{\mathrm{d}t}$ at $A=4\pi$ It is dependent upon the previous M

A1: $\frac{9}{10}\pi$ or 0.9π or $\frac{18}{20}\pi$ ……or k =0.9 etc

Question Number	Scheme	Marks
6.(a)	$x=\dfrac{3}{2}\Rightarrow a=\dfrac{\pi}{6}$	B1
	$\int y^2\mathrm{d}x=\int y^2\dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t=\int(2\sin 2t)^2 3\cos t(\mathrm{d}t)$	M1
	Uses $\sin 2t=2\sin t\cos t \Rightarrow \int(4\sin t\cos t)^2 3\cos t\ \mathrm{d}t$	dM1
	Volume = $\int \pi y^2\mathrm{d}x = 48\pi\int_0^{\left(\frac{\pi}{6}\right)}\sin^2 t\cos^3 t\ \mathrm{d}t$	A1
		(4)
(b)	$u=\sin t\Rightarrow\dfrac{\mathrm{d}u}{\mathrm{d}t}=\cos t$	B1
	$k\int\sin^2 t\cos^3 t\ \mathrm{d}t=k\int u^2\cos^2 t\ \mathrm{d}u$	
	$=k\int u^2(1-\sin^2 t)\ \mathrm{d}u=k\int u^2(1-u^2)\ \mathrm{d}u$	M1
	$=k\left[\dfrac{u^3}{3}-\dfrac{u^5}{5}\right]$	dM1
	Volume = $48\pi\left[\dfrac{u^3}{3}-\dfrac{u^5}{5}\right]_0^{\frac{1}{2}}=\dfrac{17\pi}{10}$	ddM1A1
		(5)
		(9 marks)

(a)

B1: $x=\dfrac{3}{2}\Rightarrow a=\dfrac{\pi}{6}$. This may be seen as the top limit on the integral

M1: Attempts $y^2\times\dfrac{\mathrm{d}x}{\mathrm{d}t}$

It will usually be seen within $\int y^2\dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t\to\int(...\sin 2t)^2(\pm)\delta\cos t\mathrm{d}t$, but you can condone bracketing issues and the loss of the integral signs as well as the dt

dM1: Uses $\sin 2t=2\sin t\cos t$ within their answer to $\int y^2\dfrac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t\to\lambda\int\sin^2 t\cos^3 t\ \mathrm{d}t$

Note that the previous M mark must have been scored.

Condone $\sin^2 2t\to 2\sin^2 t\cos^2 t$ for this mark

The form of the answer is given so expect to see both the integral sign and the dt

A1: Volume = $48\pi\int_0^a \sin^2 t\cos^3 t\,\mathrm{d}t$. CSO

You can follow through on their value of *a*, $a \neq \frac{3}{2}$, but a value (not necessarily correct) must be present.

The form of the answer is given so any integrals should include d*x*'s and d*t*'s correctly placed

(b) Using the substitution

B1: States $\frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$ or equivalent. It can be awarded for replacing the $\mathrm{d}t$ by $\frac{\mathrm{d}u}{\cos t}$

M1: Substitutes fully using $u = \sin t$ producing an integral just in terms of *u*. Accept $\cos^2 t = \pm 1 \pm \sin^2 t$

So award for the form $\pm k\int u^2(\pm 1 \pm u^2)\,\mathrm{d}u$

dM1: Multiplies out to form a polynomial expression of the form $\pm au^2 \pm bu^4$ and integrates each term with correct powers for their expression

ddM1: All previous method marks must have been scored in (b).
It is for using correct limits within a suitable expression
It is for substituting $\left[\frac{1}{2}, 0\right]$ into an acceptable expression and subtracting either way around.

Alternatively, it is for substituting $\left[\frac{\pi}{6}, 0\right]$ into an acceptable expression in *t* if *u* is replaced by $\sin t$

A1: $V = \frac{17\pi}{10}$ oe such as $V = \frac{51\pi}{30}$

(b) Otherwise

B1: Writes their $k\int \sin^2 t\cos^3 t\,\mathrm{d}t$ as $k\int \sin^2 t\left(1-\sin^2 t\right)\cos t\,\mathrm{d}t$

M1: Multiplies out into a form that can be integrated $\pm ...\sin^2 t\cos t \pm ...\sin^4 t\cos t$ allowing for $\cos^2 t = \pm 1 \pm \sin^2 t$

dM1: Correct integration. Look for the form $...\sin^3 t \pm ...\sin^5 t$

ddM1: Dependent upon both previous M's in part (b).

Scored for substituting $\left[\frac{\pi}{6}, 0\right]$ into an acceptable expression in *t*

A1: $V = \frac{17\pi}{10}$ oe such as $V = \frac{51\pi}{30}$

Question Number	Scheme	Marks
7(a)	$\dfrac{1}{(4-x)(2-x)} = \dfrac{A}{(4-x)} + \dfrac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \ldots \text{ or } B = \ldots$	M1
	$A = -\dfrac{1}{2},\ B = \dfrac{1}{2}$ giving $\dfrac{-\frac{1}{2}}{(4-x)} + \dfrac{\frac{1}{2}}{(2-x)}$	A1
		[2]
(b)	$\dfrac{dx}{dt} = k(4-x)(2-x),\ t \geqslant 0 \Rightarrow \displaystyle\int \dfrac{1}{(4-x)(2-x)}\, dx = \int k\, dt$	
	$\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt\ (+c)$	M1, A1
	$\{t = 0, x = 0 \Rightarrow\}\ \dfrac{1}{2}\ln 4 - \dfrac{1}{2}\ln 2 = 0 + c\ \left\{\Rightarrow c = \dfrac{1}{2}\ln 2\right\}$	M1
	$\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt + \dfrac{1}{2}\ln 2 \Rightarrow \ln\left(\dfrac{4-x}{4-2x}\right) = 2kt$	
	$\dfrac{4-x}{4-2x} = e^{2kt}$	M1
	$4 - x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4 - 4e^{2kt} = x - 2xe^{2kt}$ $\Rightarrow 4 - 4e^{2kt} = x(1 - 2e^{2kt}) \Rightarrow x = \dfrac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$ (*)	dM1, A1*
		[6]
(c)	$1 = \dfrac{4 - 4e^{0.2t}}{1 - 2e^{0.2t}} \Rightarrow 2e^{0.2t} = 3$	M1
	$\Rightarrow 0.2t = \ln\dfrac{3}{2} \Rightarrow t = \text{awrt } 2.03$	A1
		[2]
		10 marks

(a)

M1**:** For forming a correct identity leading to one value of *A* or *B*.

For example, $1 \square A(2-x) + B(4-x)$ from $\dfrac{1}{(4-x)(2-x)} = \dfrac{A}{(4-x)} + \dfrac{B}{(2-x)}$ and finds at least one of $A = \ldots$ or $B = \ldots$.

This would be implied using cover up rule and reaching one correct fraction

A1: $\dfrac{-\frac{1}{2}}{(4-x)} + \dfrac{\frac{1}{2}}{(2-x)}$ or any equivalent form**. It is not scored for just the values**

ISW after sight of a correct answer. It may be awarded in part (b)

(b)

M1: Integrates to the form $\pm\lambda\ln(4-x) \pm \mu\ln(2-x),\ \lambda \neq 0,\ \mu \neq 0$

Note that versions such as $\pm\lambda\ln(8-2x) \pm \mu\ln(4-2x),\ \lambda \neq 0,\ \mu \neq 0$ ARE correct

A1: Fully correct integration of both left and right-hand side condoning a missing constant of integration.

Look for $\frac{1}{2}\ln(4-x)-\frac{1}{2}\ln(2-x)=kt$ o.e. such as $\frac{1}{2k}\left(\ln(4-x)-\ln(2-x)\right)=t$

Note that you may see $\frac{-\frac{1}{2}}{(4-x)}$ being integrated to $\frac{1}{2}\ln(8-2x)$

Similarly, $\frac{\frac{1}{2}}{(2-x)}$ being integrated to $-\frac{1}{2}\ln(4-2x)$. These are both correct

M1: Using both $t=0$ and $x=0$ in an integrated equation containing a constant of integration and finds the value of c. If the *k* is 'moved' to the LHS, c will be found in terms of *k*.
It is dependent upon having integrated to a form equivalent to
$\pm\lambda\ln(4-x)\pm\mu\ln(2-x)=f(t)+c$ o.e.

M1: Starting from an equation of the form $\pm\lambda\ln(4-x)\pm\mu\ln(2-x)=\pm kt+c$, $\lambda,\mu,k,c\neq 0$ o.e. and applies a fully correct method to eliminate their logarithms. To score this

- the lns must be correctly combined
- all exponential work must be correct at the point when the logarithms are removed

For example, $\ln(4-x)-\ln(2-x)=kt+\ln 2\Rightarrow(4-x)-(2-x)=e^{kt+\ln 2}$ is M0

AND $\ln(4-x)-\ln(2-x)=kt+\ln 2\Rightarrow\ln\frac{(4-x)}{(2-x)}=kt+\ln 2\Rightarrow\frac{(4-x)}{(2-x)}=e^{kt+\ln 2}$ is M1 BUT $\frac{(4-x)}{(2-x)}=e^{kt}+2$ is M0

There must be a constant of integration but it need not have been evaluated.
(See below for this type of solution)

dM1: D**ependent on the previous M mark.** A complete method of rearranging to make *x* the subject. It can be awarded when the two terms in *x* are on one side of the equation and the terms independent of *x* are on the other.
So, following the award of the previous mark, look for

- cross multiplication leading to two terms in *x* and two terms independent of *x*
- collecting two *x* terms one side of the equation and two terms independent of *x* on the other

There must be a constant of integration but it need not have been evaluated.
(See below for this type of solution)

A1*: CSO. **Note that this is a given answer and all steps should be seen before awarding this mark.**

Condone a line where a bracket is missed e.g. $\ln 2 - x \leftrightarrow \ln(2-x)$ as long as it is replaced in subsequent lines

If a candidate reaches $x - 2xe^{2kt} = 4 - 4e^{2kt}$ it is acceptable to just write down $x = \dfrac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$

However, $2xe^{2kt} - x = 4e^{2kt} - 4$ must proceed to the given answer via $x = \dfrac{4e^{2kt} - 4}{2e^{2kt} - 1}$ o.e. such as $(2e^{2kt} - 1)x = 4e^{2kt} - 4$ to score the A1*

Alt (b) and the order of scoring the marks

M1, A1: $\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt + c$

$\ln(4-x) - \ln(2-x) = 2kt + C$

$\ln\dfrac{(4-x)}{(2-x)} = 2kt + C \Rightarrow \dfrac{(4-x)}{(2-x)} = e^{2kt+C}$

3rd M1 scored at this stage

As $e^{2kt+C} = e^{2kt} \times e^{C} = Ae^{2kt}$

$(4-x) = (2-x)Ae^{2kt} \Rightarrow Axe^{2kt} - x = 2Ae^{2kt} - 4 \Rightarrow x = \dfrac{2Ae^{2kt} - 4}{Ae^{2kt} - 1}$

4th M1 scored at this stage

Uses $t = 0,\ x = 0 \Rightarrow 0 = \dfrac{2A-4}{A-1} \Rightarrow A = 2$

2nd M1 scored at this stage

(c)

M1: Substitutes $x = 1$ and $k = 0.1$ into the given equation in part (c) and proceeds to $pe^{0.2t} = q,\ pq > 0$

A1: Proceeds to awrt 2.03 (Note that M1 must have been awarded to award this mark)

Alternatives exist via $\dfrac{4-x}{4-2x} = e^{2kt}$ but essentially the same method applies

SC: Answers without (sufficient) working

– there may be more space than you need.

- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

An answer of 2.03 with little or no valid working meriting the M1 mark can score SC 1,0

Question Number	Scheme	Marks
8(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$ Substitutes $t = 3$ in $x = 3^2 + 2\times 3 = 15$ B (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
8(a) ALT	Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ B (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
(b)	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\dfrac{dy}{dx} = \dfrac{3t^2 - 9}{2t + 2} \Rightarrow \dfrac{dy}{dx} = \dfrac{9}{4}$ Uses their $\dfrac{9}{4}$ with $(15, 0)$ to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 **(4)**
(c)	Substitutes $x = t^2 + 2t,\ y = t^3 - 9t$, into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\dfrac{15}{4}$ Coordinates of X are $\left(\dfrac{105}{16}, -\dfrac{1215}{64}\right)$	M1 dM1 A1 dM1A1 **(5)** **(12 marks)**

(a)

M1: States $t^3 - 9t = 0$ o.e. such as $y = 0 \Rightarrow t = 3$

No working needs to be seen for solving the equation and other values may be present including 0 and -3

A1*: Substitutes $t = 3$ in $x = t^2 + 2t$ to show that the x coordinate of B is 15 (and the y coordinate is 0)

This can be stated e.g. sub $t = 3$ in $x = t^2 + 2t \Rightarrow x = 15$

or else you need see some working such as $9 + 6$, $3^2 + 2\times 3$ o.e

It is a given answer so **you must see evidence** followed by a statement B = (15, 0) o.e. such as B $x = 15, y = 0$

So just stating $t = 3$ followed by $x = 15$ and B = (15, 0) will score M1, A0*

B1: States $A = (3,0)$ (need not see working)

Alt (a)

C. M1: Sets $t^2 + 2t = 15 \Rightarrow t = 3$

A1*: Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ and makes a comment that B is on the x-axis

B1: States $A = (3,0)$ (need not see working)

(b)

M1: Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $t = \pm 3$

or t = – 5. $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be attempted via $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{\text{Quadratic function}}{\text{Linear function}}$

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x}$ = 9/4 following M1

dM1: Uses (15,0) with the value of their gradient found using a correct method to find an equation
of the tangent.
It is dependent upon the previous M mark

A1: Achieves $9x - 4y - 135 = 0$ or any integer multiple of this

(c)

M1: Substitutes $x = t^2 + 2t,\ y = t^3 - 9t$ into their $9x - 4y - 135 = 0$ to form a simplified cubic equation in just t

dM1: This is dependent upon the previous M mark. It is for an attempt at using a non-calculator method to solve a cubic equation using the fact that $t = 3$ is a root. So, look for an attempt to divide their simplified cubic equation by (t – 3) or $t^2 - 6t + 9$

A1: Achieves $t = -\frac{15}{4}$ o.e. following correct cubic and correct factorisation. Look for prior working of

- $(t^2 - 6t + 9)(4t + 15) = 0$, which may or may not be followed by $(t-3)^2(4t+15) = 0$
- $(t-3)(4t^2 + 3t - 45) = 0$ followed by a solution of the quadratic that could be via a calculator

Note that $4t^3 - 9t^2 - 54t + 135 = 0 \Rightarrow (t-3)^2(4t+15) = 0$ is dM0, A0 unless an explanation is given alluding to the fact that $t = 3$ is a double root, so $(t-3)^2$ is a factor, and so

$4t^3 - 9t^2 - 54t + 135 = (t-3)^2 (At + B) = 0$

giving A = 4 and B = 15 via inspection

dM1: Uses their value of t to find **either** the x or y co-ordinate of X.

It is dependent upon the FIRST M only and achieving a correct value of t for their equation.

It can be implied by a 'correct' value

A1: Coordinates of X are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$ or exact equivalent, for example, x= 6.5625, y = –18.984375

We are likely to see solutions without sight of the necessary steps required by the question

Solutions lacking necessary steps are scored as follows

Alternative 1; Via a simplified equation in t

Substitutes $x = t^2 + 2t,\ y = t^3 - 9t,$ into $9x - 4y - 135 = 0$

$\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$

$\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0 \Rightarrow t = -3.75\,.$

So, X = $\left(\frac{105}{16}, -\frac{1215}{64}\right)$

Scores SC 1, 0, 0, 1, 1

M1: method shown

dM0: method not shown as presumably done via a calculator

A0: correct value for t but following dM0

dM1: For one correct coordinate (allowing either of $(6.56, -18.98)$ to 2dp) as it implies a correct method

A1: For exact answer $\left(\frac{105}{16}, -\frac{1215}{64}\right)$

Alternative 2; Scores SC 0, 0, 0, 1, 1

No simplified equation in t

Substitutes $x = t^2 + 2t,\ y = t^3 - 9t,$ into $9x - 4y - 135 = 0$

$\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$

$\Rightarrow t = -3.75\,.$

So X = $\left(\frac{105}{16}, -\frac{1215}{64}\right)$ but allow $(6.56, -18.98)$ via this method

See Alternative 1 for the award of the final two marks

Question Number	Scheme	Marks
9(a)	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ Either $A: \lambda = 1$ or $B: \lambda = 3$ leading to $a = -3,\ b = 5$	 M1 A1, A1 **(3)**
(b)	$\overrightarrow{AB} = = \pm\left(('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ ' - 3\mathbf{j}' + 5\mathbf{k})\right);\ = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1 **(2)**
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix}4\\-3\\2\end{pmatrix} - \begin{pmatrix}1\\-3\\5\end{pmatrix} = \begin{pmatrix}3\\0\\-3\end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix}-3\\0\\3\end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix}4\\2\\-2\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-3\end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1 **(3)**
(d)	$\text{Area } CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ;\ = 3\sqrt{3}$	M1, A1 **(2)**
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} + 5\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}9\\1\\1\end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} + 5\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}9\\1\\1\end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1 **(4)** **(14 marks)**

(a)
M1: Finds the value of λ at *A* or *B* but is implied by a correct value for *a* or *b*
A1: At least one of *a* or *b* correct
A1: Both *a* and *b* correct

(b)
M1: Attempts to subtract the components either way around.

If no method is shown it is implied by two correct coordinates/components (±) for their *A* and *B*

A1: Correct vector and with correct notation. Allow column vector approach, so $\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ is fine but $\begin{pmatrix} 4\mathbf{i} \\ 2\mathbf{j} \\ -2\mathbf{k} \end{pmatrix}$ is not.

(c) M1: Attempts vector *AC* or *CA* by subtracting $\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$ and their $\begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix}$ either way around.

Condone the coordinate form here, so $(-3, 0, 3)$ for vector *AC*.

If the method is not explicitly seen then award for two correct components (f.t. on their *a*)

dM1: Uses the scalar product of $\pm\overrightarrow{AB}$ and $\pm\overrightarrow{AC}$ to find the value of $\cos C\hat{A}B$

To score this mark

- Both vectors, *AB* and *AC* must have been attempted via subtraction
- The scalar product must have been attempted correctly
- The magnitudes of the vectors must have initially been attempted via a correct method

Alternatively, the cosine rule could be used following the calculation of all three sides of triangle *ABC*

A1: Correctly proceeds to $C\hat{A}B = 30^\circ$ or $\frac{\pi}{6}$ radians. Note that $\cos C\hat{A}B = -\frac{\sqrt{3}}{2}$ followed by 150° or 30° is A0

(d)

M1: Applies $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin '30^\circ$ '. The magnitudes of the vectors must have been found using a correct method.

For angles that haven't exact trig ratios, some work must be seen or implied. The demand is for an exact value for the area, so if $\cos C\hat{A}B = k$ is found in part (c) then $\sin C\hat{A}B = \sqrt{1-k^2}$ must be attempted here.

A1: $3\sqrt{3}$ but must follow correct sides with angle *CAB* = 30° or 150°

(e) There are a few different ways to achieve these coordinates so look carefully at what has been attempted

The diagram may help interpret the method used. Note that 0,0,1,1 is not possible

M1: A correct method to find one possible position of point *D*

Method 1: via the equation of *l*

Substitutes $\lambda = 5$ ie. $\lambda = \lambda_B + (\lambda_B - \lambda_A)$ OR $\lambda = -3$ ie. $\lambda = \lambda_A - 2(\lambda_B - \lambda_A)$ into the equation of line *l*

Method II: via vector *AB*

Attempts $\overrightarrow{OB} + \overrightarrow{AB} = \begin{pmatrix} 5 \\ '-1' \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} =$ OR attempts $\overrightarrow{OA} + 2\times\overrightarrow{BA} = \begin{pmatrix} 1 \\ '-3' \\ 5 \end{pmatrix} + 2\times\begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix} =$

Amongst many other alternatives include $\overrightarrow{OA} + 2\times\overrightarrow{AB}$ and $\overrightarrow{OA} - 2\times\overrightarrow{AB}$ (Use handy diagram)

Method III: Via modulus approach

Letting $\overrightarrow{OD} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and using $(AD^2) = 4\times(AB^2)$

gives $(2\lambda - 2)^2 + (\lambda - 1)^2 + (-\lambda + 1)^2 = 4\times'24'$

This leads to $6\lambda^2 - 12\lambda + 6 = 96 \Rightarrow \lambda^2 - 2\lambda - 15 = 0 \Rightarrow \lambda = 5, -3$

The M's can only be scored if $(AD^2) = 4\times(AB^2)$ is applied with AD^2 and AB^2 found using the sum of the differences squared. The resulting λ's must then be substituted into

$\overrightarrow{OD} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

for the M's to be scored. It is unlikely, under this method for 1,1,0,0 to be scored.

Method IV: via an area approach

This is similar to method III except candidates start with

area $ACD = \frac{1}{2}\times AC\times AD\times\sin BAC = 2\times\text{"ans to }(d)\text{"}$

If the earlier parts of question are correct you should see

$\frac{1}{2}\times\sqrt{18}\times\sqrt{(2\lambda-2)^2 + (\lambda-1)^2 + (1-\lambda)^2}\times\frac{1}{2} = 2\times3\sqrt{3}$ leading to $(2\lambda-2)^2 + (\lambda-1)^2 + (1-\lambda)^2 = 96$ as before. Correct processing when forming the equation in lambda is expected via this method

As with method III you should expect to see the squaring applied correctly and M's are only awarded when the resulting λ's are substituted into $\overrightarrow{OD} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

A1: One correct coordinate, either $D(9,1,1)$ **or** $D'(-7,-7,9)$ but condone the vector forms

M1: A correct method to find BOTH possible positions of point *D.*

Note that via method III both values of λ are found when the equation is solved

A1: Both possible coordinates, $D(9,1,1)$ **and** $D'(-7,-7,9)$ but condone the vector forms

Alternative route for (c) and (d)

Triangle *ABC* can be shown to be right angled at C via use of Pythagoras. This will only work if they have the correct values of *a* and *b*.

So as $\left(\sqrt{24}\right)^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{6}\right)^2 \Rightarrow$ Angle C is $90°$

Hence angle *A* = invsin $\frac{\sqrt{6}}{\sqrt{24}} = \frac{1}{2}$ which gives angle $A = 30°$

AND area *ABC* = $\frac{1}{2} \times \sqrt{18} \times \sqrt{6} = 3\sqrt{3}$

Score (c)

M1: For $\overrightarrow{AC}$ or $\overrightarrow{CA}$ which may be implied within an attempt at $\left|\overrightarrow{AC}\right|$

M1: Proves angle *A* is a right angle via Pythagoras' Theorem and uses a correct trigonometric identity (implied)

A1: Correct answer

Part (d)

M1: Use of $\frac{1}{2}bh$ but it must follow a proof of angle *A* being a right angle

A1: Correct answer