Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 28 October 2024

Morning (Time: 1 hour 30 minutes) | Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$(8 - 3x)^{-\frac{1}{3}} \qquad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

3.

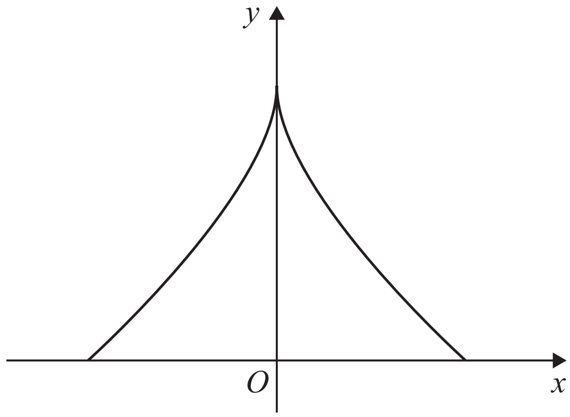

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k\operatorname{cosec}\theta \qquad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where q is a constant to be found.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

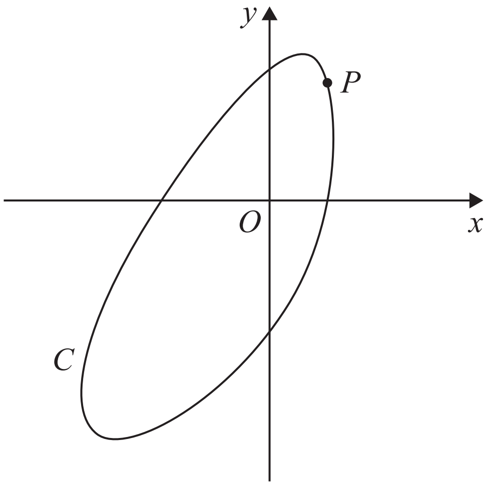

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$3x^2 + 2y^2 - 4xy + 8^x - 11 = 0$$

The point P has coordinates (1, 2).

(a) Verify that P lies on C.

(1)

(b) Find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The normal to C at P crosses the x-axis at a point Q.

(c) Find the x coordinate of Q, giving your answer in the form $a + b\ln 2$ where a and b are integers.

(3)

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P 7 6 1 8 9 A 0 1 0 3 2

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

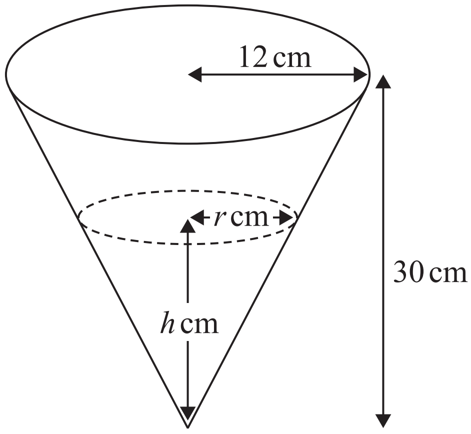

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of 2π cm^3 s^{-1}

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

Question 5 continued

(Total for Question 5 is 6 marks)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}}\,dx = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

Question 6 continued

(Total for Question 6 is 5 marks)

7.

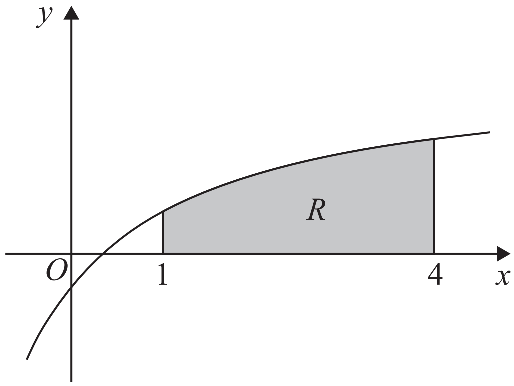

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x-1}{x+2} \qquad x > -2$$

(a) Show that

$$\frac{3x-1}{x+2} \equiv A + \frac{B}{x+2}$$

where A and B are constants to be found.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 1$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q\ln 2)$$

where p and q are rational constants.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(2)

Question 9 continued

Question 9 continued

P 7 6 1 8 9 A 0 2 8 3 2

Question 9 continued

(Total for Question 9 is 10 marks)

10.

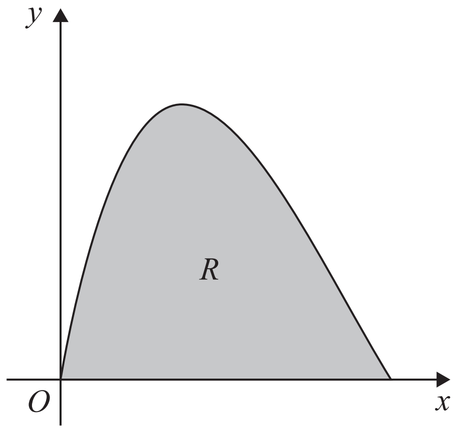

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, \mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS