# Mark Scheme (Results)

# October 2025

International Advanced Level in Pure Mathematics P3

WMA13/01A

# Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus

## Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

October 2025
Question Paper Log Number P87435A
Publication Code WMA13_01A_2510_MS

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.
3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - * – The answer is printed on the paper or ag- answer given
   - $\sqsubseteq$ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

# General Principles for Pure Mathematics Marking

*(NB specific mark schemes may sometimes override these general principles)*

**Method mark for solving 3 term quadratic:**

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$ : $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \ldots$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1 ( $x^n \to x^{n-1}$ )

2. Integration

Power of at least one term increased by 1 ( $x^n \to x^{n+1}$ )

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

**Normal marking procedure is as follows:**

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question Number | Scheme | Marks |
|---|---|---|
| **1(i)(a)** | $\int(2x+3)^{12}\mathrm{d}x=\dfrac{(2x+3)^{13}}{26}(+c)$ | M1 A1 |
| | | **(2)** |
| **(i)(b)** | $\int\dfrac{5x}{4x^2+1}\mathrm{d}x=\dfrac{5}{8}\ln\left(4x^2+1\right)(+c)$ | M1 A1 |
| | | **(2)** |

**(i) Notes**

**(a)**

**M1:** Correct form for the integration. Look for $\lambda(2x+3)^{13}$ or e.g. $\mu\left(x+\dfrac{3}{2}\right)^{13}$

Allow the index unprocessed e.g. $\lambda(2x+3)^{12+1}$

May see substitution e.g. $u=2x+3$ leading to $\lambda u^{13}$ and this would imply this mark.

**A1:** Correct integration with or without the + $c$.

Allow any equivalent correct expression simplified or unsimplified and apply isw once a correct answer is seen.

Award for e.g. $\dfrac{(2x+3)^{13}}{26}$ or $\dfrac{(2x+3)^{13}}{2\times13}$ or $\dfrac{2^{12}\left(x+\dfrac{3}{2}\right)^{13}}{13}$ or $\dfrac{4096\left(x+\dfrac{3}{2}\right)^{13}}{13}$

The index must be processed and any constants evaluated e.g. 13 not 12 + 1 but e.g. $2\times13$ is acceptable.

Ignore any spurious integral signs and/or d$x$'s and just look for a correct expression.

**(b)**

**M1:** Correct form of integration. Look for $...\ln\left(4x^2+1\right)$ or $...\ln\left|4x^2+1\right|$ but condone missing brackets e.g. $...\ln 4x^2+1$

Can also be awarded for $...\ln k\left(4x^2+1\right)$ or $...\ln k\left|4x^2+1\right|$

May see substitution e.g. $u=4x^2+1$ leading to $...\ln u$ and this would imply this mark.

**A1:** Correct integration with or without the + $c$.

Allow any equivalent correct expression simplified or unsimplified and apply isw once a correct answer is seen.

Brackets must be present and allow $\dfrac{5}{8}\ln\left(4x^2+1\right)(+c)$ or $\dfrac{5}{8}\ln\left|4x^2+1\right|(+c)$

Note that e.g. $\dfrac{5}{8}\ln\left(x^2+\dfrac{1}{4}\right)(+c)$ or $\dfrac{5}{8}\ln\left|x^2+\dfrac{1}{4}\right|(+c)$ are also correct.

Allow equivalents for $\dfrac{5}{8}$ e.g. 0.625 or e.g. $\dfrac{1}{1.6}$

Ignore any spurious integral signs and/or d$x$'s and just look for a correct expression.

| | | |
|---|---|---|
| **(ii)** | $\int \frac{t+1}{t}\,\mathrm{d}t = \int \left(1+\frac{1}{t}\right)\mathrm{d}t = t + \ln t\,(+c)$ | M1 A1 |
| | $[t+\ln t]_a^{2a} = 2a + \ln 2a - (a + \ln a) = \ln 7$ | **d**M1 |
| | $a + \ln 2 = \ln 7 \Rightarrow a = \ln\frac{7}{2}$ or $\ln 7 - \ln 2$ | A1 |
| | | **(4)** |
| | | **(8 marks)** |

**(ii) Notes**

**M1:** Writes $\frac{t+1}{t}$ as $...+\frac{1}{t}$ where $\ldots \neq 0$ and integrates $\frac{1}{t} \to \ln t$

**A1:** Correct integration with or without an arbitrary constant.

**dM1:** Applies the limits $a$ and $2a$ the right way round, subtracts and sets equal to $\ln 7$
Condone poor bracketing e.g. $2a + \ln 2a - a + \ln a = \ln 7$
**Depends on the first method mark.**

**A1:** $a = \ln\frac{7}{2}$ or e.g. $\ln 3.5$
Condone $a = \ln 7 - \ln 2$ but if this becomes $\ln 5$ score A0
Condone clear confusion with the variable if the intention is clear e.g.

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \int_a^{2a}\left(1+\frac{1}{t}\right)\mathrm{d}t = [x+\ln t]_a^{2a} \to 2a + \ln 2a - (a+\ln a) = \ln 7 \text{ etc.}$$

**Alternative attempts by parts for the first 2 marks:**

| | | |
|---|---|---|
| **I** | $\int \frac{t+1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right) - \int -\frac{1}{t^2}\left(\frac{t^2}{2}+t\right)\mathrm{d}t$ | M1 |
| | $= \frac{1}{t}\left(\frac{t^2}{2}+t\right) + \int \frac{1}{2} + \frac{1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right) + \frac{t}{2} + \ln t\,(+c)$ | A1 |

**Notes**

**M1:** Award for $\int \frac{t+1}{t}\,\mathrm{d}t = \frac{1}{t}\left(\frac{t^2}{2}+t\right) - \int -\frac{1}{t^2}\left(\frac{t^2}{2}+t\right)\mathrm{d}t$
Must be correct as shown.

**A1:** Fully correct integration. $\frac{1}{t}\left(\frac{t^2}{2}+t\right) + \frac{t}{2} + \ln t\,(+c)$ or equivalent.

| | | |
|---|---|---|
| **II** | $\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t - \int \ln t\,\mathrm{d}t$ | M1 |
| | $\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t - t\ln t + t\,(+c)$ | A1 |

**Notes**

**M1:** Award for $\int \frac{t+1}{t}\,\mathrm{d}t = (t+1)\ln t - \int \ln t\,\mathrm{d}t$
Must be correct as shown.

**A1:** Fully correct integration. $(t+1)\ln t - t\ln t + t\,(+c)$ or equivalent.

| Question Number | Scheme | Marks |
|---|---|---|
| **2(a)** | $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ | M1A1A1 |
| | $x=\alpha\Rightarrow 4\alpha\tan\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}\alpha\right)=0$ <br> $\Rightarrow 8\alpha\frac{\sin\left(\frac{1}{2}\alpha\right)}{\cos\left(\frac{1}{2}\alpha\right)}+(2\alpha^2-3)\times\frac{1}{\cos^2\left(\frac{1}{2}\alpha\right)}=0$ | M1 |
| | $8\alpha\sin\left(\frac{1}{2}\alpha\right)\cos\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)=0$ <br> $4\alpha\sin\alpha+(2\alpha^2-3)=0$ | **d**M1 |
| | $2\alpha^2-3+4\alpha\sin\alpha=0\,*$ | A1* |
| | | **(6)** |

**Notes**

**Brackets around the $\frac{1}{2}x$ are not required.**

**M1**: Attempts the product rule on $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)$ or on $y=2x^2\tan\left(\frac{1}{2}x\right)$ if they multiply out.
It is for **either** $(2x^2-3)\tan\left(\frac{1}{2}x\right)\rightarrow Ax\tan\left(\frac{1}{2}x\right)+B(2x^2-3)\sec^2\left(\frac{1}{2}x\right),\ A,B>0$
**or** for $2x^2\tan\left(\frac{1}{2}x\right)\rightarrow Ax\tan\left(\frac{1}{2}x\right)+Bx^2\sec^2\left(\frac{1}{2}x\right),\ A,B>0$

**A1:** One term correct: of $4x\tan\left(\frac{1}{2}x\right)$ or $(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ or $(2x^2)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$

**A1:** e.g. $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$.

A fully correct unsimplified or simplified derivative. $\frac{\mathrm{d}y}{\mathrm{d}x}=$ is not required.

**M1:** They need to have a derivative with both $\tan\left(\frac{1}{2}x\right)$ and $\sec^2\left(\frac{1}{2}x\right)$. It is for using

$\tan\left(\frac{1}{2}x\right)=\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and $\sec^2\left(\frac{1}{2}x\right)=\frac{1}{\cos^2\left(\frac{1}{2}x\right)}$ or $\sec^2\left(\frac{1}{2}x\right)=1+\tan^2\left(\frac{1}{2}x\right)=1+\frac{\sin^2\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$

and setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ which may be implied. This mark is for converting to an equation in $\sin\left(\frac{1}{2}x\right)$ and $\cos\left(\frac{1}{2}x\right)$ only using the correct identities which may be implied by their work.

**dM1:** Uses the correct identity $2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$.
This may be implied by their work but if the identity $\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$ is suggested, score dM0.
**Depends on the previous method mark.**

**A1*:** $2\alpha^2-3+4\alpha\sin\alpha=0$ (Allow $4\alpha\sin\alpha+2\alpha^2-3=0$).
This is a printed answer so must follow all previous marks.
There must have been no errors, including bracketing errors.
May work in $x$ or $\alpha$ (or $a$) or a mixture of both but must be $\alpha$ or $a$ for final A1

| | | | |
|---|---|---|---|
| **(b)** | $x_2 = \dfrac{3}{(2\times 0.7 + 4\sin 0.7)}$ | M1 |
| | $x_2 = 0.7544,\ x_3 = 0.7062$ | A1 |
| | | **(2)** |

**Notes**

**(b)**

**M1:** For substituting $x_1 = 0.7$, into the given iterative formula to find a value for $x_2$.

It may be implied by the sight of $x_2 = \dfrac{3}{(2\times 0.7 + 4\sin 0.7)} = \ldots$ or $x_2 =$ awrt 0.75 and also (if degrees were used) $x_2 =$ awrt 2.1

Do not condone misreads of the given iteration formula.

**A1:** $x_2 = 0.7544,\ x_3 = 0.7062$ (both awrt 4 dp)

Remember to isw so allow e.g. $x_2 = 0.75436 = 0.7543,\ x_3 = 0.7062$

| | | |
|---|---|---|
| **(c)** | Chooses interval e.g. $[0.72825, 0.72835]$ AND function, e.g. $f(x) = 2x^2 - 3 + 4x\sin x$ and attempts the value of $f(0.72825)$ and $f(0.72835)$ | M1 |
| | $2\times 0.72825^2 - 3 + 4\times 0.72825\sin 0.72825 = -0.0005$<br>$2\times 0.72835^2 - 3 + 4\times 0.72835\sin 0.72835 = 0.00026$<br>+ reason + conclusion | A1 |
| | | **(2)** |
| | | **(10 marks)** |

**Notes**

**(c)**

**M1:** Chooses

- A suitable interval e.g. $[0.72825, 0.72835]$
- A suitable function and attempts to find the value of the function at both ends.

The function may be implied by embedded values e.g.

$2(0.72825)^2 - 3 + 4(0.72825)\sin(0.72825)$ and $2(0.72835)^2 - 3 + 4(0.72835)\sin(0.72835)$

or by their working/values e.g. $0.72825 \to \frac{dy}{dx} = \ldots,\quad 0.72835 \to \frac{dy}{dx} = \ldots,$

Accept as suitable functions, $\pm(2\alpha^2 - 3 + 4\alpha\sin\alpha)$, $\pm\left(x - \frac{3}{(2x + 4\sin x)}\right)$, their $\frac{dy}{dx}$

Allow a smaller interval containing the root.

The interval does not have to be stated so the values may be seen at the point of substitution.

Condone the use of degree mode – see table below for values.

**A1:** Achieves

- Correct values to 1 sf rounded or truncated
- Gives a valid reason e.g. change of sign (or equivalent e.g. > 0, < 0)
- Minimal conclusion such as "hence root", "turning point", "proven"

There is no requirement to reference continuity and any mention of it can be ignored.

**For reference:**

| Function f | f (0.72825) | f (0.72835) |
|---|---|---|
| $2x^2 - 3 + 4x\sin x$ | −0.00051431…<br>(Degrees = −1.90227…) | 0.00026066…<br>(Degrees = −1.90197…) |
| $x - \frac{3}{(2x + 4\sin x)}$ | −0.00012487…<br>(Degrees = −1.26201…) | 0.000063278…<br>(Degrees = −1.26163…) |
| $\frac{dy}{dx}$ | −0.0002945…<br>(Degrees = −0.95117…) | 0.000149267<br>(Degrees = −0.95102…) |

**Attempts at repeated iteration score no marks in (c).**

| Question Number | Scheme | Marks |
|---|---|---|
| 3(a) | $$\begin{array}{r} x^2+5 \\ x^2+x-12\overline{)x^4+x^3-7x^2+8x-48} \\ \underline{x^4+x^3-12x^2} \\ 5x^2+8x-48 \\ \underline{5x^2+5x-60} \\ 3x+12 \end{array}$$ | M1A1 |
| | $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv x^2+5+\dfrac{3(x+4)}{(x+4)(x-3)}$ | M1 |
| | $\equiv x^2+5+\dfrac{3}{(x-3)}$ | A1 |
| | | **(4)** |

**Notes**

**M1:** Divides $x^4+x^3-7x^2+8x-48$ by $x^2+x-12$ to obtain a quadratic quotient and a linear remainder $\alpha x+\beta$ with $\alpha$ and $\beta$ non-zero.

**A1:** Correct quotient $x^2+5$ and correct remainder $3x+12$.

**M1:** Writes their answer as $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}=\text{Their Quotient}+\dfrac{\text{Their Remainder}}{(x+4)(x-3)}$

This may be implied by a correct expression following a correct quotient and remainder or by correct values for $A$ and $B$ as long as there is no incorrect work.

**A1:** Correct expression $x^2+5+\dfrac{3}{(x-3)}$ or states $A = 5$, $B = 3$

If they obtain $x^2+5+\dfrac{3}{(x+4)}$ this scores A0, even if they subsequently state $A = 5$, $B = 3$

| | **(a) Alternative 1 comparing coefficients/substitution** | |
|---|---|---|
| | $x^4+x^3-7x^2+8x-48\equiv(x^2+A)(x^2+x-12)+B(x+4)$<br>or<br>$x^4+x^3-7x^2+8x-48\equiv x^2(x^2+x-12)+A(x^2+x-12)+B(x+4)$ | M1 |
| | 2 correct equations from comparing coefficients or substituting values:<br>e.g. $x^2: A-12=-7,\quad x: A+B=8,\quad \text{const}: -12A+4B=-48$<br>or e.g. $x=3\Rightarrow 21=7B,\quad x=0\Rightarrow -48=-12A+4B$ | A1 |
| | $\Rightarrow A=\ldots$ or $B=\ldots$ | M1 |
| | $A=5, B=3$ | A1 |

**Notes**

**M1:** Multiplies up by $x^2+x-12$ or e.g. $(x-3)(x+4)$ to obtain a correct lhs and one of $(x^2+A)(x^2+x-12)$ or $B(x+4)$ on the rhs.

If $(x^2+A)(x^2+x-12)$ is expanded must see both $x^2(x^2+x-12)$ and $A(x^2+x-12)$

**A1:** Compares 2 coefficients (or may substitute 2 values) and obtains 2 correct equations.

**M1:** Solves to obtain one of $A$ or $B$.

**A1:** Both values correct.

| | (a) Alternative 2 repeated division I | |
|---|---|---|
| | $\left(x^4+x^3-7x^2+8x-48\right)\div(x-3)\to Q_1=x^3+4x^2+5x+23,\ R_1=21$<br>then<br>$\left(x^3+4x^2+5x+23\right)\div(x+4)\to Q_2=x^2+5,\ R_2=3$ | M1A1 |
| | $\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)(x-3)=x^2+5+\dfrac{21}{(x+4)(x-3)}+\dfrac{3}{x+4}$ | M1 |
| | $=x^2+5+\dfrac{21+3x-9}{(x+4)(x-3)}=x^2+5+\dfrac{3(x+4)}{(x+4)(x-3)}=x^2+5+\dfrac{3}{(x-3)}$ | A1 |

**Notes**

**M1:** Divides by $(x-3)$ first to give a cubic quotient $Q_1$ and constant remainder $R_1$.

Then divides $Q_1$ by $(x+4)$ to give a quadratic quotient $Q_2$ and constant remainder $R_2$.

**A1:** For $Q_1=x^3+4x^2+5x+23,\ R_1=21,\ Q_2=x^2+5,\ R_2=3$

**M1:** Writes g($x$) as $Q_2+\dfrac{R_1}{(x+4)(x-3)}+\dfrac{R_2}{x+4}$

**A1:** Correct expression or correct values for $A$ and $B$.

| | (a) Alternative 3 repeated division II | |
|---|---|---|
| | $\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)\to Q_1=x^3-3x^2+5x-12,\ R_1=0$<br>then<br>$\left(x^3-3x^2+5x-12\right)\div(x-3)\to Q_2=x^2+5,\ R_2=3$ | M1A1 |
| | $\left(x^4+x^3-7x^2+8x-48\right)\div(x+4)(x-3)=x^2+5+\dfrac{3}{x-3}$ | M1A1 |

**Notes**

**M1:** Divides by $(x+4)$ first to give a cubic quotient $Q_1$ and constant remainder $R_1$.

Then divides $Q_1$ by $(x-3)$ to give a quadratic quotient $Q_2$ and constant remainder $R_2$.

**A1:** For $Q_1=x^3-3x^2+5x-12,\ R_1=0,\ Q_2=x^2+5,\ R_2=3$

**M1:** Writes g($x$) as $Q_2+\dfrac{R_1}{(x+4)(x-3)}+\dfrac{R_2}{x-3}$

**A1:** Correct expression or correct values for $A$ and $B$.

| | (a) Alternative 4 substituting values | |
|---|---|---|
| | $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12}\equiv x^2+A+\dfrac{B}{(x-3)}$<br>e.g. $x=0\Rightarrow 4=A-\dfrac{B}{3}\ :\ x=1\Rightarrow\dfrac{45}{10}=1+A-\dfrac{B}{2}$ | M1A1 |
| | $4=A-\dfrac{B}{3}\ :\ \dfrac{45}{10}=1+A-\dfrac{B}{2}\Rightarrow A=...\text{ or }B=...$ | M1 |
| | $A=5,\ B=3$ | A1 |

**Notes**

**M1:** Substitutes 2 values for $x$

**A1:** Any 2 correct equations.

**M1:** Solves to obtain one of $A$ or $B$.

**A1:** Correct values for $A$ and $B$.

| | | |
|---|---|---|
| **(b)** | $$g'(x) = 2x - \frac{3}{(x-3)^2}$$ **or** $$g'(x) = \frac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$$ | M1A1ft |
| | $x = 4 \Rightarrow g'(4) = 2\times4 - \frac{3}{(4-3)^2} = 5$ | M1 |
| | $x = 4 \Rightarrow g(4) = 4^2 + 5 + \frac{3}{4-3} (= 24)$ <br> $\Rightarrow y - 24 = 5(x-4)$ **or e.g.** $y = 5x + c \Rightarrow 24 = 5\times4 + c \Rightarrow c = ...$ | **d**M1 |
| | $y = 5x + 4$ | A1 |
| | | **(5)** |
| | | **(9 marks)** |

**Notes**

**Note that candidates should not be using a calculator in this part so if there is no algebraic differentiation, they will generally score no marks.**

**M1:** Differentiates their $x^2 + A + \frac{B}{x-3}$ to obtain $2x \pm \frac{B}{(x-3)^2}$ oe e.g. $2x \pm B(x-3)^{-2}$

**or** differentiates $\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12}$ to obtain an expression of the form

$$g'(x) = \frac{\left(x^2+x-12\right)\times \text{a cubic polynomial} - \left(x^4+x^3-7x^2+8x-48\right)\times \text{a 2 term linear expression}}{\left(x^2+x-12\right)^2}$$

**A1ft:** $x^2 + A + \frac{B}{x-3} \to 2x - \frac{B}{(x-3)^2}$. Follow through on their *B* or allow the letter *B* or a made up *B*.

or $g'(x) = \frac{\left(x^2+x-12\right)\left(4x^3+3x^2-14x+8\right)-\left(x^4+x^3-7x^2+8x-48\right)(2x+1)}{\left(x^2+x-12\right)^2}$

which must be correct (nothing to follow through).

**M1:** Subtitutes $x = 4$ into their $g'(x)$ in an attempt to find a numerical gradient.
May be implied by their $g'(4)$ following an attempt to differentiate.

**dM1:** For the correct method of finding an equation of the tangent with

- $x = 4$
- *y* an attempt at g(4) which may be implied by their value or by 24
- the gradient an attempt at $g'(4)$ following an attempt to differentiate
- the values correctly placed

If $y = mx + c$ is used they must proceed to a value for *c*.
**Depends on the previous method mark.**

**A1:** $y = 5x + 4$ cao and **cso**
Note that an incorrect value for *A* in part (a) will give the correct answer in (b) but this mark is cso and should be withheld in such cases.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **4(a)** | $y=\ln(1-\cos 2x)\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2\sin 2x}{1-\cos 2x}$ | M1A1 |
| | $\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ | M1 |
| | $=\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$ | A1 |
| | | **(4)** |

**Notes**

**Note that "$\frac{\mathrm{d}y}{\mathrm{d}x}=$" is not required apart from in alternative 3 below.**

**M1:** For differentiating $\ln(1-\cos 2x)$ to obtain $\frac{\pm A\sin 2x}{1-\cos 2x}$

**A1:** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2\sin 2x}{1-\cos 2x}$ oe

**M1:** Uses the correct double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$
The identity for $\cos 2x$ may be implied by poor bracketing in the denominator e.g. $1-1-2\sin^2 x$

**A1:** Simplifies to obtain $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2\cot x$ following M1A1M1 previously.

There must be at least one intermediate correct line between $\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$ which will usually be $\frac{4\sin x\cos x}{2\sin^2 x}$ or e.g. $\frac{2\cos x}{\sin x}$

| | **(a) Alternative 1** | |
| --- | --- | --- |
| | $y=\ln(1-\cos 2x)=\ln\left(2\sin^2 x\right)\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{4\sin x\cos x}{2\sin^2 x}$ | M1A1 |
| | $\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$ | M1A1 |

**Notes**

**M1:** Writes $\ln(1-\cos 2x)$ as $\ln(2\sin^2 x)$ and differentiates to obtain $\frac{\pm A\sin x\cos x}{\sin^2 x}$

**A1:** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{4\sin x\cos x}{2\sin^2 x}$ oe

**M1:** Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\pm A\sin x\cos x}{\sin^2 x}$ and simplifies to obtain $k\cot x$

**A1:** Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2\cot x$ following M1A1M1 previously.

| | (a) Alternative 2 | |
|---|---|---|
| | $y=\ln(1-\cos 2x)=\ln\left(2\sin^2 x\right)=\ln 2+2\ln\sin x \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)0+\frac{2\cos x}{\sin x}$ | M1A1 |
| | $\frac{2\cos x}{\sin x}=2\cot x$ | M1A1 |

**Notes**

**M1:** Writes $\ln(1-\cos 2x)$ as $\ln(2\sin^2 x)$ then $\ln 2+2\ln\sin x$ and differentiates to obtain $\frac{\pm A\cos x}{\sin x}$

**A1:** $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2\cos x}{\sin x}$ oe

**M1:** Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{\pm A\cos x}{\sin x}$ and simplifies to obtain $k\cot x$

**A1:** Obtains $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2\cot x$ following M1A1M1 previously.

| | (a) Alternative 3 | |
|---|---|---|
| | $y=\ln(1-\cos 2x)\Rightarrow \mathrm{e}^y=1-\cos 2x \Rightarrow \mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x$ | M1A1 |
| | $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin 2x}{\mathrm{e}^y}=\frac{2\sin 2x}{1-\cos 2x}=\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ | M1 |
| | $\frac{4\sin x\cos x}{2\sin^2 x}=2\cot x$ | A1 |

**Notes**

**M1:** Writes $y=\ln(1-\cos 2x)$ as $\mathrm{e}^y=1-\cos 2x$ and differentiates to obtain $\mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=A\sin 2x$

**A1:** $\mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=2\sin 2x$ oe

**M1:** Divides by $\mathrm{e}^y=1-\cos 2x$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\pm A\sin 2x}{1-\cos 2x}$ and uses the correct double angle identities $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$
The identity for $\cos 2x$ may be implied by poor bracketing in the denominator e.g. $1-1-2\sin^2 x$

**A1:** Simplifies to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cot x$ following M1A1M1 previously.

There must be at least one intermediate correct line between $\frac{4\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$ and $2\cot x$ which will usually be $\frac{4\sin x\cos x}{2\sin^2 x}$ or e.g. $\frac{2\cos x}{\sin x}$

| | | |
|---|---|---|
| **(b)** | $2\cot x = 2\sqrt{3} \Rightarrow \tan x = \frac{1}{\sqrt{3}}$<br>$x = \arctan\left(\frac{1}{\sqrt{3}}\right) \Rightarrow x = \frac{\pi}{6}$ | M1A1 |
| | $y = \ln\left(1-\cos\left(\frac{2\pi}{6}\right)\right) = \ln\frac{1}{2}$ or $-\ln 2$ | M1A1 |
| | | **(8 marks)** |

**Notes**

**If $k$ = 2 is obtained fortuitously or guessed in part (a) do not allow the A marks in part (b).**

**M1:** Uses $\cot x = \frac{1}{\tan x}$ to find $\tan x$ and then arctan to find $x$ which may be implied by their value of $x$. Note that some candidates may not be able to find a value of $x$ depending on their value of $k$ from part (a) so allow this mark for e.g. $k\cot x = 2\sqrt{3} \Rightarrow \tan x = \frac{k}{2\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\frac{k}{2\sqrt{3}}\right)$ with their numerical $k$.

**A1:** $x = \frac{\pi}{6}$ following $k$ = 2 obtained correctly in part (a). (Do not accept $30°$ for this mark)

Must be evaluated e.g. not $x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Ignore additional (incorrect) values such as $x = \frac{5\pi}{6}$

**M1:** Substitutes their value of $x$ in $y = \ln(1-\cos 2x)$ to find the $y$ coordinate.

May be implied by their value and allow e.g. $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\frac{k}{2\sqrt{3}}\right)\right)\right)$ with their numerical $k$.

**A1:** $y = \ln\frac{1}{2}$ or $-\ln 2$ following $x = \frac{\pi}{6}$ or 30° from fully correct work including $k$ = 2 obtained correctly in part (a).

Must be evaluated e.g. not $y = \ln\left(1-\cos 2\left(\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)\right)$.

Withhold this mark if other values of $x$ and $y$ are given within the range.

| | | |
|---|---|---|
| | | **(4)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **5(a)** | $2\operatorname{cosec}2A = \dfrac{2}{\sin 2A}$ | B1 |
| | $\dfrac{2}{\sin 2A} - \cot A = \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A}$ | M1 |
| | $= \dfrac{2-2\cos^2 A}{2\sin A\cos A}$ | M1 |
| | $= \dfrac{2\sin^2 A}{2\sin A\cos A} = \dfrac{\sin A}{\cos A} = \tan A \quad *$ | A1* |
| | | **(4)** |

**<u>Notes</u>**

**Condone use of a different variable e.g. $\theta$ or $x$ for $A$.**

**Condone the use of mixed variables for the B and M marks as long as the intention is clear.**

**B1:** Writes $\operatorname{cosec}2A$ as $\dfrac{1}{\sin 2A}$

**M1:** Uses $\sin 2A = 2\sin A\cos A$ or e.g. $\sin 2A = \sin A\cos A + \cos A\sin A$ and $\cot A = \dfrac{\cos A}{\sin A}$ to write the lhs in terms of sin $A$ and cos $A$ only.

**M1:** For writing the lhs as a single fraction in terms of sin $A$ and cos $A$ only.

It is for writing $\dfrac{a}{b\sin A\cos A} - \dfrac{\cos A}{\sin A}$ as a single fraction with a correct common denominator (which may not be the lowest) and 2 terms in the numerator, one of which must have been "modified".

e.g. $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{2\sin A - 2\sin A\cos^2 A}{2\sin^2 A\cos A}$

is acceptable without a lowest common denominator

e.g. $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A - 2\cos^2 A}{2\sin A\cos A}$

is acceptable with a correct denominator and 2 terms in the numerator of which at least 1 has been modified

**A1*:** Completes the proof with no errors.

The 2's must be cancelled at some point but it is ok for them to just "disappear".

The $1-\cos^2 A$ in the numerator must be replaced with $\sin^2 A$ and the expression $\dfrac{\sin A}{\cos A}$ must be seen before being replaced with tan $A$ but allow this appear in the form of cancelling e.g. $\dfrac{\cancel{2}\sin^{\cancel{2}} A}{\cancel{2}\cancel{\sin A}\cos A}$. For this mark condone minor notational errors (e.g. a missing $A$) but not consistent poor notation throughout such as mixed variables or writing $\cos^2 A$ as $\cos A^2$.

**For the final 2 marks, allow a "meet in the middle" approach**

e.g. sets $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A}{\cos A}$

$\times 2\sin A\cos A \rightarrow 2-2\cos^2 A = 2\sin^2 A \Rightarrow 2\sin^2 A = 2\sin^2 A$ Hence proven.

Score the first B1M1 as in the main scheme and then M1 for e.g. multiplying through by $2\sin A\cos A$ and A1* for a fully correct proof with a (minimal) conclusion.

| | | |
|---|---|---|
| **(b)(i)** | $2\text{cosec}4\theta - \cot 2\theta = \sqrt{3} \Rightarrow \tan 2\theta = \sqrt{3} \Rightarrow \theta = ...$ | M1 |
| | $\Rightarrow \theta = \dfrac{\arctan\sqrt{3}}{2} = \dfrac{\pi}{6}$ or awrt 0.524 | A1 |

**M1:** Uses part (a) to write the given equation in form $\tan 2\theta = \sqrt{3}$ and proceeds to a value for $\theta$ by finding arctan of $\sqrt{3}$ and dividing by 2.
It may be implied by e.g. $A = 2\theta \to \tan A = \sqrt{3} \Rightarrow A = ...$ as long as they then find a value for $\theta$ by finding arctan of $\sqrt{3}$ and dividing by 2.
If a candidate restarts then they must reach $\tan 2\theta = \sqrt{3}$

**A1:** Achieves $\theta = \dfrac{\pi}{6}$ or accept awrt 0.524 but not 30°
Ignore extra solutions outside the range but withhold this mark if there are extra solutions in range. Must follow the award of M1 so correct answer only scores no marks.

| | | |
|---|---|---|
| **(ii)** | $\tan\theta + \cot\theta = 5 \Rightarrow \sin 2\theta = \dfrac{2}{5}$ | M1 |
| | $\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{2}{5}\right) =$ awrt 0.206,1.37 | M1 A1 |
| | | **(5)** |
| | | **(9 marks)** |

**M1:** Uses part (a) to deduce that $\sin 2\theta = \dfrac{2}{5}$

**M1:** Uses the correct order of operations to find at least one angle for $\theta$
It is for $\sin 2\theta = k$ where $|k| < 1$ followed by $\theta = \dfrac{1}{2}\arcsin(k) = ...$
If working is not shown you may need to check.

**A1:** For awrt 0.206 and awrt 1.37 and no other values in the range.
Accept awrt $0.0656\pi$ and awrt $0.436\pi$.
Apply isw if values are subsequently rounded further.
Answers in degrees score A0 and correct answers with no working scores no marks.

**Special cases:**

$$\tan\theta + \cot\theta = 5 \Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 5 \Rightarrow \frac{1}{\frac{1}{2}\sin 2\theta} = 5 \Rightarrow \sin 2\theta = \frac{2}{5}$$

Scores M1 for reaching $\sin 2\theta = \dfrac{2}{5}$ as in the main scheme and then **d**M1A1 as above.

**BUT**

$$\tan\theta + \cot\theta = 5 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 5 \Rightarrow \tan^2\theta - 5\tan\theta + 1 = 0$$

$$\Rightarrow \tan\theta = \frac{5 \pm \sqrt{21}}{2} \Rightarrow \theta = \text{awrt } 0.206, 1.37$$

Scores M1 for reaching $\tan\theta = \dfrac{5 \pm \sqrt{21}}{2}$ and obtains awrt 0.206 **or** awrt 1.37

and then M1 for reaching $\tan\theta = \dfrac{5 \pm \sqrt{21}}{2}$ and obtains awrt 0.206 **and** awrt 1.37

The A1 is **not** available as they have not used part (a).

| Question Number | Scheme | Marks |
|---|---|---|
| **6(a)** | $\mathrm{fg}(1) = \mathrm{f}(2) = 7$ | M1A1 |
| | | **(2)** |
| **(b)** | Either $\mathrm{g}(0) = 3$ or $\mathrm{g}(x \to \infty) \to 0.5$ | M1 |
| | $0.5 < \mathrm{g}(x) \leqslant 3$ | A1 |
| | | **(2)** |
| **(c)** | $y = \dfrac{x+9}{2x+3} \Rightarrow y(2x+3) = x+9$<br>$\Rightarrow x(2y-1) = 9-3y \Rightarrow x = \dfrac{9-3y}{2y-1}$ | M1 |
| | $\mathrm{g}^{-1}(x) = \dfrac{9-3x}{2x-1}, \quad 0.5 < x \leq 3$ | A1, B1ft |
| | | **(3)** |
| **(d)** | Attempts to find f(0) = 2×3+5=11 OR f(3) = 5 | M1 |
| | $5 < k \leq 11$ | A1 A1 |
| | | **(3)** |
| | | **(10 marks)** |

**Notes**

**(a)**

**M1:** Full method for substituting their g(1) into f.

Can also be scored for seeing $x = 1$ substituted into $2\left|3 - \dfrac{x+9}{2x+3}\right| + 5$

**A1:** 7. Sight of 7 can achieve both marks if no incorrect work is seen.
Do not accept multiple answers.

**(b)**

**M1:** Achieves the correct value for either end of the range. Sight of 0.5 or 3 or e.g. $\dfrac{9}{3}$ is sufficient.

**A1:** Correct range and correct notation. Accept $0.5 < y \leq 3$ or e.g. $0.5 < \mathrm{g} \leq 3$ or e.g. $(0.5, 3]$

**(c)**

**M1:** Full attempt to change the subject for $y = \dfrac{x+9}{2x+3}$ or $x = \dfrac{y+9}{2y+3}$

They must cross multiply and collect terms to get $x = \dfrac{ay+b}{cy+d}$ or $y = \dfrac{ax+b}{cx+d}$ oe where

$a$, $b$, $c$ and $d$ are non-zero.

**A1:** $\mathrm{g}^{-1}(x) = \dfrac{9-3x}{2x-1}$. Accept $y$ or $\mathrm{g}^{-1}$ for $\mathrm{g}^{-1}(x)$

**B1ft:** Correct domain or follow through on their answer for (b) but it must be appropriate notation in terms of $x$.
E.g. $0.5 < x \leqslant 3$ or $(0.5, 3]$
Do not follow through on $y \in \mathbb{R} \to x \in \mathbb{R}$

**(d)**

**M1:** Attempts f(0) or f(3).
Evidence could be seeing $2|3-0|+5$ or $2|3-3|+5$ or the sight of 5 or 11
It may be seen as one of the limits on $k$.

**A1:** Achieves correct limits of 5 and 11 which may be seen embedded e.g. $5 < k \lessgtr 11$

**A1:** Fully correct $5 < k \leq 11$ oe e.g. $(5, 11]$

| Question Number | Scheme | Marks |
|---|---|---|
| 7(a) | $t=0 \Rightarrow (P=)200-\dfrac{160}{15+1}=190 \Rightarrow 190\ 000$ | M1A1 |
| | | (2) |
| (b) | $e^{kt} \to ae^{kt}$ | M1 |
| | $\left(\dfrac{dP}{dt}=\right)-\dfrac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}$ | M1A1 |
| | | (3) |

**Notes**

**(a)**

**M1:** Sets $t = 0$ and uses $e^0 = 1$. It is for attempting $200-\dfrac{160}{15+1}$ but not $\dfrac{200-160}{15+1}$

This can be awarded for a correct answer.

**A1:** Correct answer only. Accept 190 000 or 190 thousand or ($P =$) 190 (bees).

The answer is an integer so do not allow awrt 190 or 190 000 i.e. no decimals.

**(b)**

**M1:** For evidence of $e^{kt} \to ae^{kt}$ where $a$ is a constant.

This may be embedded within the quotient (or product) rule or their attempt to differentiate.

**M1:** For applying the quotient rule to obtain $\left(\dfrac{dP}{dt}=\right)\pm\dfrac{(15+e^{0.8t})\times pe^{0.6t}-qe^{0.8t}\times e^{0.6t}}{(15+e^{0.8t})^2}$ or the product rule to obtain $\left(\dfrac{dP}{dt}=\right)\pm\left[Ae^{0.6t}(15+e^{0.8t})^{-1}+Be^{0.6t}(15+e^{0.8t})^{-2}\times Ce^{0.8t}\right]$

Condone invisible brackets for this mark but not for the A mark below.

**A1:** A correct unsimplified or simplified derivative.

$$\text{e.g. } \left(\frac{dP}{dt}=\right)-\frac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2} \text{ oe}$$

or e.g.

$$\left(\frac{dP}{dt}=\right)-160\times 0.6e^{0.6t}(15+e^{0.8t})^{-1}+160\times 0.8e^{1.4t}(15+e^{0.8t})^{-2} \text{ oe}$$

"$\dfrac{dP}{dt}=$" is **not** required so just look for a correct expression.

Apply isw once a correct derivative is seen.

| | | |
|---|---|---|
| **(c)** | Sets $\dfrac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}=0$<br>$\Rightarrow (15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}=0$<br>$\Rightarrow 1440e^{0.6t}=32e^{1.4t}$ or e.g. $e^{0.8t}=45$ oe<br>**or**<br>$-160e^{0.6t}(15+e^{0.8t})^{-2}\times 0.8e^{0.8t}+(15+e^{0.8t})^{-1}\times 128e^{0.6t}=0$<br>$\Rightarrow 1440e^{0.6t}=32e^{1.4t}$ or e.g. $e^{0.8t}=45$ oe | M1A1 |
| | $\Rightarrow T=\dfrac{\ln 45}{0.8}=4.76$ | **d**M1A1 |
| | | **(4)** |
| | | **(9 marks)** |

**Notes**

**Allow recovery here if signs are reversed in their derivative.**

**May work in *T* and/or *t*.**

**M1:** Sets the numerator of their quotient rule attempt = 0 or sets their product rule attempt = 0 and obtains $pe^{0.8t}=q$ or $Ae^{0.6t}=Be^{1.4t}$ or equivalent.
The denominator of their quotient rule (correct or incorrect) can be ignored in part (c).

**A1:** $1440e^{0.6t}=32e^{1.4t}$ or $e^{0.8t}=45$ or an equivalent correct equation.
Must follow correct work but allow this mark if the only error is reversed signs in their derivative.

**dM1:** Having set $\dfrac{dP}{dt}=0$ and obtained either $Ae^{\pm kt}=B$ ($k$ may be incorrect) or $Ce^{\pm\alpha t}=De^{\pm\beta t}$ where $k,\alpha,\beta\neq 0$ it is for the correct order of operations including taking ln's leading to $t = \ldots$
May be implied by their value of $t$.
It cannot be awarded for impossible equations e.g. $e^{0.8t}=-45$
**Depends on the first method mark.**

**A1:** $T=\dfrac{\ln 45}{0.8}$ or equivalent or awrt $4.76$
Must follow correct work but allow this mark if the only error is reversed signs in their derivative.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **8(a)** | $R=\sqrt{109}$ | B1 |
| | $\tan\alpha=\frac{3}{10}\Rightarrow\alpha=\text{awrt}\,16.70°$ | M1A1 |
| | | **(3)** |
| **(b)(i)** | Max height = 12+ ‘ $\sqrt{109}$ ’= awrt 22.44 (m) | B1ft |
| **(ii)** | Occurs when $30t+'16.70'=180\Rightarrow t=5.44$ | M1A1 |
| | | **(3)** |

**Notes**

**(a)**

**B1:** $R=\sqrt{109}$. Apply isw after this is seen e.g. if they convert to a decimal.
Do not allow $R=\pm\sqrt{109}$.

**M1:** Attempts to find $\alpha$. Look for $\tan\alpha=\pm\frac{3}{10}\Rightarrow\alpha=...$ but condone $\tan\alpha=\pm\frac{10}{3}\Rightarrow\alpha=...$
or if $R$ is used allow $\sin\alpha=\pm\frac{3}{R}\Rightarrow\alpha=...$ or $\cos\alpha=\pm\frac{10}{R}\Rightarrow\alpha=...$

**A1:** $\alpha=\text{awrt}\,16.70°$ but condone $\alpha=16.7°$
It is **not** for awrt 16.7 e.g. 16.67 scores A0
An answer of $\alpha=\text{awrt}\,0.29$ radians scores A0

**(b)(i)**

**B1ft:** For awrt 22.44 (m) or $12+\sqrt{109}$ following a correct value for $R$ but ft on their $R$ value so allow for 12 + their $R$ in exact form or for 12 + their $R$ as awrt 2dp (you may need to check).
Units are **not** required but if any are present they should be correct.

**(b)(ii)**

**M1:** Attempts to solve $30t+'16.70'=180$ to obtain a value for $t$.
If radians were used in part (a) then allow an attempt to solve $30t+'0.29'=\pi$
(which leads to $t$ = 0.095…) but do **not** allow mixing of degrees and radians e.g.
$30t+'0.29'=180\Rightarrow t=...$
Allow methods that use e.g. $x=30t\Rightarrow x+'16.70'=180$ etc. provided they subsequently replace $x$ with $30t$ and find $t$.
May be implied by their value for $t$.

**A1:** awrt 5.44 only.
If multiple solutions are found then awrt 5.44 must clearly be identified as their chosen solution.

| | | |
|---|---|---|
| **(c)** | $18 = 12 - \sqrt{109}\cos(30t + 16.70) \Rightarrow \cos(30t + 16.70) = -\dfrac{6}{\sqrt{109}}$ $(-0.57..)$ | M1A1 |
| | $\Rightarrow 30t + 16.70 = \arccos\left(-\dfrac{6}{\sqrt{109}}\right)(= 125.078...°) \Rightarrow t = ..$ | **d**M1 |
| | $t =$ awrt $3.61$(2dp) | A1 |
| | | **(4)** |

**Notes**

**M1:** Uses $H = 18$ in their answer to part (a) and proceeds to $\cos(30t + '16.70') = k$ where $|k| < 1$
Allow this mark for $18 = 12 - "\sqrt{109}"\cos(30t + "16.70")$ leading to $\cos(30t + '16.70') = k$ where $|k| < 1$

**A1:** $\cos(30t + '16.70') = -\dfrac{6}{\sqrt{109}}$ or awrt $-0.57...$ following through on their $\alpha$
May be implied by $30t + '16.70' =$ awrt $125°$ or awrt 2.2 radians.

**dM1:** Correct order of operations to find a **positive** value for $t$ from their **negative** value for $k$.
The order of operations must be correct from their $\cos(30t + '16.70') = k$ $(-1 < k < 0)$ and the value chosen must be in the second quadrant.
Condone use of radians if consistent with their value of $\alpha$ e.g.

$$18 = 12 - \sqrt{109}\cos(30t + "0.29") \Rightarrow \cos(30t + "0.29") = -\frac{6}{\sqrt{109}} \quad (-0.57..)$$

$$\Rightarrow 30t + "0.29" = \arccos\left(-\frac{6}{\sqrt{109}}\right) = 0.2183... \Rightarrow t = ..(0.0631...)$$

but do not allow mixing of degrees and radians.
**Depends on the first method mark.**

**A1:** awrt 3.61 and no other values following the award of M1, A1 dM1

Allow methods that use e.g. $x = 30t$ etc. provided they subsequently replace $x$ with $30t$ and find $t$.

**For reference:**
**Work and marks for a candidate working entirely in radians in (a), (b) and (c):**

(a) $R = \sqrt{109}$, $\alpha = 0.29$ — B1M1A0
(b)(i) 12+ ‘ $\sqrt{109}$ ’= awrt 22.44 m — B1
(b)(ii) $30t + 0.29 = \pi \Rightarrow t = 0.095...$ — M1A0

(c) $18 = 12 - \sqrt{109}\cos(30t + 0.29) \Rightarrow \cos(30t + 0.29) = -\dfrac{6}{\sqrt{109}}$ $(-0.57..)$ — M1A1

$\Rightarrow 30t + 0.29 = \arccos\left(-\dfrac{6}{\sqrt{109}}\right)(= 2.183...) \Rightarrow t = ..$ — M1

$\Rightarrow t = 0.063...$ — A0

| | | |
|---|---|---|
| **(d)** | $30t = 360 \Rightarrow t = ..$ or $30t = 720 \Rightarrow t = ..$ | M1 |
| | 24 minutes | A1 |
| | | **(2)** |
| **Notes** | | |
| **M1:** | Attempts to solve either $30t = 360$ or $30t = 720$ to find $t$ or else states 12 or 24 | |
| **A1:** | 24 minutes. Requires minutes or e.g. “mins”, “min”, “m”. | |
| | | **(12 marks)** |