Pearson Edexcel

# Mark Scheme (Results)

## October 2024

Pearson Edexcel International Advanced Level
In Pure Mathematics P1 (WMA11)
Paper 01

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - $*$ – The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

# General Principles for Pure Mathematics Marking

*(NB specific mark schemes may sometimes override these general principles)*

## Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2 + bx + c) = (x + p)(x + q),$ where $|pq| = |c|$ leading to $x$ = …

   $(ax^2 + bx + c) = (mx + p)(nx + q),$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x$ = …

2. Formula

   Attempt to use correct formula (with values for a, b and c)

3. Completing the square

   Solving $x^2 + bx + c = 0$ : $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x$ = …

## Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1 ( $x^n \to x^{n-1}$ )

2. Integration

   Power of at least one term increased by 1 ( $x^n \to x^{n+1}$ )

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

## Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question Number | Scheme | Marks |
|---|---|---|
| **1. (a)** | Attempts gradient = $\frac{20--4}{-5-3}=(-3)$ | M1 |
| | Attempts equation of line $y-20="-3"(x+5)$ or $y+4="-3"(x-3)$ | dM1 |
| | $y=-3x+5$ | A1 |
| | | **(3)** |
| **(b)** | Gradient $\frac{1}{3}$ or midpoint $\left(-1,8\right)$ | B1ft |
| | $y-8=\frac{1}{3}(x+1)$ | M1 |
| | $x-3y+25=0$ | A1 |
| | | **(3)** |
| | | **(6 marks)** |

(a)

M1: Attempts gradient = $\frac{\delta y}{\delta x}$ condoning slips. Look for an attempt at a difference in $y$ coordinates over a difference in $x$ coordinates. There is no requirement to simplify. They may use $20--4=m\left(-5-3\right)$ to find $m$.

dM1: Full method for equation of line using gradient and either point – as shown or may use $y="-3"x+c$ proceeding as far as finding $c$.

A1: $y=-3x+5$ The coefficients must be simplified, award A0 for $y=\frac{24}{-8}x+5$

Alt (a)

M1: Sets up two simultaneous equations using both points E.g $20=-5m+c$ and $-4=3m+c$ (allow one slip).

dM1: Solves the pair of simultaneous equations to find values for $m$ and $c$ (accept any values following two suitable equations having been set up).

A1: $y=-3x+5$

(b)

B1ft: Correct mid point or gradient, allowing follow through on the negative reciprocal of their gradient in (a)

M1: Attempts the perpendicular bisector (need not be a simplified form). Must be using ± the reciprocal of their gradient from (a) with a valid attempt at the midpoint – at least one correct coordinate (or calculation for) must be seen for the midpoint.

A1: $x-3y+25=0$ but allow any integer multiple and term in a different order (but must be on one side). Ignore if they give a label $l_2=x-3y+25=0$

| Question Number | Scheme | Marks |
|---|---|---|
| **2. (i)** | $\dfrac{3y^3\left(2x^4\right)^3}{4x^2y^4}=\dfrac{6x^{10}}{y}$ | B1, B1, B1<br><br>**(3)** |
| **(ii)** | $\dfrac{16}{\sqrt{3}+1}=a\sqrt{27}+4$ | |
| | States or uses $\sqrt{27}=3\sqrt{3}$ | B1 |
| | Correct attempt at rationalising seen e.g. $\dfrac{16}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ | M1 |
| | Correct attempt to make "$a$" the subject and make progress towards the form $p\sqrt{3}+q$ e.g.<br>$8\sqrt{3}-8=3a\sqrt{3}+4\Rightarrow 3a\sqrt{3}=8\sqrt{3}-12\Rightarrow a=\dfrac{8\sqrt{3}}{3\sqrt{3}}-\dfrac{12}{3\sqrt{3}}$ | M1 |
| | $a=-\dfrac{4}{3}\sqrt{3}+\dfrac{8}{3}$ | A1cso |
| | | **(4)**<br>**(7 marks)** |

(i)

B1: For any one of 6, $x^{10}$ or $\dfrac{1}{y}$ Allow $y^{-1}$ for $\dfrac{1}{y}$ - may be seen separately, and allow for one correctly simplified term even if a spurious extra term in the same variable is present.

B1: For any two of the above as the only term in that variable within a single expression. E.g. $2x^{10}\times\dfrac{3}{y}$ is fine for B1B1 but $2x^{10}+\dfrac{3}{y}$ is B1B0.

B1: For $\dfrac{6x^{10}}{y}$ or $6x^{10}y^{-1}$ - must be one expression. Allow with $y^1$ in the denominator.

(ii)

B1: States or uses $\sqrt{27}=3\sqrt{3}$. May be implied by working as long as it is seen before the final answer.

M1: Correct attempt at rationalisation seen anywhere. This can be on any expression with a two term surd expression in a denominator during their working. The denominator may be simplified without the multiplication being seen (and need not be correct), but the evidence of the process must be clear.

M1: Attempts to make "$a$" the subject and make progress towards the form $p\sqrt{3}+q$ $\left(p,q\neq 0\right)$ - must reach a two term expression for $a$ (but may be over common denominator), though the $\sqrt{27}$ may not have been simplified. E.g. accept for $a=\dfrac{8\sqrt{3}-12}{\sqrt{27}}$. Can be scored if a calculator was used to rationalise the surd expression but there must be an intermediate step before their final answer to show the method.

A1cso: $a=-\frac{4}{3}\sqrt{3}+\frac{8}{3}$ but isw after a correct expression. Must have score all previous marks, showing the relevant steps as outline above before the final answer is given but condone if "invisible brackets" are recovered. Accept with terms in the other order, and allow $\frac{1}{3}\left(8-4\sqrt{3}\right)$

Note that there are many ways of attempting (b). The above scheme can still be applied
A common alternative method is;

$\frac{16}{\sqrt{3}+1}=a\sqrt{27}+4\Rightarrow 16=\left(a\sqrt{27}+4\right)\left(\sqrt{3}+1\right)=9a+4+4\sqrt{3}+3\sqrt{3}a$ for B1

$\Rightarrow 12-4\sqrt{3}=\left(9+3\sqrt{3}\right)a\Rightarrow a=\frac{12-4\sqrt{3}}{9+3\sqrt{3}}\times\frac{9-3\sqrt{3}}{9-3\sqrt{3}}$ for M1

$\Rightarrow a=\frac{144-72\sqrt{3}}{54}=-\frac{4}{3}\sqrt{3}+\frac{8}{3}$ for M1 A1

Notes: The B mark can be implied by working as long as it is seen before the final answer.
So, for example, $\frac{8\sqrt{3}+4}{\sqrt{27}}\times\frac{\sqrt{27}}{\sqrt{27}}=\frac{72+12\sqrt{3}}{27}=...$ is sufficient to score the B mark.
An acceptable equivalent of rationalising the denominator of an expression for the first M is rationalising the coefficient of *a* in a rearranged expression, e.g.
$\left(9+3\sqrt{3}\right)a=12-4\sqrt{3}\rightarrow(81-27)\,a=\left(9-3\sqrt{3}\right)\left(12-4\sqrt{3}\right)$
The second M mark requires an attempt to make *a* the subject and make progress towards the form $p\sqrt{3}+q$, which means the denominator must have been reduced to a single term with two terms in the numerator. So $a=\frac{12-4\sqrt{3}}{9+3\sqrt{3}}$ is not sufficient progress. In the example above the M is not awarded until the final line where the denominator is 54.
However, once the first B and M are scored (having seen evidence before the final answer) reaching the correct answer will then be sufficient for the M (and A if all correct).
But from $\frac{12-4\sqrt{3}}{9+3\sqrt{3}}$ to $\frac{8-4\sqrt{3}}{3}$ with no further work shown is 2nd M0 (calculator use with no further progress). However, if a correct attempt to rearrange this to $-\frac{4\sqrt{3}}{3}+\frac{8}{3}$ (or with terms reversed) follows (or similar if there were errors) then the 2nd M becomes eligible as there is further work to support the method.

| Question Number | Scheme | Marks |
|---|---|---|
| **3. (a)** | $\int \mathrm{f}(x)\,\mathrm{d}x = \int x^{\frac{3}{2}} + 10x^{\frac{1}{2}} + 25x^{-\frac{1}{2}}\,\mathrm{d}x = \frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$ | M1 A1 A1 A1<br>**(4)** |
| **(b) (i)** | $\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$ | M1 A1 |
| | $\mathrm{f}'(x) = 0 \Rightarrow \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0$ AND $\times x^{\frac{3}{2}}$ | dM1 |
| | $\frac{3}{2}x^{2} + 5x^{1} - \frac{25}{2} = 0 \Rightarrow 3x^{2} + 10x - 25 = 0$ * | A1* |
| **(ii)** | $\left(x = \right)\frac{5}{3}$ only | B1<br>**(5)**<br>**(9 marks)** |

(a)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and increasing a correct power by 1 (ie must have expanded). Must be seen in part (a)

A1: For one correct term (allow un simplified)

A1: For two correct terms simplified.

A1: $\frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$ or exact equivalent. Must include the constant of integration. Condone if they write "$y$ =" and ignore any spurious integral signs or extra d$x$

(b)(i)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and decreasing a correct power by one. Must be seen or used in part (b). For attempts at the quotient or product rule look for correct denominator and one correct numerator term in $\frac{A(x+5)\sqrt{x} - B(x+5)^2 x^{-1/2}}{\left(\sqrt{x}\right)^2}$ (oe for product rule)

A1: For $\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$ **(**oe e.g from quotient rule**)** which may be left un simplified.

dM1: Sets $\alpha x^{\frac{1}{2}} + \beta x^{-\frac{1}{2}} + \chi x^{-\frac{3}{2}} = 0$ and attempts to multiply by $x^{\frac{3}{2}}$ seen in at least two terms or explicitly shown in a step before the final answer.

This may be derived in stages, first multiplying by $x^{\frac{1}{2}}$ and then by $x$. This is permissible for the M (and A) as evidence of the correct method has been shown by the stages. The dM can be scored if the index work is correct in at least two indices, and A1 if all is correct.

So $\frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{3}{2}x + 5 - \frac{25}{2}x^{-1} = 0 \Rightarrow \frac{3}{2}x^{2} + 5x - \frac{25}{2} = 0$ is sufficient for the dM1 as long as the middle step is seen with two correct powers (and the A1 if they then multiply through be 2).

Note also that dividing by $x^{-\frac{3}{2}}$ is an equivalent to multiplying by $x^{\frac{3}{2}}$.

A1*: Reaches $3x^2+10x-25=0$ showing all steps with no errors. Note that they must work correctly with the equation, if they state $f'(x)=3x^2+10x-25$ before setting equal to 0 then allow the M (if suitable method shown) but it is A0.

(b)(ii)

B1: $\left(x=\right)\frac{5}{3}$ only. The other root need not be seen, but if shown it must be subsequently rejected in some way. Note that $1.\dot{6}$ (1.6 recurring) is acceptable as the answer for B1, but 1.67 is B0.

| Question Number | Scheme | Marks |
|---|---|---|
| **4 (a)** | States or implies that $\left[\text{f}(x)=\right]kx(x-4)$ | M1 |
| | Attempts to find $k$. E.g. $-4.8=k\times2\times(2-4)\Rightarrow k=...$ | dM1 |
| | $\left[\text{f}(x)=\right]1.2x(x-4)$ | A1 |
| | | **(3)** |
| **(b)** | States or implies that $\left[\text{g}(x)=\right]\lambda x(x-4)^2$ | M1 |
| | Attempts to find $\lambda$ . E.g. $7.2=\lambda\times6\times(6-4)^2\Rightarrow\lambda=...$ | dM1 |
| | $\left[\text{g}(x)=\right]0.3x(x-4)^2$ | A1 |
| | | **(3)** |
| **(c)** | Sets their $1.2x(x-4)=0.3x(x-4)^2$ | B1ft |
| | Valid attempt to solve $1.2\cancel{x}\cancel{(x-4)}=0.3\cancel{x}(x-4)^{\cancel{2}}\Rightarrow x=4+\dfrac{1.2}{0.3}$ | M1 |
| | $x=8$ | A1 |
| | $\left(8,38.4\right)$ | A1 |
| | | **(4)** **(10 marks)** |

(a)

M1: States or implies that $\text{f}(x)=kx(x-4)$. Allow with $k$ = 1. The f($x$) need not be seen – assume they are working with f($x$) in part (a)

Alternatives include $\text{f}(x)=k(x-2)^2-4.8$ Allow with $k$ = 1 and $\text{f}(x)=ax^2+bx$ with at least one of $4a+b=0$ and/or $4a+2b=-4.8$

dM1: Attempts to find all the constants in their equation for $\text{f}(x)$ using $\left(2,-4.8\right)$. E.g. $-4.8=k\times2\times(2-4)\Rightarrow k=...$

A1: $\left[\text{f}(x)=\right]1.2x(x-4)$ o.e. such as $\left[\text{f}(x)=\right]1.2x^2-4.8x$ or $\left[\text{f}(x)=\right]1.2(x-2)^2-4.8$ and isw. Accept with ($x$ − 0). The f($x$) may be missing, it can be assumed (but use of g($x$) is A0). Allow if the full expression is given in latter parts if all the work was correct in (a) to find the constants.

(b)

M1: States or implies that $\text{g}(x)=\lambda x(x-4)^2$ Allow with $\lambda$ = 1. The g($x$) need not be seen – assume they are working with g($x$) in part (b).

Alternatives include $\text{g}(x)=ax^3+bx^2+cx$ with at least two of $64a+16b+4c=0$, $48a+8b+c=0$, and/or $216a+36b+6c=7.2$ (allow minor slips).

dM1: Attempts a full equation for $\text{g}(x)$ using $\left(6,7.2\right)$

E.g. $7.2=\lambda\times6\times(6-4)^2\Rightarrow\lambda=...$

A1: $\left[\text{g}(x)=\right]0.3x(x-4)^2$ o.e. such as $\left[\text{g}(x)=\right]0.3x^3-2.4x^2+4.8x$ and isw. Accept with ($x$ − 0). The g($x$) may be missing, it can be assumed (but use of f($x$) is A0). Allow if the full expression in used in (c) if all the work was correct in (b) to find the constants.

(c)

B1ft: Sets their $1.2x(x-4)=0.3x(x-4)^2$. It is for a "correct" ft equation on their constants, but the form (quadratic = cubic) must be correct.

M1: Valid attempt to solve their equations to find at least one non-zero solution as long as they have a cubic and quadratic equated. E.g. $1.2\cancel{x}\cancel{(x-4)}=0.3\cancel{x}(x-4)^{\cancel{2}} \Rightarrow x=4+\dfrac{1.2}{0.3}$.

Alternatively, they may expand, factor out the $x$ and solve the quadratic (allowing for slips), or even expand to a cubic (allowing slips) and solve via calculator (may need to check, and you may ignore "incorrect factorisation" if the answers are correct for their equation).

A1: $x = 8$ identified as the $x$ coordinate of $P$. A0 if they give 0,4 and 8 and do not select the correct answer.

A1: $\left(8, 38.4\right)$ or exact equivalent for 38.4

| Question Number | Scheme | Marks |
|---|---|---|
| **5 (a)** | Attempts $r\theta = 5\times1.2$<br>Perimeter =5 + 5 + 6 = 16 (km) | M1<br>A1<br>**(2)** |
| **(b)** | Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times5^2\times1.2$ | M1 |
| | Area $AOP$ = $\frac{1}{4}\times\left(\frac{1}{2}\times5^2\times1.2\right) = 3.75\text{ km}^2$ * | M1, A1*<br>**(3)** |
| **(c)** | Sets $\frac{1}{2}\times5\times OP\times\sin1.2 = 3.75 \Rightarrow OP = ...$ | M1 |
| | $OP = 1.6...$ | A1 |
| | $AP^2 = 5^2 + "1.6.."^2 - 2\times5\times"1.6.."\times\cos1.2$ | M1 |
| | $AP = 4.7\text{km}$ or 4700m | A1cso<br>**(4)**<br>**(9 marks)** |

(a)

M1: Attempts $r\theta = 5\times1.2$

A1: Achieves 16 (km)

(b)

M1: Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times5^2\times1.2$ (or $\frac{1}{2}rl = \frac{1}{2}\times5\times6$) May be implied by Area of sector = 15 seen.

M1: Attempts $\frac{1}{4}\times\left(\frac{1}{2}\times5^2\times1.2\right)$ or $\frac{3}{4}\times\left(\frac{1}{2}\times5^2\times1.2\right)$ or other full method to find the area for one of the two regions.

A1*: Achieves given answer, including units, showing all steps with no errors and full accuracy kept throughout. Score A0 if rounded values were used during working.

Approaches by verification are acceptable. The first M1 will apply the same way, a clear attempt at the sector area formula will be needed to determine the area.
The second M may then be gained for attempting
Area Sector $AOB =$ Area $R_1$ + Area $R_2 = 3\times3.75 + 3.75 = \ldots$

Then for the A mark the calculations need to both be correct (giving 15), and a conclusion made: E.g. (so) Area $R_2$ = 3.75 km$^2$.

(c)

M1: Sets $\frac{1}{2}\times5\times OP\times\sin1.2 = 3.75 \Rightarrow OP = ...$ There are some longer methods to find $OP$, so this may be scored for any complete method. E.g finds area of triangle $APB$ from difference of area triangle $OAB$ $\left(=\frac{1}{2}5^2\sin1.2\right)$ and 3.75, the uses ratio $OP$:$PB$ is same

as ratio of these areas to form and solve an equation for *OP*. Other method may be possible. Or $\frac{1}{2}OP.h = 3.75,\ h = 5\sin 1.2 \Rightarrow OP = ...$

A1: Achieves distance *OP* = awrt 1.6 (allow if called *AP*). May be implied by subsequent work.

M1: Attempts the cosine rule, or other full method, to find at least $AP^2$ using angle 1.2, *OA* = 5 and their *OP*. Condone if $AP^2$ is missing or labelled *AP*.

Again there are longer winded ways, e.g. using right angle triangle *OAX* where *X* is the base of perpendicular to *OB* through *A*, then $AX = 5\sin 1.2,\ PX = \frac{AX}{\tan 1.2} - OP$ and $AP^2 = AX^2 + PX^2$ can be used to find *AP*.

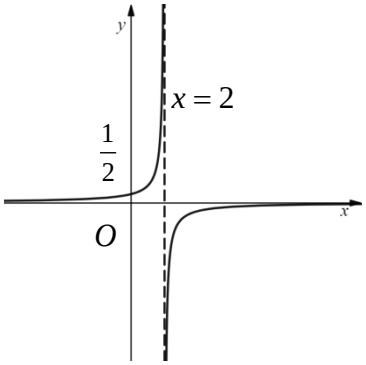
A1cso: $AP = 4.7\text{km}$ or 4700m (isw once seen if further rounding occurs). Must include the correct units and be given to nearest 100m.

Watch out for using of $\frac{1}{2} \times 5 \times OP \times 1.2 = 3.75 \Rightarrow OP = 1.25$ which also leads to 4.7km. This scores at most M0A0M1A0 despite giving the same answer.

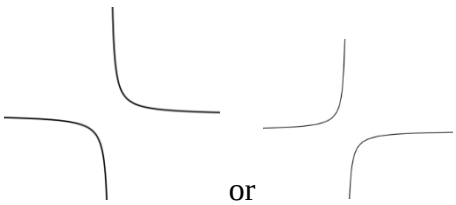
SC If $R_1$ and $R_2$ are mixed up in part (c) then score as a misread if the work is otherwise correct – so M1 for attempting $\frac{1}{2} \times 5 \times OP \times \sin 1.2 = 11.25 \Rightarrow OP = ...$ and dM1 for a correct use of the cosine rule to find *AP*.

Some methods do not find *OP*, but a different intermediate length for a triangle containing *AP*. Such methods score the M1 for the attempt to find two other side lengths for a triangle with side *AP* and the A1 for a correct length, then second M for full method to find at least $AP^2$. For example

| Question Number | Scheme | Marks |
|---|---|---|
| **5(c) Alt** | Let *X* be base of perpendicular to *OA* though *P*, then<br>$\frac{1}{2} \times 5 \times XP = 3.75 \Rightarrow XP = 1.5 \Rightarrow OX = \frac{"1.5"}{\tan 1.2}$ so<br>$AX = 5 - \frac{"1.5"}{\tan 1.2} = ...$<br>$AX = 4.416...$<br>$AP^2 = "4.416..."^2 + "1.5"^2 = ...$<br>$AP = 4.7\text{km}$ or 4700m | M1<br><br>A1<br>M1<br>A1<br>**(4)**<br>**(9 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.(a)** |  1/*x* type shape<br><br>Fully correct<br>Correct equation of vertical asymptote and *y* intercept | M1<br>A1<br>B1<br><br>**(3)** |
| **(b) (i)** | Sets $kx-4=\dfrac{1}{2-x}\Rightarrow(kx-4)(2-x)=1$ | M1 |
| | $kx^2+(-4-2k)x+9=0$ | A1 |
| | Attempts use $b^2-4ac\ldots0\ \Rightarrow(-4-2k)^2-4k\times9\ldots0$ | dM1 |
| | $\Rightarrow 4k^2-20k+16\ldots0\Rightarrow k^2-5k+4\ldots0$ * | A1* |
| **(ii)** | $(k-1)(k-4)\ldots0\Rightarrow k\ ,,\ 1,\ k\ldots4$ | M1, A1 |
| | | **(6)**<br>**(9 marks)** |

(a)

M1: Correct shape for a $\dfrac{1}{x}$ type graph. Look for  or

Must have two branches, be generous with curves that bend back on themselves if the intent is clear.

A1: Fully correct shape and position. Do not accept curves that clearly bend back on themselves, but "pen flicks" at the end of a curve can be condoned.

B1: Correct equation (must be an equation) of asymptote and *y* intercept. The graph must be asymptotic here and in the correct places (e.g. the equation must match the placement), though it need not be fully correct – as long as these features are shown. Information stated on the graph takes precedence but if coordinates or equation are missing, allow if they are stated in the text. Accept ½ labelled on the *y*-axis for the intercept. Condone (½ , 0) if it is in the correct position on the axis.

(b)(i)

M1: Sets $kx-4=\dfrac{1}{2-x}\Rightarrow(kx-4)(2-x)=1$

A1: $kx^2+(-4-2k)x+9=0$ with terms collected. This may be implied by later work, e.g. $a$ = ..., $b$ =…, $c$ =… listed or used. The "=0" may be implied.

dM1: Attempts use $b^2-4ac\ldots0 \Rightarrow(-4-2k)^2-4k\times9\ldots0$ with their coefficients to set up a quadratic inequality in $k$. Must be the correct direction of inequality but allow > or …. Use of $b^2 - 4ac = 0$ will score dM0 unless clearly recovered before stating that answer.

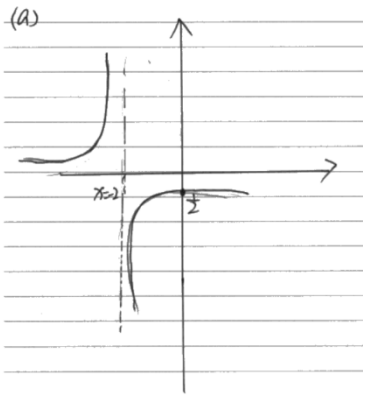
A1*: Achieves the given answer of $k^2-5k+4\ldots0$ with no errors and sufficient working shown – the bracket should be expanded before the final answer is reached. A0 if $x$ used in place of $k$.

(b)(ii)

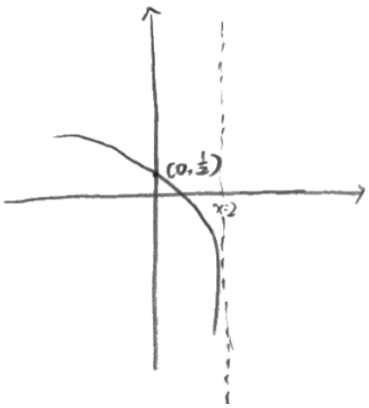
M1: Solves to find the critical values – may be implied by sight of 1 and 4 used in their answer but must have two values from a valid attempt to solve. Allow from attempts at their "$b^2-4ac$"...0 with any inequality or equality.

A1: $k$,, $1, k\ldots4$ Accept with "and" or "or" used. Accept interval notation $(k\in)\,(-\infty,1]\cup[4,\infty)$ - must be correct union if set notation used. Must be the final answer, but allow if recovery from "incorrect" inequalities when solving for the critical values. A0 if $x$ used instead of $k$. Note $4$,, $k$,, $1$ is A0.

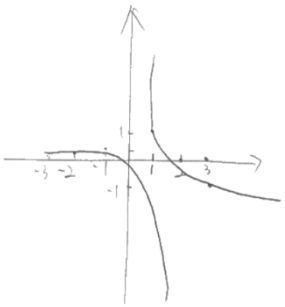
Some examples of graphs:



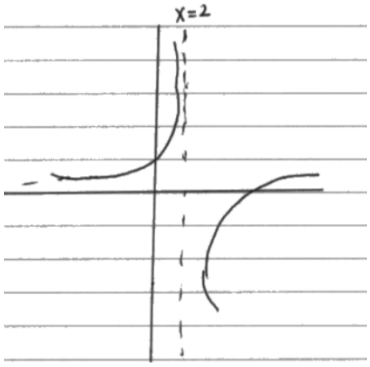

M1A0B0 intent clear for first M. Wrong position, A0, incorrect asymptote and intercept, B0.



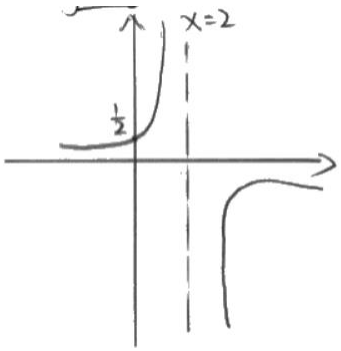

M0A0B1 Only one branch, but asymptote and intercept correct.



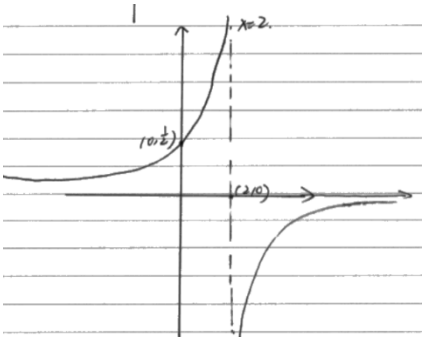
M1A0B0 this intent is acceptable for the shape, but not fully correct as "overlapping" so A0. Intercept/asymptotes incorrect.



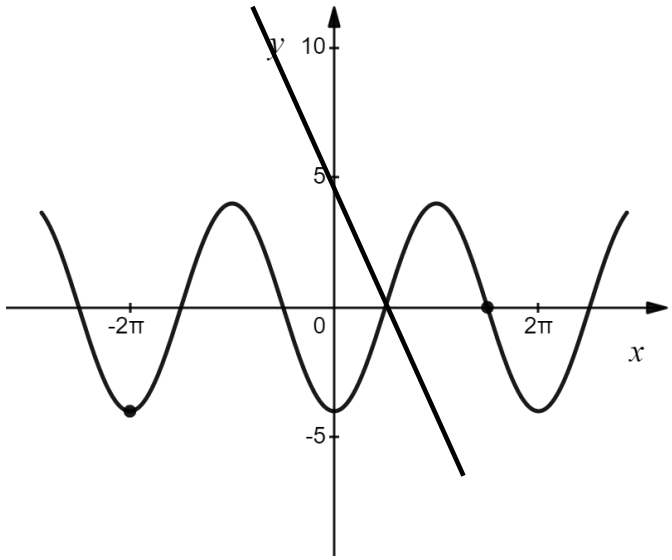

M1A0B0 Intent clear for the First M, but clearly bending back on itself, so A0. No intercept given.




M1A0B1 Intent clear for first M, clearly bends down in fourth quadrant, A0. Asymptote and intercept correct.




M1A1B1 Shape and position correct, the slight "pen flick" is condoned. Intercept and vertical asymptote correctly labelled.

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)(i)** | $P=(-2\pi,-4)$ | B1, B1 |
| **(ii)** | $Q=\left(\frac{3\pi}{2},0\right)$ | B1<br>**(3)** |
| **(b) (i)** | $k=7$ | B1 |
| **(ii)** | $(2\pi,3)$ | M1 A1<br>**(3)** |
| **(c)** |  <br>One root as there is one point of intersection | M1A1<br>**(2)**<br>**(8 marks)** |

NB: Watch for the answers being stated next to the question rather than in the answer space.
(a)(i) Allow all marks in (a) if *P* and *Q* are mislabelled but the coordinates are correct.

B1: For one of $(-2\pi,-4)$ Condone/allow one of $(-360°,-4)$ Condone for this B mark if the coordinates are reversed.

B1: For both of $(-2\pi,-4)$ (in correct order).

(a)(ii)

B1: $(Q=)\left(\frac{3\pi}{2},0\right)$

(b)(i)

B1: For $k=7$

(b)(ii)

M1: For one of $(2\pi,3)$ **or** follow through on the negative of their *x* coordinate of *P* or their $-4+$ their *k* Condone/allow one of $(360°,3)$ and condone for the M if the coordinates are given in the wrong order.

A1: $(2\pi,3)$ Allow $(360°,3)$ if B marks in (a) were lost due solely due to working in degrees.

(c)

M1: Draws a line with negative gradient passing through (0, 5) and cutting the graph at least once, with a consistent number of solutions for their sketch stated.

A1: Correct line drawn (intersecting at graph on the $x$ axis) with correct deduction (one root) and reason (intersects only once).

Note use of degrees only will score a maximum of B1B0 B0 B1 M1A1 M1A0, though if the $\frac{10}{\pi}$ is converted to degrees resulting in a "correct" graph, send to review for consideration.

| Question Number | Scheme | Marks |
|---|---|---|
| **8 (a)** | States or implies that gradient of tangent is 24 | B1 |
| | Solves $f'(3)=24$ to find $k$. E.g $4\times3^2+k\times3+3=24\Rightarrow k=...$ | M1 |
| | $3k+39=24\Rightarrow k=\dfrac{24-39}{3}=-5$ * | A1* |
| | | **(3)** |
| **(b)** | $f'(x)=4x^2-5x+3\Rightarrow f(x)=\dfrac{4}{3}x^3-\dfrac{5}{2}x^2+3x+c$ | M1 A1 |
| | Substitutes $x=3, y=-\dfrac{3}{24}+5$ into $y=f(x)$ to find "$c$" | dM1 |
| | $f(x)=\dfrac{4}{3}x^3-\dfrac{5}{2}x^2+3x-\dfrac{141}{8}$ | A1 |
| | | **(4)** |
| | | **(7 marks)** |

Mark this question as a whole, some may try finding f($x$) first in part (a), so award positively.

(a)

B1: States or implies that gradient of tangent (to the curve at $P$) is 24. This may occur within the use of the perpendicular condition, e.g. $f'(3)\times-\dfrac{1}{24}=-1$ implies B1.

M1: Attempts to solve $f'(3)=\pm24$ or $\dfrac{1}{24}$ (or equivalent equations) to find $k$. Accept as minimum at least one unsimplified equation in $k$ before the given answer is stated. Alternatively, allow for substitution of $k=-5$ and $x=3$ into f'($x$) and simplifying to a value to compare with the gradient (verification method).

A1*: Shows that $k=-5$ with at least one correct equation with simplified terms (not necessarily all gathered) and no incorrect work shown before stating the printed answer. By verification all steps must be correct with conclusion given.

(b)

M1: Integrates to a form $px^3+qx^2+rx$, with at least $p,q\neq0$ with no requirement for + $c$. Allow with $k$ used in place of −5 for this mark.

A1: $f(x)=\dfrac{4}{3}x^3-\dfrac{5}{2}x^2+3x$ with no requirement for + $c$

dM1: Substitutes $x=3, y="-\dfrac{3}{24}+5"$ into $y=f(x)$ and finds a value for "$c$". Must have had a constant of integration.

A1: $\left(f(x)=\right)\dfrac{4}{3}x^3-\dfrac{5}{2}x^2+3x-\dfrac{141}{8}$ Accept with $y=$… instead of f($x$), or even with no label at all.

| Question Number | Scheme | Marks |
|---|---|---|
| **9 (a)** | $x \ldots -5$ | B1 **(1)** |
| **(b)** | $\mathrm{f}(x) = (x+5)\left(3x^2 - 4x + 20\right) = 3x^3 + 11x^2 + 100$ | M1 |
| | $\mathrm{f}'(x) = 9x^2 + 22x$ | M1 A1cso **(3)** |
| **(c)** | Finds $\mathrm{f}'(-4) = 9\times\left(-4\right)^2 + 22\times -4 = \left(56\right)$ | M1 |
| | Sets $\mathrm{f}'(x) = "9x^2 + 22x" = "56"$ | dM1 |
| | $9x^2 + 22x - 56 = 0 \Rightarrow x = \frac{14}{9}, \left(-4\right)$ | ddM1 A1cso **(4)** |
| **(d)(i)** | $\left(-1, 84\right)$ | B1 |
| **(ii)** | $\left(-4, 336\right)$ | B1 **(2) (10 marks)** |

Note: mark the question as a whole – do not be concerned if there is incorrect part labelling.

(a)

B1: Correct range of values given, $x \ldots -5$ or exact equivalent in set notation. Must be clearly identified as the answer, do not isw if a different range is given later but allow if $x = -5$ is stated as well as the range.

(b)

M1: Attempts to multiply out to form a cubic (which may be unsimplified), achieving at least $3x^3 + .... \pm 100$ with at least one $x$ or $x^2$ term between, and allow if there are extra constant terms between too.

M1: Differentiates to form a quadratic. Power decreased by 1 at least twice.

A1cso: $\mathrm{f}'(x) = 9x^2 + 22x$ Must have come from a correct expansion.

The product rule may be attempted:

M1: For $\mathrm{f}'(x) = (1)(3x^2 - 4x + 20) + (x+5)(Ax + B)$

M1: Expands and simplifies to a quadratic as long as a sum of terms was attempted.

A1: $\mathrm{f}'(x) = 9x^2 + 22x$

(c)

M1: Finds $\mathrm{f}'(-4)$ for their $\mathrm{f}'(x)$

dM1: Sets $\mathrm{f}'(x) = "9x^2 + 22x" = "56"$ with their derivative and value.

ddM1: Solves their $"9x^2 + 22x = 56"$ which must be a 3 term quadratic. May be implied by the correct answer for their equation.

A1cso: For $x = \frac{14}{9}$ from fully correct work. Ignore references to $x = -4$ if left in but A0 if any erroneous extra solutions are given. The derivative must have been correct. Note the same answer will follow if they have a constant term in their (otherwise correct) derivative.

(d)

(i) B1: $(-1, 84)$ (ii) B1: $(-4, 336)$

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