Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Pure Mathematics P1 (WMA11) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

- **B** marks are unconditional accuracy marks (independent of M marks)

- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given

- ⎾ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:

    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where} |pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx = 12\times\frac{x^4}{4} + \frac{1}{6}\times 2x^{\frac{1}{2}} - \frac{3}{2}\times\frac{x^{-3}}{-3}$	M1
	$= 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c$	A1A1A1A1
		(5)
		(5 marks)

M1 Applies $\int x^n\,dx \to x^{n+1}$ for at least one index.

The index must be processed so allow for $x^3 \to x^4$ or $\frac{1}{\sqrt{x}} \to x^{\frac{1}{2}}$ or $\frac{1}{x^4} \to x^{-3}$

A1 One correct term simplified or + c. Look for one of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$ or the + c.

A1 Two correct terms simplified or one correct simplified with + c.

Look for two of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$, $+c$

A1 Three correct terms simplified or two correct simplified with + c.

Look for three of $3x^4$, $+\frac{1}{3}x^{\frac{1}{2}}$, $+\frac{1}{2}x^{-3}$, $+c$

A1 $3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c$ all correct and simplified and on one line.

Allow simplified equivalents such as $\frac{1}{3}x^{\frac{1}{2}} \leftrightarrow \frac{1}{3}\sqrt{x}$ and $\frac{1}{2}x^{-3} \leftrightarrow \frac{1}{2x^3}$

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

E.g. $\int 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c\,dx$, $3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + c = 0$

Question Number	Scheme	Marks
2.	$y=3x^5+4x^3-x+5 \Rightarrow \left(\frac{dy}{dx}=\right)15x^4+12x^2-1$	M1 A1
	$15x^4+12x^2-1=2 \Rightarrow 15x^4+12x^2-3=0$	dM1
	$\Rightarrow 3(5x^2-1)(x^2+1)=0$ o.e	ddM1
	$\Rightarrow x=\pm\frac{1}{\sqrt{5}}$ o.e.	A1
		(5)
		(5 marks)

M1 Attempts to differentiate with $x^n \to x^{n-1}$ for one correct power

Allow for $x^5 \to x^4$ or $x^3 \to x^2$ or $x \to 1$

A1 $\left(\frac{dy}{dx}=\right)15x^4+12x^2-1$ which may be left unsimplified. Just look for a correct expression

i.e. no need to see $\frac{dy}{dx}=...$

dM1 Sets their $\frac{dy}{dx}=2$ and collects terms to one side to obtain a 3TQ in x^2

Depends on first method mark.

ddM1 Correct attempt to solve 3TQ in x^2. This may be by factorising, using the quadratic formula, or completing the square (see general guidance). The attempt to factorise must be consistent with their 3TQ.

The correct quadratic with the correct answers just written down scores M0

Must be solving for x^2 not x to obtain at least one value for x^2.

Depends on both previous method marks.

A1 $x=\pm\frac{1}{\sqrt{5}}$ or exact equivalent such as $\pm\frac{\sqrt{5}}{5}$, $\pm\sqrt{\frac{3}{15}}$ and isw once the correct answers are seen.

Must see both values so $x=\frac{1}{\sqrt{5}}$ is A0.

Ignore any attempts to find the y coordinates.

Question Number	Scheme	Marks
3.(i)	$\frac{3}{x} > 4 \Rightarrow 3x > 4x^2 \Rightarrow x(4x-3) < 0 \Rightarrow 0, \frac{3}{4}$	B1
	$0 < x < \frac{3}{4}$	M1 A1
		(3)
(ii)	$y - 0 = 3(x+5)$	B1
	E.g. $y < 2x^2 - 50,\ y > 3x + "15"$	M1
	E.g. $y < 2x^2 - 50,\ y > 3x + 15,\ x < -5$	A1
		(3)
		(6 marks)

(i)

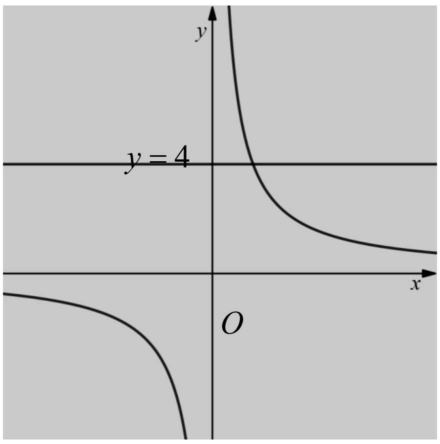
B1 For the two critical values 0 and $\frac{3}{4}$

M1 Chooses the inside region for their critical values

A1 $0 < x < \frac{3}{4}$ Award for exact equivalents such as $x > 0$ and $x < \frac{3}{4}$ or $\left\{x : x > 0 \cap x < \frac{3}{4}\right\}$

(Note they may deduce that $x > 0$ as $\frac{3}{x} > 4$ then solve to find $x < \frac{3}{4}$ which combined gives $0 < x < \frac{3}{4}$)

....



B1: Sketches BOTH graphs. May only see right had branch of
hyperbola.

M1: Chooses the inside region between 0 and their solution to $\frac{3}{x} = 4$

A1: $0 < x < \frac{3}{4}$

....

Special Case which is very common:

B1: States $x < \frac{3}{4}$ only

....

(ii)

B1 Correct equation for l E.g. $y - 0 = 3(x + 5)$. This may be implied by e.g. sight of $y > 3x + 15$ or e.g. $y = 3x + k$ and $k = 15$

M1 Two of $y < 2x^2 - 50$, $y > 3x + "15"$, $x < a$ where $-5 \leq a \leq 6.5$
Follow through their straight line provided it has a gradient of 3 with a numerical "15".
Also allow two of $y \leq 2x^2 - 50$, $y \geq 3x + 15$, $x \leq a$ where $-5 \leq a \leq 6.5$
Also allow $3x + 15 < y < 2x^2 - 50$ or $3x + 15 \leq y \leq 2x^2 - 50$ as 2 inequalities.
Do **not** allow inequalities in terms of R e.g. $R < 2x^2 - 50$, $R > 3x + 15$. This scores M0.

A1 Fully defines region. E.g. $y < 2x^2 - 50$, $y > 3x + 15$, $x < a$ where $-5 \leq a \leq 6.5$
Also allow $y \leq 2x^2 - 50$, $y \geq 3x + 15$, $x \leq a$ where $-5 \leq a \leq 6.5$
If set notation is used, then they must use "$\cap$" between any of their inequalities rather than "$\cup$".
Condone attempts as long as the intention is clear.
E.g.
$\{x, y \in \mathbb{R} : y < 2x^2 - 50 \cap y > 3x + 15 \cap x < a\}$, $\{x, y \in \mathbb{R} : y < 2x^2 - 50, y > 3x + 15, x < a\}$
are acceptable.

Note regarding consistency for the A1 if −5 is used:

$y < 2x^2 - 50$, $y > 3x + 15$ must go with $x < -5$
$y \leq 2x^2 - 50$, $y \geq 3x + 15$ must go with $x \leq -5$

If $-5 < a \leq 6.5$ is used then $x < a$ or $x \leq a$ is acceptable.

Question Number	Scheme	Marks
4.(a)(i)	$(90,-1)$	B1 B1
(ii)	225	B1
		(3)
(b)	One of $-1<p<0$, $p=1$	M1
	Both $-1<p<0$, $p=1$	A1
		(2)
		(5 marks)

(a) (i)

B1 One coordinate correct in the correct position. E.g $(180,-1)$

Award for $x=90$ or $y=-1$. Condone $x=90°$ and condone for $\frac{180}{2}$ for 90

B1 Fully correct $(90,-1)$ with or without brackets.

Award for $x=90$ and $y=-1$. Condone $x=90°$ and condone for $\frac{180}{2}$ for 90

Special Case: If they give $(-1,\ 90)$ or $\left(-1,\ \frac{\pi}{2}\right)$ rather than $(90,-1)$ allow B1B0

but e.g. $(-1,\ 180)$ scores B0B0 (2 errors)

Special case: If the 90 and –1 are clearly indicated on the axes for Q on the sketch, score B1B0

(a)(ii)

B1 225 Condone $225°$. Also allow (225, 0) or (225°, 0) or 225 or 225° marked at *R* on the diagram but if there is any ambiguity, the body of the script takes precedence.

For candidates who use radians in part (a), penalise this once on the first occurrence so e.g.

(a)(i) $\left(\frac{\pi}{2},-1\right)$ (a)(ii) $\frac{5\pi}{4}$ scores B1B0 B1

(a)(i) $\left(-1,\ \frac{\pi}{2}\right)$ (a)(ii) $\frac{5\pi}{4}$ scores B0B0 B1 (2 errors)

(b)

M1 One of the "correct" pair, but ft on "–1" from (a).

For example if they believe that (a)(i) is $(180,-2)$ M1 can be awarded for either $-2<p<0$ or $p=2$

Condone the use of y rather than p for this method mark e.g. condone $-1<y<0$ or $y=1$

A1 Fully correct in terms of *p*

A significant number of candidates are writing their answers within the question so be sure to check answers appearing there.

Question Number	Scheme	Marks
5(a)	$3y-2x=30 \Rightarrow m=\frac{2}{3}$	B1
	$y=-\frac{3}{2}(x-24),\ y=-\frac{3}{2}x+36,\ 2y+3x=72,\ \frac{y-0}{x-24}=-\frac{3}{2}$	M1 A1ft
	Full method to find one co-ordinate of P E.g. Solves $\frac{2}{3}x+10=-\frac{3}{2}(x-24)$	M1
	Coordinates of P $(12,18)$	A1
		(5)
(b)	$y=0,\ 3y-2x=30 \Rightarrow x=...$	M1 (B1 on EPEN)
	Area BPA = $\frac{1}{2}\times(24+"15")\times"18"=351$	dM1 A1 cso
		(3)
		(8 marks)

(a)

B1 Gradient of $l_1=\frac{2}{3}$ seen or implied

M1 Full method for equation of l_2. This involves an equation using the point $(24,0)$ with a gradient using the negative reciprocal of their $\frac{2}{3}$. If they use $y=mx+c$ then must reach as far as $c=\dots$

A1ft Correct equation of normal in any form e.g. $y=-\frac{3}{2}(x-24)$ but condone poor notation here and allow e.g. $l_2=-\frac{3}{2}(x-24)$ provided this is used appropriately to solve simultaneously. Follow through their negative reciprocal gradient.

M1 Full method to find one coordinate of P

Note that we allow a calculator to be used here e.g.
$3y-2x=30,\ 2y+3x=72 \Rightarrow x=...$ or $y=...$

A1 Coordinates of P $(12, 18)$ or $x=12, y=18$

Alt (a)

Note that the first three marks can be scored via

B1 Attempts l_2 via $2y+3x=c$

M1 Full method for equation of l_2. This involves an equation using the point $(24,0)$ and $2y+3x=c$

A1 $2y+3x=72$

(b)

M1(B1 on EPEN): Uses $y = 0$ in l_1 in an attempt to find the x coordinate of B.
Checking any working for this mark but it may be implied by $x = \pm 15$

dM1 Correct attempt at area BPA using their -15 and 18. This requires

$$\frac{1}{2}\times\left(24+"15"\right)\times"18" \text{ or equivalent}$$

correct work with their values e.g. $\frac{1}{2}\times\sqrt{(24-12)^2+18^2}\times\sqrt{(12+15)^2+18^2}$

Or e.g. "shoelace method"

$$\frac{1}{2}\begin{vmatrix}-15 & 24 & 12 & -15\\ 0 & 0 & 18 & 0\end{vmatrix} = \frac{1}{2}\left|0+24\times18+0-0-0+15\times18\right|$$

Depends on the first mark and depends on their B being a point on the x-axis.
The working for the area takes precedence over any diagrams they have drawn so if the working is correct but e.g. their diagram has points in the wrong positions, award the marks.
There may be other methods e.g. Hero's formula or use of trigonometry – if you are unsure if such attempts deserve credit , use review.

A1 351 cso

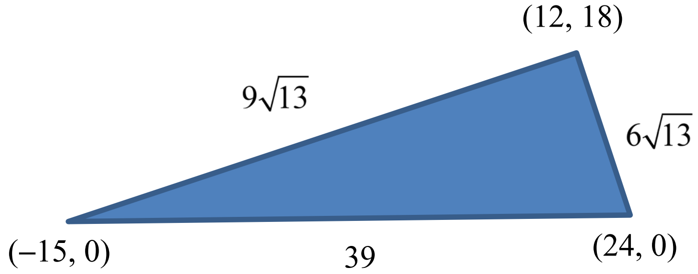
Common errors seen in marking:

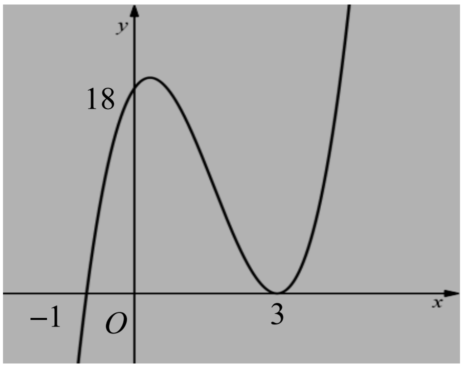
$$3y-2x=30 \Rightarrow l_1 : m=\frac{3}{2} \rightarrow l_2 : y=-\frac{2}{3}(x-24) \rightarrow P\left(\frac{36}{13},\frac{184}{13}\right),\ B\left(-\frac{20}{3},\ 0\right)$$

$$3y-2x=30 \Rightarrow l_1 : m=-\frac{2}{3} \rightarrow l_2 : y=\frac{3}{2}(x-24) \rightarrow P\left(\frac{276}{13},\frac{-54}{13}\right),\ B(15,\ 0)$$

With a correct triangle area method such attempts will often score (a) B0M1A1ftM1A0 (b) M1M1A0

Triangle for reference:



Question Number	Scheme		Marks
6.(a)		 Positive cubic shape anywhere with 1 maximum and 1 minimum	M1
		Positive cubic shape that at least reaches the x-axis at $x = -1$ and with a minimum on the x-axis at $x = 3$	A1
		y intercept at 18. Must correspond with their sketch	B1
	For the intercepts allow as numbers as above or allow as coordinates e.g. (18, 0), (0, –1), (0, 3) as long as they are marked in the correct place.		
			(3)
(b)	E.g. $(2x+2)(x^2-6x+9)=\ldots$		M1
	$= 2x^3-10x^2+6x+18$		A1 A1
			(3)
(c)	$\left(\text{f}'(x)=\right)6x^2-20x+6$		B1ft
	$\text{f}'\left(\frac{1}{3}\right)=6\left(\frac{1}{3}\right)^2-20\left(\frac{1}{3}\right)+6$		M1
	$\text{f}'\left(\frac{1}{3}\right)=0$		A1
	$y=\frac{512}{27}$		A1
			(4)
			(10 marks)

(a)

M1 Correct shape for a $y = +x^3$ graph. Do not be too concerned if the "ends" become vertical or even go beyond the vertical slightly. Condone with no axes and condone cusp like appearance for the turning points.

A1 $y = +x^3$ shape, intersects (or at least reaches the x-axis) at -1, minimum at $x = 3$ but must not stop or cross at $x = 3$

B1 y intercept at 18

You can ignore the position of the maximum i.e. it may be to the left of or right of or on the *y*-axis.

(b) Mark (b) and (c) together.

M1 Attempts to multiply out.

E.g. Look for an attempt to square $(x-3)$ to obtain $x^2 \pm 6x \pm 9$ and then an attempt to multiply by $(x+1)$ or $(2x+2)$ or an attempt to multiply $(x+1)$ or $(2x+2)$ by $(x - 3)$ and then multiply the result by $(x - 3)$

Condone slips e.g. attempting $(2x + 1)(x - 3)^2$ but expect to see an expression of the required form

A1 Two correct terms of $2x^3 - 10x^2 + 6x + 18$

A1 Fully correct $2x^3 - 10x^2 + 6x + 18$. (Ignore any spurious "= 0")

Special case: if they obtain $2x^3 - 10x^2 + 6x + 18$ but then attempt to "simplify" as e.g. $\mathrm{f}(x) = x^3 - 5x^2 + 3x + 9$ then score A1A0 **but note that all marks are available in (c) in such cases.**

(c)

B1ft Correctly differentiates their $2x^3 - 10x^2 + 6x + 18$. Allow follow through but only from a 4 term cubic.

Allow use of product rule e.g.

$\mathrm{f}(x) = 2(x+1)(x-3)^2 \rightarrow \mathrm{f}'(x) = 2(x-3)^2 + 4(x+1)(x-3)$

You can condone poor notation so just look for the correct or correct ft expression.

M1 Attempts $\mathrm{f}'\left(\frac{1}{3}\right)$

A1 Correctly achieves $\mathrm{f}'\left(\frac{1}{3}\right) = 0$. Must follow a correct derivative but allow this mark if they have e.g. divided their derivative by 2 before substituting or e.g. if they have divided their expanded f(x) by 2 before differentiating so they have $\mathrm{f}'(x) = 3x^2 - 10x + 3$ or if only their constant in their expansion in (b) is incorrect

The working may be minimal so allow e.g. $\mathrm{f}'\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2 - 20\left(\frac{1}{3}\right) + 6 = 0$ or even $\mathrm{f}'\left(\frac{1}{3}\right) = 0$ on its own following a correct derivative. Just look for the answer of 0 (e.g. they may call it y).

A1 $y = \frac{512}{27}$ or exact equivalent. All previous marks must have been scored in (c).

Acceptable alternative to show $\mathrm{f}'\left(\frac{1}{3}\right) = 0$:

$\mathrm{f}'(x) = 6x^2 - 20x + 6 = 0 \Rightarrow (6x-2)(x-3) = 0 \Rightarrow \mathrm{f}'(x) = 0$ at $x = \frac{1}{3} \Rightarrow \mathrm{f}'\left(\frac{1}{3}\right) = 0$

Score M1 for attempting to solve quadratic (usual rules) and A1 for deducing $\mathrm{f}'\left(\frac{1}{3}\right) = 0$

Question Number	Scheme	Marks
7.(a)	$OB^2 = 0.6^2 + 1.4^2 - 2\times 0.6\times 1.4\cos 2 \Rightarrow OB^2 = ...$ or $OB = ...$	M1
	$OB = 1.738$	A1
	$\frac{\sin AOB}{1.4} = \frac{\sin 2}{"1.738"} \Rightarrow AOB = 0.822$ or e.g. $\frac{\sin ABO}{0.6} = \frac{\sin 2}{"1.738"} \Rightarrow ABO = 0.319... \Rightarrow AOB = \pi - 2 - 0.319...$	dM1
	$\theta = 2\times AOB = 2\times 0.822 = 1.64*$	A1*
		(4)
(b)	Attempts $0.6\times\alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $2\times\pi\times 0.6 - 0.6\times 1.64$	M1
	$0.6\times(2\pi - 1.64) + 2.8 = 5.6\text{ m}$	A1
		(2)
(c)	Attempts $\frac{1}{2}\times 0.6^2\times\alpha$ with $\alpha = 2\pi - 1.64$ or $\alpha = \pi - 1.64$ or Attempts $\pi\times 0.6^2 - \frac{1}{2}\times 0.6^2\times 1.64$	M1
	Attempts $0.6\times 1.4\sin 2$	M1
	Full method $\frac{1}{2}\times 0.6^2\times(2\pi - 1.64) + 0.6\times 1.4\sin 2 = 1.6\text{ m}^2$	ddM1 A1
		(4)
		(10 marks)

In part (a), candidates may not be careful with the use of θ. E.g. in their working, their θ may be angle AOB which they correctly double to get 1.64 – condone this poor notation and give credit if the intention is clear.
Example:

$$OB^2 = 0.6^2 + 1.4^2 - 2\times 0.6\times 1.4\cos 2 \Rightarrow OB = 1.73756...$$
$$\cos\theta = \frac{0.6^2 + 1.738^2 - 1.4^2}{2\times 0.6\times 1.738} \Rightarrow \theta = 0.822$$
$$\theta = 2\times 0.822 = 1.64$$

Is acceptable for full marks in (a)

(a)

M1 Attempts the cosine rule to get OB or OB^2 **seen in part (a) only**

A1 OB = awrt 1.74 or truncated as 1.7 or e.g. 1.73 (may be implied)

dM1 Attempts the sine rule with the sides and angles in the correct positions in an attempt to find AOB or 1/2 AOB. **This must be a full attempt including using inverse sin to find the angle.**

This may be achieved by attempting the sine rule to find angle *ABO* first and then using the angle sum. This requires use of the sine rule with the sides and angles in the correct positions in an attempt to find *ABO* followed by $\pi-2-"0.319..."$ or e.g. $2\pi-4-2\times"0.319..."$

Alternatively uses the cosine rule to find angle *AOB*

E.g. $1.4^2=0.6^2+1.738^2-2\times0.6\times1.738\cos\frac{\theta}{2}\Rightarrow\frac{\theta}{2}=...$

Depends on the first method mark.

A1* Fully correct work leading to the given answer with no obvious rounding errors.
E.g. if they obtain angle *AOB* = 0.827 following correct a method then state 2×0.827 = 1.64 this mark should be withheld.

Alternatives working backwards:

$\theta=1.64\Rightarrow\frac{\theta}{2}=0.82,\ \frac{BC}{\sin 0.82}=\frac{0.6}{\sin(\pi-2-0.82)}\Rightarrow BC=1.4$ so $\theta=1.64$

Or $\theta=1.64\Rightarrow\frac{\theta}{2}=0.82,\ \frac{1.4}{\sin 0.82}=\frac{OC}{\sin(\pi-2-0.82)}\Rightarrow OC=0.6$ so $\theta=1.64$

M1: Finds $\frac{\theta}{2}$ and uses angle sum of triangle and sine rule
A1: Correct sine rule statement
dM1: Rearranges for *BC* or *OC*. **Depends on first method mark.**
A1: Fully correct work to obtain *BC* = 1.4 or *OC* = 0.6 with minimal conclusion e.g. tick, QED etc.

(b)

M1 Attempts $0.6\times\alpha$ with an allowable α
For an allowable angle accept $2\pi-1.64$ (awrt 4.64) or $\pi-1.64$ (awrt 1.50)
An alternative is to find the circumference and subtract the minor arc *AC*
For reference the correct value is 2.78… which may imply the method (3.7699… – 0.984)

A1 $0.6\times(2\pi-1.64)+2.8=$ awrt 5.6 m. Condone lack of units

(c) **In general the marks in part (c) are M1: Attempting the major sector, M2 attempting the kite area or half of it e.g. area of triangle *AOB* but not e.g. area of triangle *OCA* (they would need to find the area of triangle *ABC* as well, or half of both of these), dM3: A complete and correct method for the total area, A1: awrt 1.6**

M1 **This mark is for an attempt at the sector area *OCXA*:**
E.g. Attempts $\frac{1}{2}\times0.6^2\times\alpha$ with $\alpha=2\pi-1.64$ or $\alpha=\pi-1.64$
An alternative is to find the area of the circle and subtract the area of the minor sector
For reference the correct value is 0.835… which may imply the method (1.130… – 0.2952)

M1 **This mark is for an attempt at the kite *ABCO* (or half of it):**
Examples: Attempts $0.6\times1.4\sin 2$ which may be part of $\frac{1}{2}\times0.6\times1.4\sin 2$

Attempts *OCA* + *ABC* e.g. $\frac{1}{2}\times0.6^2\sin1.64+\frac{1}{2}\times1.4^2\sin(2\pi-4-1.64)$

Attempts $"1.74"\times 0.6\sin\frac{\theta}{2}$ which may be part of $\frac{1}{2}\times"1.74"\times 0.6\sin\frac{\theta}{2}$

ddM1 **This mark is for a complete and correct attempt at the total area and depends on <u>both</u> previous method marks:**

e.g. $\frac{1}{2}\times 0.6^2\times(2\pi-1.64)+0.6\times 1.4\sin 2$ o.e

A1 awrt 1.6 m^2 Condone a lack of units

Alternative which doesn't follow the above but is equivalent:

$$\text{Area} = \underline{\pi\times 0.6^2-\frac{1}{2}\times 0.6^2(1.64-\sin 1.64)}+\underline{\underline{\frac{1}{2}\times 1.4^2\sin(2\pi-4-1.64)}}$$

Award <u>M1</u> for the attempt at the major segment and $\underline{\underline{\text{M1}}}$ for the attempt at triangle *ABC* (or half of it) then ddM1A1 as above.

Some lengths and angles for reference:

$OB^2 = 3.019\ldots$
$OB = 1.7375\ldots$
$AC^2 = 0.7697\ldots$
$AC = 0.8773\ldots$
Angle $ABO = 0.322$
Angle $COA = 4.64\ldots$

Solutions where candidates change to degrees NB 2 radians is 114.591559…°		
(a)	$OB^2 = 0.6^2 + 1.4^2 - 2\times 0.6\times 1.4\cos 114.59\ldots \Rightarrow OB^2 = \ldots$ or $OB = \ldots$	M1
	$OB = 1.738$	A1
	$\dfrac{\sin AOB}{1.4} = \dfrac{\sin 114.59\ldots}{"1.738"} \Rightarrow AOB = 0.822$ or e.g. $\dfrac{\sin ABO}{0.6} = \dfrac{\sin 114.59\ldots}{"1.738"} \Rightarrow ABO = 18.3 \Rightarrow AOB = 180 - 114.59\ldots - 18.3$	M1
	$\theta = 2\times AOB = 2\times 47.1 = 94.2° = 1.64*$	A1*
		(4)
(b)	Attempts $\dfrac{\alpha}{360}\times 2\times \pi\times 0.6$ with $\alpha = 360 - \text{awrt } 94°$ or Attempts $2\times\pi\times 0.6 - \dfrac{\text{awrt } 94°}{360}\times 2\times\pi\times 0.6$	M1
	$\dfrac{360 - \text{awrt } 94}{360}\times 2\times\pi\times 0.6 + 2.8 = 5.6\text{ m}$	A1
		(2)
(c)	Attempts $\dfrac{\alpha}{360}\times\pi\times 0.6^2$ with $\alpha = 360 - \text{awrt } 94°$ or Attempts $\pi\times 0.6^2 - \dfrac{\text{awrt } 94°}{360}\times\pi\times 0.6^2$	M1
	Attempts e.g. $0.6\times 1.4\sin 114.59\ldots$	M1
	Full method e.g. $\dfrac{360 - \text{awrt } 94°}{360}\times\pi\times 0.6^2 + 0.6\times 1.4\sin 114.59\ldots = 1.6\text{ m}^2$	dM1 A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
8.(a)(i)	$4+12x-3x^2 = a \pm 3(x+c)^2$ or $a+b(x\pm 2)^2$	M1
	Two of $16-3(x-2)^2$ or two of $a=16, b=-3, c=-2$	A1
	$16-3(x-2)^2$	A1
(ii)	Coordinates $M=(2,16)$	B1ft B1ft
		(5)
(b)	States or implies that l_2 has equation $y = "8"x+k$	M1
	Sets $4+12x-3x^2 = "8x"+k$ and proceeds to 3TQ	dM1
	Correct 3TQ $3x^2-4x+k-4=0$	A1
	Attempts to use $b^2-4ac=0$ to find k	ddM1
	$k=\frac{16}{3} \Rightarrow y=8x+\frac{16}{3}$	A1
		(5)
		(10 marks)

(a)(i)

M1 For attempting to complete the square. Look for $b=\pm 3$ or $c=\pm 2$

A1 Two correct constants or two correct integers from $16-3(x-2)^2$

A1 $16-3(x-2)^2$ ($16-3(2-x)^2$ scores M1A1A0)

Alternative by comparing coefficients:

$$a+b(x+c)^2 = a+b(x^2+2xc+c^2) = bx^2+2bcx+a+bc^2$$

$$bx^2+2bcx+a+bc^2 \equiv 4+12x-3x^2$$

$$b=-3$$

$$2bc=12 \Rightarrow c=-2$$

$$a-12=4 \Rightarrow a=16$$

Score M1 for expanding $a+b(x+c)^2$ and compare x^2 coefficients to find a value for b

(NB this can be deduced directly and would score the M mark for $b=\pm 3$ as above)

A1: Continues the process and compares x coefficients to find both $b=-3$ and $c=-2$

A1: $a=16$

(a)(ii)

B1ft Either $x=2$ or $y=16$ but follow through on their $\mathbf{16}-3(x-\mathbf{2})^2$ where $a \neq 0$

B1ft Both $x=2$ and $y=16$ but follow through on their $(-c, a)$ from $\boldsymbol{a}+b(x+\boldsymbol{c})^2$ where $b \neq \pm 1$

For correct or correct ft coordinates the wrong way round e.g. (16, 2) score SC B1 B0 but apply isw if the correct or correct ft answers are seen as $x = \ldots, y = \ldots$

(b)

M1 States or implies that l_2 has equation $y="8"x+k,\ k\neq 0$

Follow through on their $y = "\frac{a}{c}"x+k$ or on $y=\left(\frac{y\text{ coordinate of their } M}{x\text{ coordinate of their } M}\right)x+k$

dM1 Sets $4+12x-3x^2 = "8x"+k$ and proceeds to 3TQ

A1 Correct 3TQ $3x^2-4x+k-4=0$ (The "= 0" may be implied by subsequent work)

ddM1 Attempts to use $b^2-4ac=0$ to find *k*.

A1 $k=\dfrac{16}{3} \Rightarrow y=8x+\dfrac{16}{3}$. Condone just $k=\dfrac{16}{3}$ if $y=8x+k$ was mentioned as the equation for l_2

Alternative for part (b)

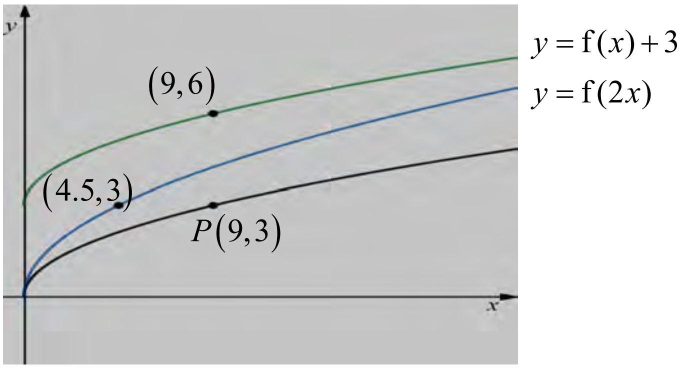
M1 Attempts to differentiate $4+12x-3x^2$ ($x^n \rightarrow x^{n-1}$ at least once) and sets equal to their 8

dM1 Solves for *x* and proceeds to find the coordinates of point of contact

A1 Tangent meets curve at $\left(\dfrac{2}{3},\dfrac{32}{3}\right)$ o.e.

ddM1 Substitutes their $\left(\dfrac{2}{3},\dfrac{32}{3}\right)$ in their $y="8"x+k$ to find *k*.

A1 $y=8x+\dfrac{16}{3}$

Question Number	Scheme	Marks
9(a)		
	One correct sketch drawn and labelled correctly	M1
	One correct sketch drawn and labelled and with correct point	A1
	Completely correct sketches with both points	A1
		(3)
(b)	Sets $\sqrt{x}+3=\sqrt{2x}$	B1
	$3=\left(\sqrt{2}-1\right)\sqrt{x}$	M1
	$\sqrt{x}=\dfrac{3}{\left(\sqrt{2}-1\right)}\times\dfrac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}=3\left(\sqrt{2}+1\right)$ *	A1*
		(3)
(c)	$\sqrt{x}=3\left(\sqrt{2}+1\right)\Rightarrow x=9\left(\sqrt{2}+1\right)^2=...$	M1
	$\Rightarrow x=9\left(3+2\sqrt{2}\right),\ y=3\sqrt{2}+6$	A1, B1
		(3)
		(9 marks)

(a) **Check <u>all 3</u> diagrams and score the best <u>single</u> diagram unless the candidate clearly indicates which one they want marked by e.g. crossing out the other(s).**

M1 One correct curve drawn and labelled correctly:
For $\mathrm{f}(2x)$ the curve should start at O and be above and remain above $\mathrm{f}(x)$ and not head back towards it significantly i.e. at least maintain the same gap.
For $\mathrm{f}(x)+3$ the curve should start on the positive y-axis and be approximately the same shape as $\mathrm{f}(x)$

A1 One correct curve drawn as above and **labelled** and with **correct point** for that curve.
The point does **not** have to be in the correct relative position – just look for values.

A1 Completely correct sketches with both points correct and at least one correctly labelled – you can assume the other is the other. Allow $\mathrm{f}(2x)$ to cross $\mathrm{f}(x)+3$ as long as it is beyond (9, 6) but with no other intersections for $x>0$

<u>The coordinates for the transformed *P* must be indicated on the sketch or if they are away from the sketch it must be clear which curve they relate to.</u>

For examples see below.

If you are in any doubt use review.

(b)

B1 Correct equation $\sqrt{x}+3=\sqrt{2x}$

M1 Writes $\sqrt{2x}$ as $\sqrt{2}\sqrt{x}$ and proceeds to collect terms in $\sqrt{x}$

Note that this may be achieved via e.g.

$$\sqrt{x}+3=\sqrt{2x}\Rightarrow\sqrt{x}+3=\sqrt{2}\sqrt{x}\Rightarrow 1+\frac{3}{\sqrt{x}}=\sqrt{2}$$

Or e.g. $\sqrt{x}+3=\sqrt{2x}\Rightarrow\sqrt{x}+3=\sqrt{2}\sqrt{x}\Rightarrow\frac{1}{3}+\frac{1}{\sqrt{x}}=\frac{\sqrt{2}}{3}$

A1* Proceeds to the given answer showing at least the steps

$$\sqrt{x}=\frac{3}{\left(\sqrt{2}-1\right)}\times\frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}=3\left(\sqrt{2}+1\right)$$

or e.g. $3=\left(\sqrt{2}-1\right)\sqrt{x}\Rightarrow 3\left(\sqrt{2}+1\right)=\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)\sqrt{x}=\sqrt{x}$

Attempts using e.g. $\sqrt{x}+3=\frac{\sqrt{x}}{2}$ score no marks in part (b)

Alternative:

B1 Correct equation $\sqrt{x}+3=\sqrt{2x}$

M1 $\sqrt{x}+3=\sqrt{2x}\Rightarrow x+6\sqrt{x}+9=2x\Rightarrow x-6\sqrt{x}-9=0$

$$x-6\sqrt{x}-9=0\Rightarrow\sqrt{x}=\frac{6\pm\sqrt{36+36}}{2}$$

Squares both sides and collects terms to obtain a 3TQ in $\sqrt{x}$ and attempts to solve for $\sqrt{x}$

e.g. using quadratic formula

A1* $\frac{6\pm\sqrt{36+36}}{2}=3\pm3\sqrt{2}=3\left(\sqrt{2}\pm1\right)\Rightarrow\sqrt{x}=3\left(\sqrt{2}+1\right)$

Simplifies and reaches the printed answer. If they give both answers score A0 but there is no requirement to explain why the other answer is rejected.

(c)

M1 Attempts to square the given expression to find x. Condone a slip on the 3 (it may remain 3) but must result in an expression of the form $\alpha+\beta\sqrt{2}$. (Working need not be shown as long as this condition is met)

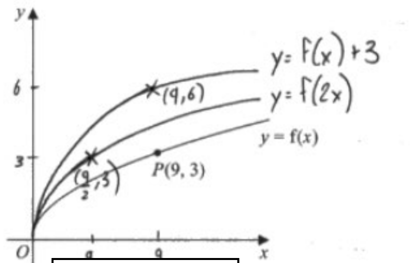
A1 $x=9\left(3+2\sqrt{2}\right)$ oe such as $x=27+18\sqrt{2}$

B1 $y=3\sqrt{2}+6$

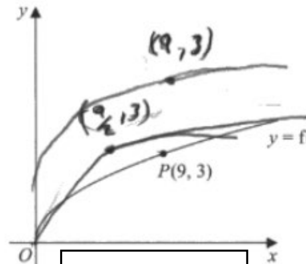
Note that $y=\sqrt{18\sqrt{2}+27}+3$ is correct but is not simplified so scores B0

Note that working for (c) must be seen in (c) i.e. do not allow working for (c) to be credited in parts (a) and (b) unless the answers are copied into (c)

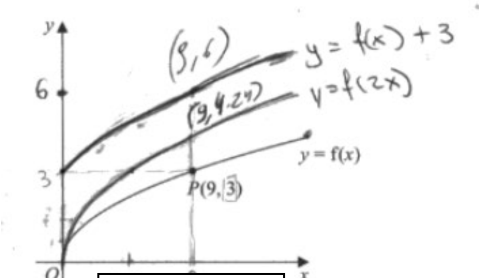
Example marking for sketches for 9(a):



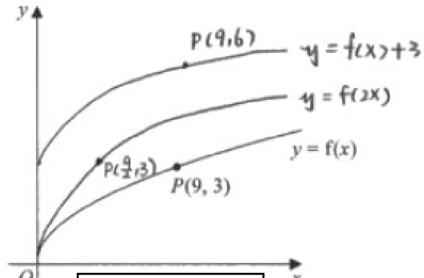
M1A1A0



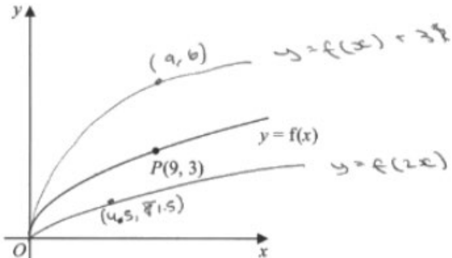
M0A0A0



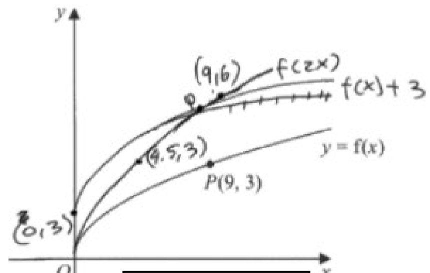
M1A1A0



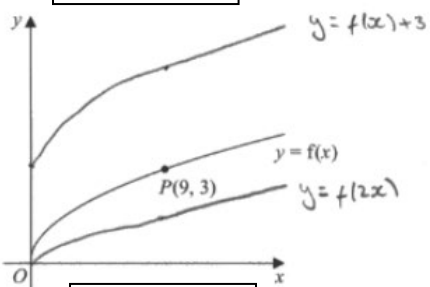
M1A1A1



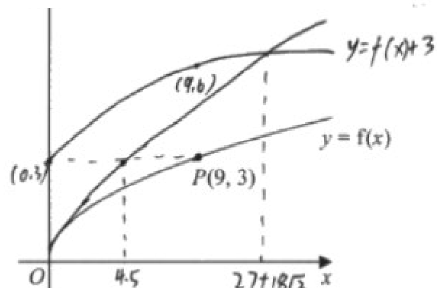
M0A0A0



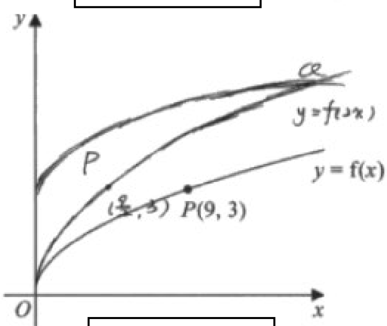
M1A1A0



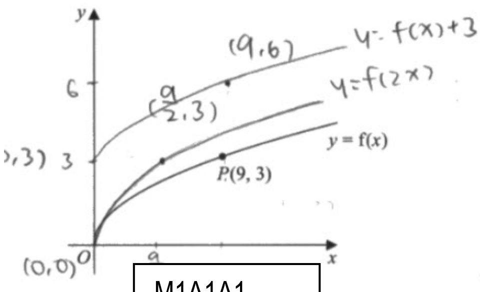
M1A0A0



M1A1A1



M1A1A0



M1A1A1

Question Number	Scheme	Marks
10(a)	$\mathrm{f}'(x)=ax-12x^{\frac{1}{3}} \Rightarrow \mathrm{f}''(x)=a-4x^{-\frac{2}{3}}$	B1
	Sets $\mathrm{f}''(27)=0 \Rightarrow 0=a-4\times\frac{1}{9} \qquad \Rightarrow a=\frac{4}{9}$	M1 A1
		(3)
(b)	$\mathrm{f}'(x)=ax-12x^{\frac{1}{3}} \Rightarrow \left(\mathrm{f}(x)=\right)\frac{1}{2}ax^{2}-9x^{\frac{4}{3}}+c$	M1 A1ft
	Substitutes $x=1, \mathrm{f}(x)=-8 \Rightarrow c=\ldots$	dM1
	$\left(\mathrm{f}(x)=\right)\frac{2}{9}x^{2}-9x^{\frac{4}{3}}+\frac{7}{9}$	A1
		(4)
		(7 marks)

Mark (a) and (b) together

Do not be too concerned with notation e.g. in (a) when they differentiate they may call it f′(*x*) or e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}$

(a)

B1 States or uses $\mathrm{f}''(x)=a-4x^{-\frac{2}{3}}$ which may be unsimplified

M1 Sets their $\mathrm{f}''(27)=0$ and proceeds to a value for *a*.

It is dependent upon one correct term in $\mathrm{f}''(x)$ e.g. *a* or $-4x^{-\frac{2}{3}}$ (oe)

A1 $a=\frac{4}{9}$

(b)

M1 Integrates $ax-12x^{\frac{1}{3}}$ with one term correct e.g. $\frac{1}{2}ax^{2}$ or $-\frac{12x^{\frac{4}{3}}}{\frac{4}{3}}$ with the indices processed.

A1ft $\mathrm{f}'(x)=ax-12x^{\frac{1}{3}} \Rightarrow \left(\mathrm{f}(x)=\right)\frac{1}{2}ax^{2}-9x^{\frac{4}{3}}+c$ follow through on *a* or a numerical *a* or a "made up" *a* but must include + *c*

Allow simplified or unsimplified so allow e.g. $\frac{\frac{4}{9}x^{2}}{2}-\frac{12x^{\frac{4}{3}}}{\frac{4}{3}}+c$

dM1 Substitutes $x=1$ and $\mathrm{f}(x)=-8$ to obtain a value for *c*. Must have numerical *a* now. **Depends on the first M mark.**

A1 $\left(\mathrm{f}(x)=\right)\frac{2}{9}x^{2}-9x^{\frac{4}{3}}+\frac{7}{9}$. Allow equivalent correct fractions for $\frac{2}{9}, \frac{7}{9}$ or recurring decimals e.g. $0.\dot{2}$, $0.\dot{7}$ with clear dots over the 2 and 7.

Allow e.g. $\sqrt[3]{x^4}$ for $x^{\frac{4}{3}}$

Look for a correct expression so no need to see f(x) = … and isw if necessary.

Note that a fairly common error is to obtain $a = -\frac{4}{9}$ in part (a) leading to $c = \frac{11}{9}$ in part (b)

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