Pearson Edexcel

Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level In Mechanics M3 (WME03) Paper 01

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January 2023
Question Paper Log Number P74312A
Publications Code WME03_01_MS_2301

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for this paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>‘M’ marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

‘A’ marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

‘B’ marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- * – The answer is printed on the paper
- – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking
(But note that specific mark schemes may sometimes override these general prinicíples)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF

- Use of g = 9.81 should be penalised once per (complete) question.
  N.B. Over-accuracy or under-accuracy of correct answers should only be penalized *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton’s Second Law (Equation of Motion)
NEL	Newton’s Experimental Law (Newton’s Law of Impact
HL	Hooke’s Law
SHM	Simple harmonic motion
PCLM	Principle of conversation of linear momentum
RHS, LHS	Right hand side, left hand side

Question Number	Scheme	Marks
1(a)	$\pi\int_0^1 (1+\sqrt{x})^2 \mathrm{d}x$	M1
	$=\pi\left[x+\frac{4}{3}x^{\frac{3}{2}}+\frac{1}{2}x^2\right]_0^1$	A1
	$=\frac{17\pi}{6}\ \mathrm{m}^3$ * including units	A1*
		(3)
(b)	$\pi\int_0^1 x(1+\sqrt{x})^2 \mathrm{d}x$	M1
	$=\pi\left[\frac{1}{2}x^2+\frac{4}{5}x^{\frac{5}{2}}+\frac{1}{3}x^3\right]_0^1$	A1
	$=\frac{49\pi}{30}$	A1
	$\bar{x}=\dfrac{\frac{49\pi}{30}}{\frac{17\pi}{6}}$	dM1
	$=\frac{49}{85}\ \mathrm{m}$ * including units	A1 *
		(5)
		(8)

Notes

NB: Penalise missing units maximum of once per question.

(a)	
M1	Use of $\pi\int_0^1 (1+\sqrt{x})^2 \mathrm{d}x$. Limits not needed. π is required.
A1	Correct integration – limits not needed
A1*	Correct given answer correctly obtained. Must include units. Limits must be seen (sight of substitution is not required). Accept $\frac{17}{6}\pi\ \mathrm{m}^3$
(b)	
M1	Use of $\pi\int_0^1 x(1+\sqrt{x})^2 \mathrm{d}x$. Limits not needed ($\pi$'s will cancel so it may not be seen)
A1	Correct integration – limits not needed
A1	Correct unsimplified with or without π (may see $\frac{1}{2}+\frac{4}{5}+\frac{1}{3}-0$)
dM1	Correct expression with their numerator (consistent π - seen in neither or both)
A1*	Correct given answer correctly obtained. Must include units.

Question Number	Scheme	Marks	
2.	$F\cos\alpha = mg$	M1	A1
	$F\sin\alpha = T$		A1
	$T = \dfrac{2mgx}{l}$ or $T = \dfrac{2mg(AB-l)}{l}$	M1	
	$\dfrac{3}{4}mg = \dfrac{2mgx}{l}$	dM1	
	$AB = \dfrac{11l}{8}$	A1	
		(6)	

Notes	
M1	Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)
A1	Correct vertical equation
A1	Correct horizontal equation (A2 for $T = mg\tan\alpha$ from triangle of forces)
M1	Hooke's Law. Must clearly be an extension and not *AB*. Since *x* is not defined in the question, other extensions may be used including ($AB - l$) or xl where *x* is found to be the constant $\frac{3}{8}$.
dM1	Substitute trig (not necessarily correctly) to produce an equation in '*x*' (and *l*) only, dependent on previous M's and on having two equations.
A1	Cao Accept $1.375l$, $1.4l$, $1.38l$

Question Number	Scheme	Marks
3(a)	Slant height, $l = \sqrt{\left(\frac{7a}{4}\right)^2 + (6a)^2} \;\; (= \frac{25a}{4})$	M1
	Masses Square $\quad 16a^2$ Circle $\quad \pi\left(\frac{7a}{4}\right)^2$ Conical shell $\quad \pi \times \frac{7a}{4} \times \frac{25a}{4}$ Total $\quad \left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right]$	B1 square B1 circle B1ft (shell and total)
	Distances Square 0, Circle 0, Conical shell $2a$: Total $\bar{x}$	B1
	$\pi \times \frac{7a}{4} \times \frac{25a}{4} \times 2a = \left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right]\bar{x}$	M1 A1
	$\bar{x} = \frac{175\pi a}{(63\pi + 128)}$ *	A1*
		(8)
3(b)	$\tan\alpha = \frac{2a}{\left(\frac{175\pi a}{(63\pi+128)}\right)}$	M1
	$\tan\alpha = \frac{126\pi + 256}{175\pi} \quad$ (or $\frac{2(63\pi+128)}{175\pi}$)	A1
		(2)
		(10)

Notes	
(a)	
M1	Use of Pythagoras (unsimplified). May be seen on the diagram.
B1	Mass/area of square
B1	Mass/area of circle
B1 ft	Mass/area of conical shell and total. A common error is to use $6a$ as slant height, only ft on their calculated slant height. May derive conical shell formula from area of a sector.
B1	All distances correct
M1	Dimensionally correct moments equation. Must have correct number of terms including an attempt to subtract the circle. Condone a slip with an 'a' in one term.
A1	Correct equation (no ft)
A1*	Given answer correctly obtained. Condone missing brackets from denominator and terms reversed.
(b)	
M1	Allow reciprocal. Must use $2a$ and given $\bar{x}$.
A1	Cao Exact fraction required.

Question Number	Scheme	Marks
4(a)	$a = v\dfrac{dv}{dx}$	M1
	$= \dfrac{3}{2}(2x+1)^{\frac{1}{2}} \times 2 \times (2x+1)^{\frac{3}{2}} = 3(2x+1)^2$	A1
	$3(2x+1)^2 = 243$	M1
	$x = 4$	A1
		(4)
4(b)	$(2x+1)^{\frac{3}{2}} = \dfrac{dx}{dt}$ **OR** $a = 3v^{\frac{4}{3}} = \dfrac{dv}{dt}$	M1 A1
	$\int dt = \int (2x+1)^{-\frac{3}{2}} dx$ $\quad$ $\int 3dt = \int v^{-\frac{4}{3}} dv$	M1
	$t = -(2x+1)^{-\frac{1}{2}} (+C)$ $\quad$ $3t + (C) = -3v^{-\frac{1}{3}}$	A1
	$t = 0,\ x = 0 \Rightarrow C = 1$ $\quad$ $t = 0,\ x = 0 \Rightarrow v = 1 \Rightarrow C = -3$ and obtain an equation in v and t only.	M1
	$v = \dfrac{1}{(1-t)^3}$	A1
		(6)
		(10)

Notes

(a)	
M1	Use of $a = v\dfrac{dv}{dx}$ or $a = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$. Evidence of differentiation, power decreasing by 1. Should see a product of terms to imply 'use of'.
A1	Correct differentiation
M1	Independent. Use their result from differentiation and put a = 243 then solve for x
A1	Cao If -5 is seen then it must be rejected or 4 must be clearly identified.
(b)	
M1	Use of $v = \dfrac{dx}{dt}$ to obtain DE in x and t **OR** Use of $a = \dfrac{dv}{dt}$ to obtain DE in v and t
A1	Correct equation
M1	Separate and integrate (evidence of integration, power increasing by 1)
A1	Correct integration, condone missing C
M1	Use $t = 0$, $x = 0$ to obtain a value of C and obtain an equation in v and t *only*.
A1	Cao Accept $v = (1-t)^{-3}$ or $v = \dfrac{-1}{(t-1)^3}$ or $v = -(t-1)^{-3}$
	Note: No marks in (b) for use of a = 243

Question Number	Scheme	Marks
5(a)	Use of cosine rule on triangle *APB* **OR** trig. on 'half' of the triangle *APB* to find one relevant angle.	M1
	Given answers correctly obtained.*	A1*
		(2)
5(b)	$T_A \cos 30^o + T_B \cos 60^o = mg$	M1 A1
	$T_A \sin 30^o + T_B \sin 60^o = mr\omega^2$	M1A1A1
	$r = a\sin 60^o$ (or $r = a\sqrt{3}\cos 30$ or $r = a\dfrac{\sqrt{3}}{2}$)	B1
	Solve for T_A	dM1
	$T_A = \frac{1}{2}m\sqrt{3}(2g - a\omega^2)$ *	A1*
		(8)
5(c)	Attempt to obtain one inequality on ω^2	M1
	Correct inequality	A1
	Attempt to obtain another inequality on ω^2 and use both to obtain answer	M1
	$\dfrac{2g}{3a} < \omega^2 < \dfrac{2g}{a}$ *	A1 *
		(4)
		(14)

Notes

(a)	
M1	Either complete method to obtain one relevant angle.
A1*	Correct GIVEN angles correctly obtained. Sufficient annotation/justification leading to both given answers eg Stating $\angle OBP = 2 \times \angle OAP$ alone is not sufficient – additional annotation or justification is required. Use of triangles to verify is acceptable.
(b)	
M1	Resolve vertically, dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.
A1	Correct equation
M1	Equation of motion horizontally: dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.
A1	Correct equation, with at most one error. If $r\omega^2$ is never seen, this is an A error.
A1	Correct equation
B1	Cao If this is seen in (a) it must be used in (b) for this mark.
dM1	Solve for T_A in terms of *m*, *a*, *g* and ω
A1*	Given answer correctly obtained. Must see exactly.
(c)	
M1	Correct use of either $T_A > 0$ or their $T_B > 0$ oe to obtain one inequality on ω^2. Could be their expression for either Tension > 0.
A1	Correct inequality
M1	Use both $T_A > 0$ and their $T_B > 0$ to form inequalities in attempt to obtain answer. Could be their expression for either Tension > 0. Note: $T_B = \frac{3}{2}ma\omega^2 - mg$
A1*	Given answer correctly obtained

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - mgl$ or $mgl - \frac{1}{2}mv^2$ seen or implied	B1
	Use of EPE	M1
	$\frac{mg}{2l}l^2$	A1
	$\frac{mg}{2l}(l\sqrt{2}-l)^2$	A1
	$\frac{1}{2}mv^2 + \frac{mg}{2l}(l\sqrt{2}-l)^2 = mgl + \frac{mg}{2l}l^2$	M1
	Solve for v^2	dM1
	$v^2 = 2gl\sqrt{2}$ *	A1*
		(7)
(b)	$T = \frac{mg(l\sqrt{2}-l)}{l} = mg(\sqrt{2}-1)$	M1 A1
	$\pm N + T\cos 45^o = \frac{mv^2}{l}$	M1A1A1
	$\pm N + mg(\sqrt{2}-1)\times\frac{\sqrt{2}}{2} = \frac{m}{l}\times 2gl\sqrt{2}$	dM1
	$N = \frac{1}{2}mg(5\sqrt{2}-2)$ *	A1*
		(7)
		(14)

Notes	
(a)	
B1	Difference between KE and GPE, seen either way round.
M1	Use of EPE formula at top or at *B*
A1	Correct EPE at top
A1	Correct EPE at B
M1	Use of conservation of energy, with 1 GPE, 1 KE and 2 EPE terms, condone sign errors
dM1	Solve for v^2, dependent on previous M
A1*	Exact given answer correctly obtained
(b)	
M1	Use of Hooke's Law at *B* – this may appear in an attempted equation of motion
A1	Correct unsimplified tension at *B*
M1	Equation of motion at *B* horizontally with correct terms, condone sign errors
A1	Correct equation with at most one error
A1	Correct equation
dM1	Sub for *T* and v^2. Dependent on both previous M marks
A1*	Given answer correctly obtained (exactly). If $N = -\frac{1}{2}mg\left(5\sqrt{2}-2\right)$ then clear justification is required to reach the given answer eg use of 'magnitude' or modulus signs.

Question Number	Scheme	Marks
7(a)	$T_A - T_B = m\ddot{x}$	M1
	$\frac{2mg}{l}\left(\frac{2l}{3}-x\right)-\frac{mg}{l}\left(\frac{4l}{3}+x\right)=m\ddot{x}$ or $\frac{mg}{l}\left(\frac{4l}{3}-x\right)-\frac{2mg}{l}\left(\frac{2l}{3}+x\right)=m\ddot{x}$.	dM1A1
	$-\frac{3g}{l}x=\ddot{x}$, so SHM	A1
	$T=\frac{2\pi}{\sqrt{\frac{3g}{l}}}=2\pi\sqrt{\frac{l}{3g}}$ *	M1 A1*
		(6)
7(b)	$\frac{1}{2}l\times\sqrt{\frac{3g}{l}}$ or $\frac{1}{2}\sqrt{3gl}$ or $\sqrt{\frac{3gl}{4}}$ oe	B1
		(1)
7(c)	$\frac{3g}{2}$ or $1.5g$	B1
		(1)
7(d)	$x=a\cos\omega t \Rightarrow v=-a\omega\sin\omega t$	M1
	$-\frac{3}{4}\sqrt{gl}=-a\omega\sin\omega t$ to find t	M1A1
	Solve for t	M1
	$t=\frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe	A1
		(5)
		(13)

Notes	
(a)	
M1	Equation of motion in a *general* position, allow a for acceleration, correct no. of terms, condone sign errors.
dM1	Use Hooke's Law to sub for the two tensions, allow a for acceleration. Extensions must be different and of the form $(d \pm x)$ where d is a multiple of l.
A1	Correct unsimplified equation, allow a for acceleration.
A1	Correct equation using $\ddot{x}$ for acceleration.
M1	Use of $\frac{2\pi}{\omega}$ Their ω from their equation of motion, which must be in terms of x.
A1*cso	Given answer correctly obtained – this includes proof of SHM **with conclusion** and correct expression for the period.
(b)	
B1	Cao Speed at O so must be positive. Unsimplified, ignore errors from subsequent 'simplifying' of surds.
(c)	
B1	Cao Max acceleration so must be positive.

(d)	
Main	
M1	Use of $x = a\cos\omega t$ to obtain $v = -a\omega\sin\omega t$ Substitution for a and ω is not required.
M1	Use $v = -a\omega\sin\omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $-\frac{3}{4}\sqrt{gl} = -a\omega\sin\omega t$
A1	Correct equation in t only
M1	Solve to find the required time, t
A1	Cao for required time.
ALT 1	
M1	Use of $x = a\sin\omega t$ to obtain $v = a\omega\cos\omega t$ Substitution for a and ω is not required.
M1	Use $v = a\omega\cos\omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $\frac{3}{4}\sqrt{gl} = a\omega\cos\omega t$
A1	Correct equation in t only
M1	Solve to find t and then subtract from $\frac{1}{4}$ period to find the required time. Eg $t = \frac{\pi}{6}\sqrt{\frac{l}{3g}} \Rightarrow$ required time = $\frac{1}{4}\left(2\pi\sqrt{\frac{l}{3g}}\right) - \frac{\pi}{6}\sqrt{\frac{l}{3g}} = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$
A1	Cao for required time, $t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe
ALT2	
M1	Use of $x = a\cos\omega t$ or use of $x = a\sin\omega t$. Substitution for a and ω is not required.
M1	Using $v^2 = \omega^2(a^2 - x^2)$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in x only. $\left(-\frac{3}{4}\sqrt{gl}\right)^2 = \omega^2(a^2 - x^2)$
A1	Correct equation in x only. (Solution leads onto the first M mark in (d))
M1	Solves for t and then completes the method to find the required time. e.g. $\frac{l}{4} = \frac{l}{2}\cos\left(\sqrt{\frac{3g}{l}}t\right)$ or quarter period with sin method.
A1	Cao for required time, $t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe
SPECIAL CASE where *a* = ½ is clearly stated as amplitude and consistently used in (b) (c) & (d)	
(b)	B1 $\frac{1}{2}\sqrt{\frac{3g}{l}}$
(c)	B1 $\frac{3g}{2l}$
(d)	Maximum M1 M1 A0 M0 A0

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