Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Tuesday 11 June 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. The hyperbola H has

- foci with coordinates $\left(\pm\dfrac{13}{2}, 0\right)$
- directrices with equations $x = \pm\dfrac{72}{13}$
- eccentricity e

Determine

(a) the value of e
(3)

(b) an equation for H, giving your answer in the form $px^2 - qy^2 = r$, where p, q and r are integers.
(3)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & 0 \end{pmatrix}$$

Given that $\mathbf{M}$ has exactly two distinct eigenvalues λ_1 and λ_2 where $\lambda_1 < \lambda_2$

(a) determine a normalised eigenvector corresponding to the eigenvalue λ_1

(6)

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, where μ is a scalar parameter.

The transformation T is represented by $\mathbf{M}$.

The line l_1 is transformed by T to the line l_2

(b) Determine a vector equation for l_2, giving your answer in the form $\mathbf{r} \times \mathbf{b} = \mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are constant vectors.

(3)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

P 7 5 7 1 7 A 0 7 3 2

Turn over

3. $$y = \operatorname{arsinh}\left(\sqrt{x^2 - 1}\right) \qquad x > 1$$

(a) Prove that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 - 1}}$

(3)

$$f(x) = \frac{1}{3}\operatorname{arsinh}\left(\sqrt{x^2 - 1}\right) - \arctan x \qquad x > 1$$

(b) Determine the exact values of x for which $f'(x) = 0$

(4)

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Question 3 continued

(Total for Question 3 is 7 marks)

4. (a) Use the definitions of hyperbolic functions in terms of exponentials to show that

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B$$

(3)

(b) Hence express $10\sinh x + 8\cosh x$ in the form $R\sinh(x + \alpha)$ where $R > 0$, giving α in the form $\ln p$ where p is an integer.

(4)

(c) Hence solve the equation

$$10\sinh x + 8\cosh x = 18\sqrt{7}$$

giving your answer in the form $\ln\left(\sqrt{7} + q\right)$ where q is a rational number to be determined.

(2)

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Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. $$4x^2 + 4x + 17 \equiv (2x + p)^2 + q$$

where p and q are integers.

(a) Determine the value of p and the value of q

(2)

Given that

$$\frac{8x+5}{\sqrt{4x^2+4x+17}} \equiv \frac{1}{\sqrt{4x^2+4x+17}} + \frac{Ax+B}{\sqrt{4x^2+4x+17}}$$

where A and B are integers,

(b) write down the value of A and the value of B

(1)

(c) Hence use algebraic integration to show that

$$\int_{\frac{1}{3}}^{1} \frac{8x+5}{\sqrt{4x^2+4x+17}}\,\mathrm{d}x = k + \frac{1}{2}\ln k$$

where k is a rational number to be determined.

(5)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The line l is the normal to E at the point $P(5\cos\theta, 3\sin\theta)$ where $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$5x\sin\theta - 3y\cos\theta = 16\sin\theta\cos\theta$$

(4)

Given that

- l intersects the y-axis at the point Q
- the midpoint of the line segment PQ is M

(b) determine the exact maximum area of triangle OMP as θ varies, where O is the origin.

You must justify your answer.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

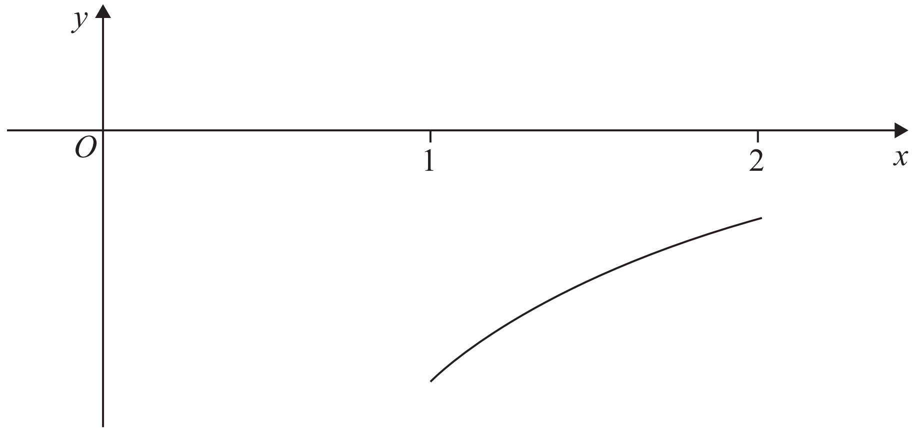

Figure 1

Figure 1 shows the curve with equation

$$y = \ln\left(\tanh\frac{x}{2}\right) \qquad 1 \leqslant x \leqslant 2$$

(a) Show that the length, s, of the curve is given by

$$s = \int_1^2 \coth x \, dx$$

(4)

(b) Hence show that

$$s = \ln\left(e + \frac{1}{e}\right)$$

(4)

Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. $$I_n = \int_0^k x^n (k - x)^{\frac{1}{2}} \, \mathrm{d}x \qquad n \geqslant 0$$

where k is a positive constant.

(a) Show that

$$I_n = \frac{2kn}{3 + 2n} I_{n-1} \qquad n \geqslant 1$$

(5)

Given that

$$\int_0^k x^2 (k - x)^{\frac{1}{2}} \, \mathrm{d}x = \frac{9\sqrt{3}}{280}$$

(b) use the result in part (a) to determine the exact value of k.

(4)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 9 marks)

9. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where s and t are scalar parameters.

(a) Determine a Cartesian equation for Π_1

(3)

The plane Π_2 has vector equation $\mathbf{r}.\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 1$

(b) Determine a vector equation for the line of intersection of Π_1 and Π_2

Give your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where **a** and **b** are constant vectors and λ is a scalar parameter.

(4)

The plane Π_3 has Cartesian equation $4x - 3y - z = 0$

(c) Use the answer to part (b) to determine the coordinates of the point of intersection of Π_1, Π_2 and Π_3

(3)

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Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS