Mark Scheme (Results)

Summer 2019

Pearson Edexcel International A Level
In Further Pure Mathmatics F3
(WFM03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme		Marks
1.(a)	$ae=6, \quad a^2\left(e^2-1\right)=9$	Both correct equations needed but need not be shown explicitly	B1
	$e=\dfrac{6}{a} \Rightarrow 36-a^2=9 \Rightarrow a=...$ Or $a=\dfrac{6}{e} \Rightarrow 36-\dfrac{36}{e^2}=9 \Rightarrow e=\dfrac{2\sqrt{3}}{3}$ $\Rightarrow a=\dfrac{6}{e}=...$	Eliminates e from their 2 equations to obtain an equation in a and solves for a or a^2 **or** Eliminates a from their 2 equations to find e and then finds a	M1
	$a=\sqrt{27}$ or $3\sqrt{3}$	Correct exact value. $a=\pm\sqrt{27}$ is A0 unless $-\sqrt{27}$ is rejected.	A1
			(3)
(b)	$e^2-1=\dfrac{9}{27} \quad e^2=\dfrac{36}{27} \quad e=\dfrac{2}{\sqrt{3}}$ or $\dfrac{2\sqrt{3}}{3}$ Finds a numerical value for e **<u>using a correct identity</u>**. This mark can be awarded if e has been found in part (a) or may be seen as part of their calculation to find $\dfrac{a}{e}$		M1
	$(x=)(\pm)\dfrac{a}{e}=...$	Obtains a numerical value for x using a correct equation for at least one directrix with their a and e (signs can be ignored)	M1
	$x=\pm\dfrac{9}{2}$	**Two** correct **equations**. Allow equivalents but must be **simplified**. So allow $x=\pm\dfrac{9}{2}$, $x=\pm 4.5$	A1
			(3)
			[Total 6]

Note: Use of $b^2=a^2\left(1-e^2\right)$ can score (a) B0 M1 A0 and (b) M0 (if used again) M1 A0

Question Number	Scheme		Marks
2 (a)(i)	$2\cosh^2 x-1=2\frac{\left(e^x+e^{-x}\right)^2}{4}-1=\frac{\left(e^{2x}+2e^x\times e^{-x}+e^{-2x}\right)}{2}-1$ Substitutes the correct definition for cosh$x$ into the rhs and squares - full expansion must be seen but allow 2 for $2e^x\times e^{-x}$		M1
	$=\frac{\left(e^{2x}+e^{-2x}\right)}{2}+1-1=\cosh 2x*$	Correct completion with no errors seen.	A1
	Working from left to right:		
	$\cosh 2x=\frac{\left(e^{2x}+e^{-2x}\right)}{2}=\frac{\left(e^x+e^{-x}\right)^2-2}{2}$ Uses the correct definition for cosh2x on lhs and expresses in terms of $\left(e^x+e^{-x}\right)^2$.		M1
	$2\cosh^2 x-1*$	Correct completion with no errors seen.	A1
(ii)	$2\sinh x\cosh x=2\frac{\left(e^x-e^{-x}\right)}{2}\times\frac{\left(e^x+e^{-x}\right)}{2}=...$ Use both correct definitions on rhs and attempts to multiply $2\sinh x\cosh x=\frac{1}{2}\left(e^x-e^{-x}\right)\left(e^x+e^{-x}\right)=...\text{scores M0}$ as the definitions for sinh$x$ and cosh$x$ have not been seen		M1
	$\frac{\left(e^{2x}-e^{-2x}\right)}{2}=\sinh 2x*$	Correct completion with no errors seen.	A1
	Working from left to right:		
	$\sinh 2x=\frac{\left(e^{2x}-e^{-2x}\right)}{2}=\frac{\left(e^x+e^{-x}\right)\left(e^x-e^{-x}\right)}{2}$ Uses the correct definition for sinh2x on lhs and uses the difference of 2 squares.		M1
	$2\sinh x\cosh x*$	Correct completion with no errors seen.	A1

If they work from both ends then a clear link must be established as a conclusion e.g. lhs = rhs, tick QED etc.

(b)	$2\cosh^2 x - 1 - 7\cosh x + 7 = 0$	Use the identity for cosh2x	M1
	$2\cosh^2 x - 7\cosh x + 6 = 0 \Rightarrow (2\cosh x - 3)(\cosh x - 2) = 0 \Rightarrow \cosh x = ...$ Solve their 3TQ in cosh x (the usual rules for solving can be applied if necessary)		M1
	$\cosh x = \frac{3}{2}, 2$	Correct answers, both needed	A1
	$\cosh x = \alpha \Rightarrow x = \ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ **or** $\frac{e^x + e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0$ or $\frac{e^x + e^{-x}}{2} = \frac{3}{2} \Rightarrow e^{2x} - 3e^x + 1 = 0$ $\Rightarrow e^x = \frac{4 \pm \sqrt{12}}{2}$ or $e^x = \frac{3 \pm \sqrt{5}}{2}$ $\Rightarrow x = \ln ...$ Changes at least one arcosh to ln form either using the correct ln form of arcosh or by returning to the correct exponential form of cosh and solving a quadratic in e^x. (Note that returning to exponentials is more likely to give all 4 answers below)		M1
	$x = \ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right),\ -\ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)\left(\text{or } \ln\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right)\right),$ $\ln\left(2 + \sqrt{3}\right), -\ln\left(2 + \sqrt{3}\right)\left(\text{or } \ln\left(2 - \sqrt{3}\right)\right)$ All 4 correct, must be exact logarithms but can be any equivalent to those shown **with brackets**. Allow unsimplified if necessary and apply isw e.g. allow $\ln\left(\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1}\right)$ for $\ln\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$		A1
			(5)
			[Total 9]

	Alternative for 2(b) using exponentials:	
	$\cosh 2x - 7\cosh x = -7 \Rightarrow \frac{e^{2x}+e^{-2x}}{2} - 7\left(\frac{e^{x}+e^{-x}}{2}\right) = -7$ $\Rightarrow e^{4x} - 7e^{3x} + 14e^{2x} - 7e^{x} + 1 = 0$ Substitutes the correct exponential forms and forms quartic in e^x	M1
	$e^{4x} - 7e^{3x} + 14e^{2x} - 7e^{x} + 1 = 0 \Rightarrow \left(e^{2x} - 4e^{x} + 1\right)\left(e^{2x} - 3e^{x} + 1\right) = 0$ $\Rightarrow e^{x} = \frac{4 \pm \sqrt{12}}{2}$ or $e^{x} = \frac{3 \pm \sqrt{5}}{2}$ M1: Attempts to solve one of their quadratics in e^x which has come from their quartic in e^x to **obtain exact values** for e^x A1: For at least 2 **exact** values of e^x	M1A1
	$\Rightarrow e^{x} = \frac{4 \pm \sqrt{12}}{2}$ or $e^{x} = \frac{3 \pm \sqrt{5}}{2}$ $\Rightarrow x = \ln ...$ Change at least one exponential form to ln form	M1
	$\Rightarrow x = \ln\left(\frac{4 \pm \sqrt{12}}{2}\right)$ and $x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$ All 4 correct, must be exact logarithms but can be any equivalent to those shown with brackets if necessary but e.g. they would not be required in the above forms.	A1

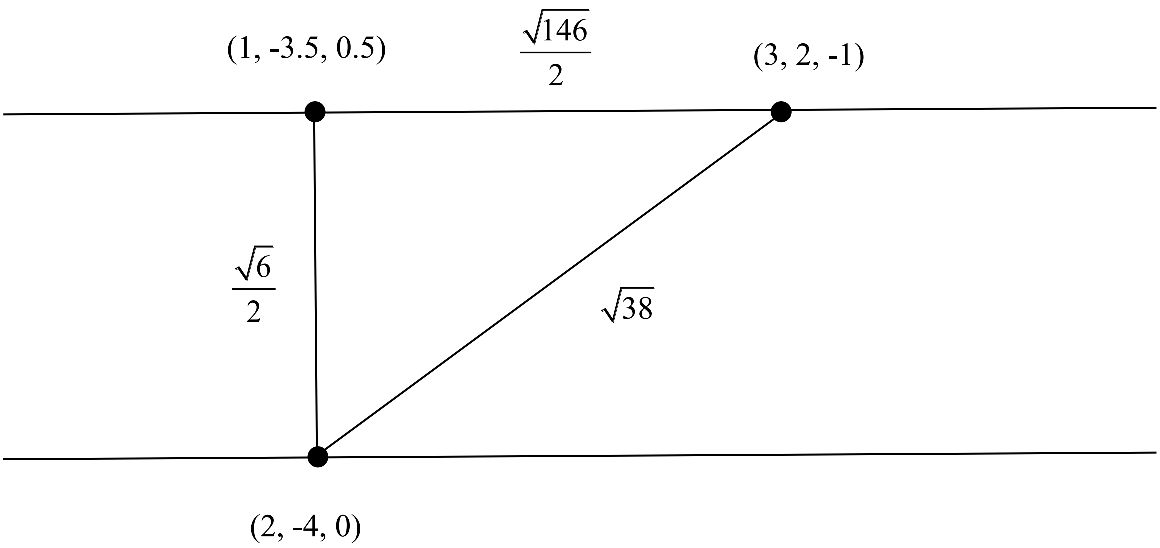
Question Number	Scheme		Marks
3(a)	$8+4x+x^2=(x+2)^2+4$	Correct completion of the square	B1
	$\int \frac{1}{(x+2)^2+4}\,dx=\frac{1}{2}\arctan\frac{x+2}{2}\quad(+c)$ M1: For obtaining $k\arctan f(x)$ Accept other notation for arctan e.g. artan, $\tan^{-1}$ etc. A1: Correct result oe e.g. $\frac{1}{2}\arctan\left(\frac{x}{2}+1\right)(+c)$ **(must see brackets in this case)** The constant of integration is **not** required		M1A1
	May see substitution e.g. $x+2=2\tan u\Rightarrow\int\frac{1}{(x+2)^2+4}\,dx=\int\frac{1}{4\tan^2u+4}2\sec^2u\,du$ $=\frac{1}{2}\int du=\frac{1}{2}u(+c)=\frac{1}{2}\arctan\left(\frac{x+2}{2}\right)(+c)$ For M1 this requires a complete method using a **correct substitution** and including the reversal of the substitution and A1 as already defined Accept other notation for arctan e.g. artan, $\tan^{-1}$ etc.		
			(3)

(b)	$8-4x-x^2=12-(x+2)^2$	For an attempt to complete the square. Allow $8-4x-x^2=\alpha-(\pm x\pm 2)^2$ **Where α > 0**	M1
		$12-(x+2)^2$	A1
	$\int \frac{1}{\sqrt{\{12-(x+2)^2\}}}\,\mathrm{d}x=\arcsin\frac{x+2}{\sqrt{12}}\quad(+c)$ M1: For obtaining $\alpha \arcsin \mathrm{g}(x)$ Accept other notation for arcsin e.g. arsin, $\sin^{-1}$ etc. A1: Correct result oe e.g. $\arcsin\frac{x+2}{2\sqrt{3}}\quad(+c)$ The constant of integration is **not** required Accept other notation for arcsin e.g. arsin, $\sin^{-1}$ etc.		M1A1
	May see substitution e.g. $x+2=\sqrt{12}\sin u \Rightarrow \int \frac{1}{\sqrt{12-(x+2)^2}}\,\mathrm{d}x=\int \frac{1}{\sqrt{12-12\sin^2 u}}\sqrt{12}\cos u\,\mathrm{d}u$ $=\int \mathrm{d}u=u(+c)=\arcsin\left(\frac{x+2}{\sqrt{12}}\right)(+c)$ For M1 this requires a complete method using a **correct substitution** and including the reversal of the substitution and A1 as already defined		
	May see substitution e.g. $x+2=\sqrt{12}\cos u \Rightarrow \int \frac{1}{\sqrt{12-(x+2)^2}}\,\mathrm{d}x=\int \frac{-1}{\sqrt{12-12\cos^2 u}}\sqrt{12}\sin u\,\mathrm{d}u$ $=-\int \mathrm{d}u=u(+c)=-\arccos\left(\frac{x+2}{\sqrt{12}}\right)(+c)$ For M1 this requires a complete method using a correct substitution and including the reversal of the substitution and A1 as already defined Accept other notation for arccos e.g. arcos, $\cos^{-1}$ etc.		
			(4)
			[Total 7]

Question Number	Scheme		Marks
4(a)	$\dfrac{dy}{dx}=\sinh\dfrac{x}{3}$	Correct expression for dy/dx seen explicitly or used	B1
	$\int\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx=\int\sqrt{1+\sinh^2\left(\dfrac{x}{3}\right)}dx=\int\cosh\left(\dfrac{x}{3}\right)dx$ Uses the correct formula for arc length and reaches: $k\int\pm\cosh\left(\dfrac{x}{3}\right)dx$		M1
	$=3\sinh\left(\dfrac{x}{3}\right)$	Correct integration	A1
	$\text{length}=\left[3\sinh\left(\dfrac{x}{3}\right)\right]_{-3a}^{3a}=3(\sinh a-\sinh(-a))=\ldots$ **or** $\text{length}=2\left[3\sinh\left(\dfrac{x}{3}\right)\right]_{0}^{3a}=2\times3(\sinh a-\sinh(0))=\ldots$ Correct use of limits – in the second case, the lower (0) limit does not have to been seen used		M1
	= 6sinh a	Correct expression	A1
	Do not be overly concerned if a "sinh" becomes "sin" or there is a missing "dx" along the way but the final answer must be correct.		
			(5)
(b)	$6\sinh a=12\Rightarrow\sinh a=2\Rightarrow x_Q=3a=3\text{arsinh}2$ Uses their arc length in terms of sinh a and the 12 to find a in terms of arsinh or ln and multiplies by 3		M1
	$=3\ln\left(2+\sqrt{5}\right)$	Correct answer **<u>including brackets</u>** and no other answers.	A1
			(2)
(c)	$y_Q=3\cosh a=3\sqrt{1+\sinh^2 a}=3\sqrt{1+"2"^2}$ or $y_Q=3\cosh a=3\cosh\left(\ln\left(2+\sqrt{5}\right)\right)=3\left(\dfrac{e^{\ln\left(2+\sqrt{5}\right)}+e^{-\ln\left(2+\sqrt{5}\right)}}{2}\right)=\ldots$ Use the curve equation with x = 3a or x = -3a and their value for a or sinh a to obtain a numerical **value** for y_Q i.e. no e's or ln's or cosh's etc.		M1
	$=3\sqrt{5}$	Cao	A1
			(2)
			[Total 9]

Question Number	Scheme		Marks
5(a)	$(\mathbf{i}+2\mathbf{k})\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=\mathbf{i}(0-2)-\mathbf{j}(3-4)+\mathbf{k}(1-0)$ Attempt the correct vector product between the direction vectors If no method is shown at least 2 "components" should be correct		M1
	$-2\mathbf{i}+\mathbf{j}+\mathbf{k}$	Any multiple of this vector	A1
			(2)
(a) Way 2	$\mathbf{n}=\begin{pmatrix}1\\a\\b\end{pmatrix}\Rightarrow\begin{pmatrix}1\\a\\b\end{pmatrix}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=0,\begin{pmatrix}1\\a\\b\end{pmatrix}\bullet\begin{pmatrix}2\\1\\3\end{pmatrix}=0\Rightarrow a=-\frac{1}{2},b=-\frac{1}{2}$ Correct method leading to values for a and b		M1
	$-2\mathbf{i}+\mathbf{j}+\mathbf{k}$	Any multiple of this vector	A1

Useful Diagram:



Mark (b) and (c) together

(b)	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$(\mathbf{i}+6\mathbf{j}-\mathbf{k})\bullet(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-2+6-1$	Attempts scalar product between the direction of l and the normal to the plane.	M1
	$\sin\alpha$ or $\cos(90°-\alpha)=\dfrac{-2+6-1}{\sqrt{6}\times\sqrt{38}}$ $\left(\text{NB }\sqrt{6}\times\sqrt{38}=2\sqrt{57}\right)$	$\sin...$ or $\cos...=\pm\left(\dfrac{-2+6-1}{\sqrt{6}\times\sqrt{38}}\right)$	A1
	$\alpha=(11.45...=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1
			(4)
(b) Way 2	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$\lvert\mathbf{i}+6\mathbf{j}-\mathbf{k}\rvert=\sqrt{1^2+6^2+1^2}\left(=\sqrt{38}\right)$ Followed by a complete method to find the perpendicular distance from A to the plane i.e. as in part (c) $\left(\dfrac{3}{\sqrt{6}}\right)$ or finds the distance from (3, 2, -1) to the intersection of the perpendicular from A with the plane (1, -3.5, 0.5) $\left(\dfrac{\sqrt{146}}{2}\right)$ and then uses correct trigonometry to find the sin or cos or tan (would need both distances) to find the required angle.		M1
	$\sin\alpha=\dfrac{\frac{3}{\sqrt{6}}}{\sqrt{38}},\quad\cos\alpha=\dfrac{\frac{\sqrt{146}}{2}}{\sqrt{38}},\quad\tan\alpha=\dfrac{\frac{3}{\sqrt{6}}}{\frac{\sqrt{146}}{2}}$		A1
	$\alpha=(11.45...=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1
(b) Way 3	l has direction $\mathbf{i}+6\mathbf{j}-\mathbf{k}$	Any multiple of this vector	B1
	$(\mathbf{i}+6\mathbf{j}-\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=7\mathbf{i}+\mathbf{j}+13\mathbf{k}$ Attempts vector product between the direction of l and the normal to the plane. If no method is shown at least 2 "components" should be correct		M1
	$\sin\alpha=\dfrac{\sqrt{169+49+1}}{\sqrt{6}\times\sqrt{38}}$	Correct value for sin…	A1
	$\alpha=(11.45...=)11°$	For 11 (degrees symbol not required). **Do not isw and mark their final answer.**	A1

(c) **Way 1**	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation. Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(2\mathbf{i}-4\mathbf{j}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-8$ $\Rightarrow d=\left\|\dfrac{-8-(-5)}{\sqrt{2^2+1^2+1^2}}\right\|=...$		M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
			(4)
(c) **Way 2**	Dist (2, -4, 0) to $(3,2,-1)=\sqrt{(3-2)^2+(2+4)^2+1^2}=...$ Correct attempt to find the distance between (2, - 4, 0) and (3, 2, 1)		M1
	$=\sqrt{38}$	Correct distance	A1
	d $="\sqrt{38}"\sin\alpha="\sqrt{38}"\dfrac{3}{\sqrt{6}\times\sqrt{38}}=...$	Uses correct trigonometry to find the required distance	M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
(c) **Way 3**	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=(3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k})=-5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation. Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$\Rightarrow d=\left\|\dfrac{-2\times2+1(-4)+(0)+5}{\sqrt{2^2+1^2+1^2}}\right\|=...$	Uses a correct formula for the distance. (Allow ± their -5)	M1
	$d=\dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1

(c) Way 4	Let P be (3, 2, -1) so $\mathbf{AP} = \mathbf{i} + 6\mathbf{j} - \mathbf{k}$ $(\mathbf{i}+6\mathbf{j}-\mathbf{k})\bullet(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -2+6-1$ M1: Forms the vector AP and calculates scalar product with normal vector A1: Correct scalar product for their normal i.e. 3 or a multiple of 3 depending on their normal (may be unsimplified)		M1A1
	$\Rightarrow d = \left\lvert \dfrac{-2+6-1}{\sqrt{2^2+1^2+1^2}} \right\rvert = \ldots$	Uses a correct formula for the distance.	M1
	$d = \dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
(c) Way 5	Π has equation: $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = (4\mathbf{i}+2\mathbf{j}+\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -5$ or $\mathbf{r}.(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = (3\mathbf{i}+2\mathbf{j}-\mathbf{k}).(-2\mathbf{i}+\mathbf{j}+\mathbf{k}) = -5$ M1: Forms scalar product of a point in the plane with their normal vector A1: Correct equation Allow this mark if -5 (**or the equivalent multiple of -5 for their normal**) is obtained i.e. do not need to see the equation of the plane explicitly		M1A1
	$(2\mathbf{i}-4\mathbf{j})+\lambda(-2\mathbf{i}+\mathbf{j}+\mathbf{k}),\ 2x-y-z=5$ $\Rightarrow 4-4\lambda+4-\lambda-\lambda = 5 \Rightarrow \lambda = \dfrac{1}{2}$ $\Rightarrow d = \dfrac{1}{2}\lvert -2\mathbf{i}+\mathbf{j}+\mathbf{k} \rvert = \dfrac{\sqrt{2^2+1^1+1^2}}{2}$ Requires a complete method: Uses the parametric form of the line through (-2, 4, 0) perpendicular to the plane and substitutes into the equation of the plane to find the value of the parameter and uses this correctly to find the required distance.		M1
	$d = \dfrac{\sqrt{6}}{2}$	Correct distance in any equivalent exact form e.g. $\dfrac{3}{\sqrt{6}}$	A1
			[10]

Question Number	Scheme		Marks
6 (a) Mark (i) and (ii) together	$\begin{vmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{vmatrix} = -2(-3)(-2)-4(-3)=0 \Rightarrow 5$ is an eigenvalue This mark is for demonstrating that 5 is an eigenvalue as above and **requires a conclusion**. This may also be done by **substituting** λ = 5 into their CE to obtain 0 **with a conclusion** or by performing long division and obtaining a remainder of 0 **with conclusion**. However if the candidate forms and solves the CE and correctly obtains λ = 5 this is sufficient without a conclusion. The conclusion can be minimal e.g. "proven", "QED", tick etc.		B1 **M1 on ePEN**
	$\begin{vmatrix} 3-\lambda & 0 & 1 \\ 1 & 2-\lambda & 2 \\ 4 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)(3-\lambda)-4(2-\lambda)$ M1: Attempts determinant of **A** - λ**I** Whichever method is chosen e.g. row/column/Sarrus, the expression should be similar to the above e.g. • Row 2: $(2-\lambda)\{(3-\lambda)(3-\lambda)-4\}$ • Row 3: $4(0-(2-\lambda))+(3-\lambda)(3-\lambda)(2-\lambda)$ • Column 1 = as Row 1 • Column 2 = as Row 2 • Column 3 = as Row 3 A1: Correct equation including " = 0" which may be implied by an attempt to solve. NB Correct expanded cubic is $\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$		M1A1
	$(2-\lambda)(\lambda-5)(\lambda-1)=0 \Rightarrow \lambda = ...$ Attempts to solve cubic – **may just be seen from calculator**		M1
	$\lambda = 2,\ 1,\ \ (5)$	Other eigenvalues 2 and 1	A1
			(5)
(b)	$\begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 2 \\ 4 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} 3-5 & 0 & 1 \\ 1 & 2-5 & 2 \\ 4 & 0 & 3-5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ Demonstrates the understanding that 5 is an eigenvalue Either of these statements is sufficient		M1
	$3x+z=5x$ $x+2y+2z=5y$ $4x+3z=5z$	Multiplies out to obtain at least 2 **correct** equations	M1
	$\therefore$ Eigenvector is $\begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$	Allow any multiple of this vector e.g. $\begin{pmatrix} 1 \\ \frac{5}{3} \\ 2 \end{pmatrix}$ will be common. Allow *x* =…, *y* =…, and *z* =… where *x*, *y* and *z* were seen in a vector earlier and isw.	A1
			(3)

(c)	$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots$ **or** $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$ Attempts to multiply one of the given vectors by **M** to find at least one image		M1 **B1 on ePEN**
	$\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2\\1\\-3\end{pmatrix}=\ldots$ **and** $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots$ Attempts to multiply both of the given vectors by **M** to find both images		M1
	Note that an attempt at $\begin{pmatrix}3&0&1\\1&2&2\\4&0&3\end{pmatrix}\begin{pmatrix}2+\mu\\1+2\mu\\-3-\mu\end{pmatrix}=\cdots$ scores both M's provided this results in the parametric image		
	$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **or** $\begin{pmatrix}2\\3\\1\end{pmatrix}$	One correct	A1 **M1 on ePEN**
	$\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\3\\1\end{pmatrix}$	Both correct	A1
	$\begin{pmatrix}3+2\mu\\-2+3\mu\\-1+\mu\end{pmatrix}$ scores both A marks		
	$(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}$ where $\mathbf{c}=3\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $\mathbf{d}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ Or $(\mathbf{r}-(3\mathbf{i}-2\mathbf{j}-\mathbf{k}))\times(2\mathbf{i}+3\mathbf{j}+\mathbf{k})=\mathbf{0}$ Correct equation in the correct form. Follow through their vectors but they must be correctly placed and **depends on both method marks.**		A1ft
			(5)
			[Total 13]

Note that candidates may transform 2 points on l_1 and then use the transformed points to find the direction of l_2. In this case the second M and the second A1 will only be scored when the direction of l_2 is found and then the final mark becomes available.

Question Number	Scheme	Marks
7(a)	$I_n = \int \cosh^n x \,\mathrm{d}x = \int \cosh x \cosh^{n-1} x \,\mathrm{d}x$ $(I_n =) \int \cosh x \cosh^{n-1} x \,\mathrm{d}x$ seen explicitly or used	B1
	$I_n = \pm\cosh^{n-1} x \sinh x \pm k\int \cosh^{n-2} x \sinh^2 x \,\mathrm{d}x$ Uses parts in the correct direction to obtain an expression of the above form	M1
	$I_n = \cosh^{n-1} x \sinh x - \int (n-1)\cosh^{n-2} x \sinh^2 x \mathrm{d}x$ Correct expression	A1
	$I_n = \cosh^{n-1} x \sinh x - \int (n-1)\cosh^{n-2} x\left(\cosh^2 x - 1\right)\mathrm{d}x$ Use of $\sinh^2 x = \pm\cosh^2 x \pm 1$ **Dependent on the previous method mark**	**d**M1
	$I_n = \cosh^{n-1} x \sinh x - (n-1)I_n + (n-1)I_{n-2}$ Sub for I_n and I_{n-2} and collect terms **Dependent on both previous method marks**	**dd**M1
	$nI_n = \sinh x \cosh^{n-1} x + (n-1)I_{n-2}$ *	A1cso
Do not be overly concerned with notational errors e.g. if a "cosh" becomes a "cos" or a "sinh" becomes a "sin" and is then recovered or if e.g. a cosh^2*x* appears as cosh*x*^{2} but is then recovered or if the odd "*x*" or "d*x*" disappears etc. as long as the intention is clear. However, if there are any obvious errors such as sign errors then the final mark should be withheld.		
		(6)
(a) Way 2	$I_n = \int \cosh^n x \,\mathrm{d}x = \int \cosh^2 x \cosh^{n-2} x \,\mathrm{d}x$ $(I_n =) \int \cosh^2 x \cosh^{n-2} x \,\mathrm{d}x$ seen explicitly or used	B1
	$= \int \left(1 + \sinh^2 x\right)\cosh^{n-2} x \,\mathrm{d}x$ Use of $\cosh^2 x = \pm\sinh^2 x \pm 1$	M1
	$= \int \cosh^{n-2} x \,\mathrm{d}x + \int \sinh x \sinh x \cosh^{n-2} x \,\mathrm{d}x$	
	$= \left(\int \cosh^{n-2} x \,\mathrm{d}x +\right)\sinh x \dfrac{\cosh^{n-1} x}{n-1} - \int \cosh x \dfrac{\cosh^{n-1} x}{n-1}\mathrm{d}x$ M1: Integrates sinh *x*sinh *x*cosh^{n-2} x to obtain an expression of the form $\pm P \sinh x \cosh^{n-1} x \pm Q\int \cosh x \cosh^{n-1} x \,\mathrm{d}x$ **Dependent on the previous method mark** A1: For $... + \sinh x \dfrac{\cosh^{n-1} x}{n-1} - \int \cosh x \dfrac{\cosh^{n-1} x}{n-1}\mathrm{d}x$ Mark in the order on ePEN so that the dM1 is the A1 on ePEN and the A1 is the M2 on ePEN	**d**M1A1 **Note that these 2 marks are reversed for this way.**
	$\Rightarrow (n-1)I_n = (n-1)I_{n-2} + \sinh x \cosh^{n-1} x - I_n$ Sub for I_n and I_{n-2} and collect term **Dependent on both previous method marks**	**dd**M1
	$nI_n = \sinh x \cosh^{n-1} x + (n-1)I_{n-2}$ *	A1cso

(b)	$\int \cosh^4 x\,dx = \frac{1}{4}\left[\cosh^3 x\sinh x + 3I_2\right]$ or $4\int \cosh^4 x\,dx = \cosh^3 x\sinh x + 3I_2$ Applies the reduction formula to I_4	M1
	$\int \cosh^4 x\,dx = \frac{1}{4}\left[\cosh^3 x\sinh x + 3\left(\frac{1}{2}\left[\cosh x\sinh x + I_0\right]\right)\right]$ or $4\int \cosh^4 x\,dx = \cosh^3 x\sinh x + 3\times\frac{1}{2}\left[\cosh x\sinh x + I_0\right]$ Attempts to use the reduction formula again to obtain I_2 in terms of I_0 May be seen embedded in their I_4 **This is a method mark so allow confusion with the constants.** This mark can also be scored by an attempt to integrate $\cosh^2 x$: Either $\int \cosh^2 x\,dx = \frac{1}{2}\int (\pm\cosh 2x \pm 1)\,dx = \alpha\sinh 2x + \beta x$ or $\int \cosh^2 x\,dx = \int \left(\frac{e^x + e^{-x}}{2}\right)^2 dx = \frac{1}{4}\int \left(e^{2x} + 2 + e^{-2x}\right)dx = \frac{1}{8}e^{2x} + \frac{1}{2}x - \frac{1}{8}e^{-2x}$	M1
	<u>Note that the final 2 A marks are only to be awarded once I_0 has been evaluated and substituted and they depend on having scored at least one method mark.</u> **Examples:** • $\int \cosh^4 x\,dx = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{8}\cosh x\sinh x + \frac{3}{8}x(+c)$ • $\int \cosh^4 x\,dx = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{16}\sinh 2x + \frac{3}{8}x(+c)$ • $\int \cosh^4 x\,dx = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{32}e^{2x} - \frac{3}{32}e^{-2x} + \frac{3}{8}x(+c)$ A1: $\frac{1}{4}\cosh^3 x\sinh x + 1$ other correct term But note that $\frac{3}{32}e^{2x} - \frac{3}{32}e^{-2x}$ counts as 1 term A1: Fully correct Correct answer – **constant of integration not required.** **NOTE:** $\frac{1}{4}\left[\cosh^3 x\sinh x + \frac{3}{2}\left(\sinh x\cosh x + I_0\right)\right]$ scores M1M1A0A0 $\frac{1}{4}\left[\cosh^3 x\sinh x + \frac{3}{2}\left(\sinh x\cosh x + x\right)\right]$ scores M1M1A1A1	A1A1
		(4)
		[Total 10]

Note that part (b) can be done in reverse order, in which case the method marks are awarded the other way round e.g.

$$\left(I_0 = \int \mathrm{d}x = x\right)$$

Second M: $I_2 = \frac{1}{2}\left[\sinh x \cosh x + x\right]$ **or as defined in the main scheme**

First M: $I_4 = \frac{1}{4}\left[\sinh x \cosh^3 x + 3\left(\frac{1}{2}\left[\cosh x \sinh x + x\right]\right)\right]$

$$\int \cosh^4 x \, \mathrm{d}x = \frac{1}{4}\cosh^3 x \sinh x + \frac{3}{8}\cosh x \sinh x + \frac{3}{8}x\,(+c)$$

A1A1: As defined in the main scheme

Question Number	Scheme	Marks
8(a) **Way 1**	$\dfrac{x^2}{a^2}+\dfrac{(mx+c)^2}{b^2}=1=\ldots$ Eliminates y from the equation of the ellipse and attempts to expand $(mx+c)^2$	M1
	$\dfrac{x^2}{a^2}+\dfrac{m^2x^2+2cmx+c^2}{b^2}=1$ Correct equation with the $(mx+c)^2$ expanded correctly	A1
	$4a^4m^2c^2=4a^2\left(b^2+a^2m^2\right)\left(c^2-b^2\right)$ Uses discriminant = 0 or equivalent **Dependent on the first method mark**	**d**M1
	$a^2m^2c^2=b^2c^2-b^4+a^2m^2c^2-a^2m^2\times b^2$ $c^2=b^2+a^2m^2$ * Complete to obtain the GIVEN result with no errors seen. At least one intermediate step should be shown.	A1 cso
		(4)
(a) **Way 2**	$m=\dfrac{b\cos\theta}{-a\sin\theta}\Rightarrow y-b\sin\theta=\dfrac{b\cos\theta}{-a\sin\theta}\left(x-a\cos\theta\right)$ $\Rightarrow y=\dfrac{b\cos\theta}{-a\sin\theta}x+\dfrac{b}{\sin\theta}\Rightarrow m=\dfrac{b\cos\theta}{-a\sin\theta},\quad c=\dfrac{b}{\sin\theta}$ M1: Forms the equation of a general tangent and "extracts" c and m A1: Correct c and m	M1A1
	$b^2+a^2m^2=b^2+a^2\left(\dfrac{b\cos\theta}{-a\sin\theta}\right)^2$ — Substitutes their m into $b^2+a^2m^2$ or equivalent work. **Dependent on the first method mark**	dM1
	$b^2+a^2m^2=\dfrac{b^2\cos^2\theta+b^2\sin^2\theta}{\sin^2\theta}=\dfrac{b^2}{\sin^2\theta}=c^2$ $b^2+a^2m^2=c^2$ * Completes correctly with conclusion e.g. tick, QED etc.	A1

(b) Way 1	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is })\left(-\frac{c}{m},\ 0\right)\ (B \text{ is })(0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or – here)	B1
	$= -\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or – here)	A1
	$\frac{\mathrm{d}A}{\mathrm{d}m} = \frac{b^2}{2}m^{-2} - \frac{a^2}{2}$ or $\frac{\mathrm{d}A}{\mathrm{d}m} = -\frac{2m \times 2a^2m - 2\left(b^2+a^2m^2\right)}{4m^2}$ Differentiates wrt m (must be correct differentiation for their A). **Dependent on the previous M**		dM1
	At min $m^2 = \frac{b^2}{a^2}\quad m = (\pm)\frac{b}{a}$	Equate their derivative to 0 and solves for m^2 or m. **Dependent on all the previous M's**	ddM1
	Min area $= -\frac{b^2+b^2}{-\frac{2b}{a}} = ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1
			(7)
			[Total 11]
(b) Way 2	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is })\left(-\frac{c}{m},\ 0\right)\ (B \text{ is })(0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or – here)	B1
	$= -\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or – here)	A1
	$A = -\frac{(am+b)^2 - 2amb}{2m}$	Correct completion of the square in the numerator. **Dependent on the previous M**	dM1
	$= ab - \frac{(am+b)^2}{2m}$ is minimum when $am + b = 0$ Correct argument for establishing the minimum **Dependent on all the previous M's**		ddM1
	Min area $= ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

(b) Way 3	$x=0\Rightarrow y=\dfrac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\dfrac{b}{\sin\theta}\right)$ **or** $y=0\Rightarrow y=\dfrac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\dfrac{a}{\cos\theta}\right)$ Correct value for one of the intercepts or correct coordinates.		B1
	$x=0\Rightarrow y=\dfrac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\dfrac{b}{\sin\theta}\right)$ **and** $y=0\Rightarrow y=\dfrac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\dfrac{a}{\cos\theta}\right)$ Correct values for both intercepts or correct coordinates.		B1
	Area ΔOAB $A=\dfrac{1}{2}\dfrac{a}{\cos\theta}\dfrac{b}{\sin\theta}$	Correct method for the area using their intercepts	M1
		Correct area (may be unsimplified)	A1
	$\dfrac{\mathrm{d}A}{\mathrm{d}\theta}=\dfrac{-2ab\left(\cos^2\theta-\sin^2\theta\right)}{4\sin^2\theta\cos^2\theta}=0\Rightarrow\theta=\dfrac{\pi}{4}$ Adopts a correct strategy for finding θ at the minimum **Dependent on the previous M**		dM1
	$A_{\min}=\dfrac{ab}{2\sin\dfrac{\pi}{4}\cos\dfrac{\pi}{4}}$	Uses their value for θ to find the minimum value. **Dependent on all the previous M's**	ddM1
	$=ab\ \left(\text{units}^2\right)$	Cso	A1
	Alternative for last 3 marks:		
	Area ΔOAB $A=\dfrac{ab}{2\sin\theta\cos\theta}=\dfrac{ab}{\sin 2\theta}$ And the minimum will occur when sin 2θ is maximum i.e. when sin 2θ = 1 Score this mark for a valid argument for determining the minimum **Dependent on the previous M**		dM1
	$A_{\min}=\dfrac{ab}{1}$	Completes the process of finding the minimum. **Dependent on all the previous M's**	ddM1
	$=ab\ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

<table>
<tr><td rowspan="7">(b)
Way 4</td><td colspan="2">$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\dfrac{c}{m}$ or $(A \text{ is })\left(-\dfrac{c}{m},\ 0\right)$ $(B \text{ is })(0,\ c)$
Correct values for the intercepts or correct coordinates.</td><td>B1</td></tr>
<tr><td>Area $\Delta OAB = -\dfrac{c^2}{2m}\left(\text{or } \dfrac{c^2}{2m}\right)$</td><td>Correct expression for the area
(allow + or – here)</td><td>B1</td></tr>
<tr><td rowspan="2">$= \dfrac{ac^2}{2\sqrt{c^2-b^2}}\left(=-\dfrac{ac^2}{2\sqrt{c^2-b^2}}\right)$</td><td>Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate m.</td><td>M1</td></tr>
<tr><td>Correct expression.
(allow + or – here)</td><td>A1</td></tr>
<tr><td colspan="2">$\dfrac{\mathrm{d}A}{\mathrm{d}c} = \dfrac{2ac \times 2\sqrt{c^2-b^2} - 2ac^3\left(c^2-b^2\right)^{-\frac{1}{2}}}{4\left(c^2-b^2\right)}$
Differentiates wrt c.
Dependent on the previous M</td><td>dM1</td></tr>
<tr><td colspan="2">At min $2c^2 - 2b^2 - c^2 = 0 \Rightarrow c^2 = 2b^2$
Equate their derivative to 0 and solves to obtain c in terms of b
Dependent on all the previous M’s</td><td>dM1</td></tr>
<tr><td>Min area $= \dfrac{2ab^2}{2\sqrt{2b^2-b^2}} = ab \ \left(\text{units}^2\right)$</td><td>Correct completion with no errors.</td><td>A1</td></tr>
</table>

There will be other valid methods in part (b). Generally, the first 4 marks are for obtaining an expression for the area of *AOB* and then applying the result in part (a) to enable progress to be made in establishing the minimum or for using the general tangent in terms of θ to find the intercepts and hence the area of *AOB*. The final 3 marks are for selecting and implementing a correct strategy for proving that the minimum area is *ab*.

There may also be other valid methods in part (a).

If you are in any doubt whether a particular method is valid then please seek advice from your Team Leader.

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