Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Monday 26 June 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 4 8 2 6 0 A 0 1 3 6

Pearson

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1. Solve the equation

$$18\cosh x + 14\sinh x = 11 + e^x$$

Give your answers in the form $\ln a$, where a is rational.

(5)

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Question 1 continued

Q1

(Total 5 marks)

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2.

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & a \\ 2 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -2 & 3 \\ 1 & 2 & b \end{pmatrix}$$

where a and b are constants.

(a) Write down $\mathbf{A}^{\mathrm{T}}$ in terms of a.

(1)

(b) Calculate **AB**, giving your answer in terms of a and b.

(2)

(c) Hence show that

$$(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$$

(3)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 6 marks)

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3. Given that

$$y = x - \operatorname{artanh}\left(\frac{2x}{1+x^2}\right)$$

(a) show that

$$1 - \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1-x^2}$$

where k is a constant to be found.

(4)

(b) Hence, or otherwise, show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\left(1 - \frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 0$$

(4)

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Question 3 continued

Q3

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4.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(a) Show that 6 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(4)

(b) Find a normalised eigenvector corresponding to the eigenvalue 6

(4)

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

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5.

$$I_n = \int \text{cosec}^n x \, dx, \quad 0 < x < \frac{\pi}{2}, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{1}{n-1} \cot x \, \text{cosec}^{n-2} x$$

(5)

(b) Hence, or otherwise, find

$$\int \text{cosec}^4 x \, dx$$

giving your answer in terms of $\cot x$.

(4)

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Question 5 continued

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Question 5 continued

Q5

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6. The hyperbola H has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

and the ellipse E has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where $a > b > 0$

The line l is a tangent to **hyperbola** H at the point $P(a\sec\theta, b\tan\theta)$, where $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$bx\sec\theta - ay\tan\theta = ab$$

(4)

Given that the point F is the focus of **ellipse** E for which $x > 0$ and that the line l passes through F,

(b) show that l is parallel to the line $y = x$

(5)

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Question 6 continued

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Question 6 continued

Q6

(Total 9 marks)

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7. (a) Find

$$\int \frac{5 + x}{\sqrt{4 - 3x^2}} \, dx$$

(5)

(b) Hence find the exact value of

$$\int_0^1 \frac{5 + x}{\sqrt{4 - 3x^2}} \, dx$$

giving your answer in the form $p\pi\sqrt{3} + q$, where p and q are rational numbers to be found.

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 8 marks)

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8. The curve C has parametric equations

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta, \quad 0 \leqslant \theta \leqslant 2\pi$$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(a) Show that

$$S = 2\pi\sqrt{2}\int_0^{2\pi} (1 - \cos\theta)^{\frac{3}{2}} \mathrm{d}\theta$$

(4)

(b) Hence find the exact value of S.

(6)

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Question 8 continued

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Question 8 continued

Q8

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9 With respect to a fixed origin O, the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.

(a) Find the volume of the tetrahedron $ABCD$.

(4)

The plane Π contains the points A, B and C.

(b) Find a cartesian equation of Π.

(4)

The point T lies on the plane Π.

The line DT is perpendicular to Π.

(c) Find the exact coordinates of the point T.

(4)

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Question 9 continued

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Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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