Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level in Further Pure Mathematics 3 (WFM03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required..

Question Number	Scheme	Notes	Marks
1.	$1+2\sinh^2 x - 7\sinh x = 5$	Replaces cosh2x by $1+2\sinh^2 x$ or replaces cosh2x with $\cosh^2 x + \sinh^2 x$ and then $\cosh^2 x$ with $1+\sinh^2 x$. There must be no incorrect identities used.	M1
	$2\sinh^2 x - 7\sinh x - 4 = 0$	Correct quadratic	A1
	$(2\sinh x+1)(\sinh x-4)=0 \Rightarrow \sinh x=$	Attempt to solve 3TQ in sinh$x$ (usual rules)	M1
	$\sinh x = -\frac{1}{2}, 4$	Both (allow un-simplified e.g. $\frac{7\pm 9}{4}$)	A1
	$\text{arsinh}x = \ln\left(x+\sqrt{x^2+1}\right)$	Use of the **correct** log form of arsinh	M1
	This mark may also be gained by using the exponential form of sinh$x$ and attempting to solve to give x in terms of ln		
	$x=\ln\left(-\frac{1}{2}+\sqrt{\frac{5}{4}}\right),\ \ln\left(4+\sqrt{17}\right)$	A1: One correct exact value of x. Allow equivalent exact answers which may be un-simplified. A1: Both values correct and exact and no incorrect values. Allow equivalent exact answers which may be un-simplified. **Condone missing brackets.**	A1, A1
	Correct work giving $x=\ln\left(-\frac{1}{2}\pm\sqrt{\frac{5}{4}}\right),\ \ln\left(4\pm\sqrt{17}\right)$ would generally lose the final mark		
			(7)
			Total 7
	Alternative:		
	$\left(\frac{e^{2x}+e^{-2x}}{2}\right)-7\left(\frac{e^{x}-e^{-x}}{2}\right)=5$	M1: Substitutes the correct exponential definitions for cosh2x and sinh$x$ A1: Correct expression	M1A1
	$e^{4x}-7e^{3x}-10e^{2x}-7e^{x}+1=0$	M1: Multiplies by e^{2x} A1: Correct quartic in e^x	M1A1
	$\left(e^{2x}+e^{x}-1\right)\left(e^{2x}-8e^{x}-1\right)=0 \Rightarrow e^{x}=...$ $\Rightarrow x=...$	Solves their quartic as far as $e^x = ...$ and then converts to give x in terms of ln. There must be a recognisable attempt to solve a quartic with at least 4 terms as e.g. the product of two 3TQ's in e^x.	M1
	$x=\ln\left(-\frac{1}{2}+\sqrt{\frac{5}{4}}\right),\ \ln\left(4+\sqrt{17}\right)$	A1: One correct exact value of x. Allow equivalent exact answers which may be un-simplified. A1: Both values correct and exact and no incorrect values. Allow equivalent exact answers which may be un-simplified. **Condone missing brackets**	A1, A1

Question Number	Scheme	Notes	Marks
2.	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$		
(a)	$\frac{12^2}{a^2}-\frac{5^2}{b^2}=1$ **and** $b^2=a^2\left(\left(\frac{\sqrt{21}}{4}\right)^2-1\right)$	Substitutes the given point into the hyperbola with the 12 and 5 correctly positioned and substitutes the given value of e into the **correct** eccentricity equation.	M1
	$b^2=\frac{5}{16}a^2 \Rightarrow \frac{144}{a^2}-\frac{80}{a^2}=1 \Rightarrow a \text{ or } a^2=...$ or $\frac{45}{b^2}-\frac{25}{b^2}=0 \Rightarrow b \text{ or } b^2=...$	Solves simultaneously to obtain a value for a or a^2 or b or b^2	M1
	$a=8,\ b=\sqrt{20}$	Allow equivalents for $\sqrt{20}$ e.g. $2\sqrt{5}$ or awrt 4.47. **Do not allow ± in either case**	A1, A1
			(4)
(b)	$(\pm ae, 0)=(\pm 2\sqrt{21}, 0)$	Both (follow through their a). **Must be coordinates.**	B1ft
			(1)
			Total 5
	Alternative to (a):		
	$12=a\sec\theta$, $5=b\tan\theta$ **and** $b^2=a^2\left(\left(\frac{\sqrt{21}}{4}\right)^2-1\right)$ $\frac{b}{a}=\frac{\sqrt{5}}{4},\ \frac{b}{a}=\frac{5}{12}\operatorname{cosec}\theta \Rightarrow \operatorname{cosec}\theta=\frac{5}{3\sqrt{5}}$	Substitutes the given value of e into the **correct** eccentricity equation and substitutes the given point into the correct parametric form and eliminates a and b	M1
	$\operatorname{cosec}\theta=\frac{5}{3\sqrt{5}} \Rightarrow a=...$ or $b=...$	Solves to obtain a value for a or b	M1
	$a=8,\ b=\sqrt{20}$	Allow equivalents for $\sqrt{20}$ e.g. $2\sqrt{5}$ or awrt 4.47. **Do not allow ± in either case**	A1, A1

Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix}0&1&9\\1&4&k\\1&0&-3\end{pmatrix}\begin{pmatrix}7\\19\\1\end{pmatrix}=\lambda\begin{pmatrix}7\\19\\1\end{pmatrix}$	Correct statement	M1
	$7-3=\lambda$ or $28=7\lambda \Rightarrow \lambda=4$	Correct eigenvalue	A1
			(2)
(b)	$7+4\times19+k=4\times19 \Rightarrow k=-7$ *	M1: Uses y component to establish an equation for k A1*: Correct k	M1A1*
			(2)
(c)	$\begin{vmatrix}0-\lambda&1&9\\1&4-\lambda&-7\\1&0&-3-\lambda\end{vmatrix}=0$		
	$\lambda(4-\lambda)(3+\lambda)+(3+\lambda)-7+9(\lambda-4)=0$ or $-7+9(\lambda-4)-(3+\lambda)[\lambda(\lambda-4)-1]$	M1: Correct characteristic equation method (allow sign errors only) A1: Correct equation in any form	M1A1
	$(4-\lambda)[\lambda(3+\lambda)-1-9]=0$	NB $\lambda^3-\lambda^2-22\lambda+40=0$	
	$(\lambda-2)(\lambda+5)=0 \Rightarrow \lambda=2,-5$	A1: $\lambda=2$ or $\lambda=-5$ A1: $\lambda=2$ and $\lambda=-5$	A1A1
			(4)
(d) Way 1	$\begin{pmatrix}0&1&9\\1&4&-7\\1&0&-3\end{pmatrix}\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}q+9r\\p+4q-7r\\p-3r\end{pmatrix}$	Multiplies by **M** to obtain a vector in terms of p, q and r	M1
	$\begin{pmatrix}q+9r\\p+4q-7r\\p-3r\end{pmatrix}=\begin{pmatrix}-6\\21\\5\end{pmatrix}$	Correct equations	A1
	$p=2,\ q=3,\ r=-1$	M1: Solves simultaneously to obtain at least one of p, q or r. **Dependent on the previous method mark.** A1: Correct answers	dM1A1
	Correct equations followed by correct answers scores full marks in part (d)		
			(4)
(d) Way 2	$\begin{pmatrix}p\\q\\r\end{pmatrix}=\mathbf{M}^{-1}\begin{pmatrix}-6\\21\\5\end{pmatrix}=\frac{1}{40}\begin{pmatrix}12&-3&43\\4&9&-9\\4&-1&1\end{pmatrix}\begin{pmatrix}-6\\21\\5\end{pmatrix}$	M1: An appreciation that $p\mathbf{i}+q\mathbf{j}+r\mathbf{k}=\mathbf{M}^{-1}(-6\mathbf{i}+21\mathbf{j}+5\mathbf{k})$ A1: Correct inverse	M1A1
	$\begin{pmatrix}p\\q\\r\end{pmatrix}=\frac{1}{40}\begin{pmatrix}12&-3&43\\4&9&-9\\4&-1&1\end{pmatrix}\begin{pmatrix}-6\\21\\5\end{pmatrix}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$	M1: Multiplies their inverse by the given vector. **Dependent on the previous method mark.** A1: Correct vector	dM1A1
			(4)
			Total 12

Question	Scheme		Notes	Marks
4(a)	$\int \cosh^n x\,dx = \int \cosh x \cosh^{n-1} x\,dx$		Writes $\cosh^n x$ as $\cosh x \cosh^{n-1} x$	B1
	$\int \cosh x \cosh^{n-1} x\,dx = \sinh x \cosh^{n-1} x - \int (n-1)\cosh^{n-2} x \sinh^2 x\,dx$ M1: Parts in the correct direction (if the method is unclear or formula not quoted only allow sign errors) A1: Correct expression			M1A1
	$= \sinh x \cosh^{n-1} x - (n-1)\int \cosh^{n-2} x\left(\cosh^2 x - 1\right)dx$ Writes $\sinh^2 x$ as $\cosh^2 x - 1$			dM1
	$= \sinh x \cosh^{n-1} x - (n-1)\int \cosh^n x\,dx + (n-1)\int \cosh^{n-2} x\,dx$			
	$= \sinh x \cosh^{n-1} x - (n-1)\mathrm{I}_n + (n-1)\mathrm{I}_{n-2}$		Substitutes for I_n and I_{n-2}	ddM1
	$(1+n-1)\mathrm{I}_n = \sinh x \cosh^{n-1} x + (n-1)\mathrm{I}_{n-2}$			
	$n\mathrm{I}_n = \sinh x \cosh^{n-1} x + (n-1)\mathrm{I}_{n-2}$ *		Correct answer with at least one intermediate line of working and no errors seen	A1*
	Condone omission of "dx" and the occasional invisible x throughout but the final answer must be as printed.			**(6)**
(b)	$I_5 = \frac{1}{5}\left[\sinh x \cosh^4 x\right]_0^{\ln 2} + \frac{4}{5}\mathrm{I}_3$ **or** $I_3 = \frac{1}{3}\left[\sinh x \cosh^2 x\right]_0^{\ln 2} + \frac{2}{3}I_1$ One application of reduction formula (I_5 in terms of I_3 **or** I_3 in terms of I_1)			M1
	$I_5 = \frac{1}{5}\left[\sinh x \cosh^4 x\right]_0^{\ln 2} + \frac{4}{5}\mathrm{I}_3$ **and** $I_3 = \frac{1}{3}\left[\sinh x \cosh^2 x\right]_0^{\ln 2} + \frac{2}{3}I_1$ Second application of reduction formula (I_5 in terms of I_3 **and** I_3 in terms of I_1)			M1
	$I_1 = \int_0^{\ln 2} \cosh x\,dx = \left[\sinh x\right]_0^{\ln 2} = \frac{3}{4}$		$I_1 = \frac{3}{4}$	B1
	$= \frac{1}{5}.\frac{3}{4}.\left(\frac{5}{4}\right)^4 + \frac{4}{5}\left(\frac{1}{3}.\frac{3}{4}.\left(\frac{5}{4}\right)^2 + \frac{2}{3}.\frac{3}{4}\right)$			
	$\int_0^{\ln 2} \cosh^5 x\,dx = \frac{5523}{5120}$		Must be exact	A1
	NB $I_5 = \frac{1}{5}\sinh x \cosh^4 x + \frac{4}{15}\sinh x \cosh^2 x + \frac{8}{15}I_1$ could score M1M1B0A0			
				(4)
				Total 10
(a) Way 2	$\int \cosh^n x\,dx = \int \cosh^2 x \cosh^{n-2} x\,dx = \int \left(1+\sinh^2 x\right)\cosh^{n-2} x\,dx$ Writes $\cosh^n x = (1+\sinh^2 x)\cosh^{n-2} x$			B1
	$\int \sinh x \sinh x \cosh^{n-2} x\,dx = \frac{1}{n-1}\sinh x \cosh^{n-1} x - \frac{1}{n-1}\int \cosh^n x\,dx$ M1: Parts in the correct direction (if the method is unclear or formula not quoted only allow sign errors) A1: Correct expression			M1A1
	$\int \cosh^n x\,dx = \int \cosh^{n-2} x\,dx + \frac{1}{n-1}\sinh x \cosh^{n-1} x - \frac{1}{n-1}\int \cosh^n x\,dx$ Adds I_{n-2} to their integration by parts			dM1
	$I_n = I_{n-2} + \frac{1}{n-1}\sinh x \cosh^{n-1} x - \frac{1}{n-1}I_n$		Substitutes for I_n and I_{n-2}	ddM1
	$(n-1)I_n = (n-1)I_{n-2} + \sinh x \cosh^{n-1} x - I_n$			
	$n\mathrm{I}_n = \sinh x \cosh^{n-1} x + (n-1)\mathrm{I}_{n-2}$ *		Correct answer with at least one intermediate line of working and no errors seen	A1*

Question Number	Scheme	Notes	Marks
5(a)	$\dfrac{x^2}{25}+\dfrac{(mx+c)^2}{9}=1$	Uses E and L to obtain an equation in one variable	M1
	$9x^2+25\left(m^2x^2+2cmx+c^2\right)=225$		
	$\left(25m^2+9\right)x^2+50cmx+25c^2-225=0$	Correct quadratic with terms collected	A1
	$b^2=4ac\Rightarrow(50cm)^2=4\left(25m^2+9\right)\left(25c^2-225\right)$	Use of $b^2=4ac$	M1
	$2500c^2m^2=4\left(625c^2m^2-5625m^2+225c^2-2025\right)$		
	$225c^2-5625m^2=2025$		
	$c^2-25m^2=9\,*$	Achieves printed answer with no errors	A1*
	Use of the unproved general case $a^2m^2+b^2=c^2$ scores no marks		
			(4)
	See end of scheme for alternatives		
(b)	$c=4-3m\Rightarrow(4-3m)^2-25m^2=9$	Uses the point (3, 4) and solves simultaneously to obtain an equation in one variable	M1
	$16m^2+24m-7=0$		
	$(4m-1)(4m+7)=0\Rightarrow m=\dfrac{1}{4},\ -\dfrac{7}{4}$	M1: Solves their quadratic to obtain 2 values for m A1: Correct values	M1A1
	$m=\dfrac{1}{4}\Rightarrow c=\dfrac{13}{4},\ m=-\dfrac{7}{4}\Rightarrow c=\dfrac{37}{4}$	Finds at least one value for c	M1
	$y=\dfrac{1}{4}x+\dfrac{13}{4},\ y=-\dfrac{7}{4}x+\dfrac{37}{4}$	Correct equations	A1
			(5)
(b) Way 2	$m=\dfrac{4-c}{3}\Rightarrow c^2-25\left(\dfrac{4-c}{3}\right)^2=9$	Uses the point (3, 4) and solves simultaneously to obtain an equation in one variable	M1
	$16c^2-200c+481=0$		
	$(4c-37)(4c-13)=0\Rightarrow c=\dfrac{37}{4},\ \dfrac{13}{4}$	M1: Solves their quadratic to obtain 2 values for c A1: Correct values	M1A1
	$c=\dfrac{13}{4}\Rightarrow m=\dfrac{1}{4},\ c=\dfrac{37}{4}\Rightarrow m=-\dfrac{7}{4}$	Finds at least one least one value for m	M1
	$y=\dfrac{1}{4}x+\dfrac{13}{4},\ y=-\dfrac{7}{4}x+\dfrac{37}{4}$	Correct equations	A1
	Generally if candidates assume (3, 4) lies on the ellipse they score no marks		
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
6	$x = 2\cos\theta - \cos 2\theta,\ \ y = 2\sin\theta - \sin 2\theta$		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -2\sin\theta + 2\sin 2\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta} = 2\cos\theta - 2\cos 2\theta$	Correct derivatives	B1, B1
	$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2 =$ $4\sin^2\theta - 8\sin\theta\sin 2\theta + 4\sin^2 2\theta + 4\cos^2\theta - 8\cos\theta\cos 2\theta + 4\cos^2 2\theta$ Squares **and adds** their derivatives		M1
	$= 8 - 8(\cos 2\theta\cos\theta + \sin 2\theta\sin\theta)$		
	$= 8 - 8\cos(2\theta - \theta) = 8(1 - \cos\theta)$ *	M1: Use of at least one correct trig identity A1*: Correct proof with **no errors**	M1A1*
			(5)
(b)	$S = 2\pi\int y\sqrt{\left(\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2\right)}\,\mathrm{d}\theta$		
	$S = 2\pi\int(2\sin\theta - \sin 2\theta)\sqrt{8(1-\cos\theta)}\,\mathrm{d}\theta$	Substitutes $y = 2\sin\theta - \sin 2\theta$ and $8(1-\cos\theta)$ into a correct formula	M1
	$= 2\pi\int 2\sin\theta(1-\cos\theta)\sqrt{8(1-\cos\theta)}\,\mathrm{d}\theta$		
	$= 8\pi\sqrt{2}\int \sin\theta(1-\cos\theta)^{\frac{3}{2}}\,\mathrm{d}\theta$	Processes to reach an integrand of the form $k\sin\theta(1-\cos\theta)^{\frac{3}{2}}$	M1
	$= 8\sqrt{2}\,\pi\left[\frac{2}{5}(1-\cos\theta)^{\frac{5}{2}}\right]_0^{\pi}$	Integrates to obtain an expression of the form $\alpha(1-\cos\theta)^{\frac{5}{2}}$. Dependent on the previous method mark. (May be done by substitution)	dM1
	$= 8\pi\sqrt{2}\left(\frac{2}{5}(2)^{\frac{5}{2}} - 0\right)$	Use of limits 0 and π and subtracts Dependent on all previous method marks.	dddM1
	$= \frac{128}{5}\pi$	Allow equivalents e.g. 25.6π	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$(3,\ 3,\ -2),\quad \frac{x-1}{2}=\frac{y-2}{-1}=\frac{z+1}{4}$		
	$2\mathbf{i}-\mathbf{j}+4\mathbf{k},\ 2\mathbf{i}+\mathbf{j}-\mathbf{k}$	2 correct vectors lying in Π_1	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -1 & 4\\ 2 & 1 & -1\end{vmatrix}=\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1: Attempt normal vector using 2 vectors lying in Π_1. If the method is unclear, at least 2 components should be correct. A1: Correct normal (any multiple)	M1A1
	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}3\\3\\-2\end{pmatrix}=-9+30-8=13$	Attempt scalar product with a point lying in the plane . **Dependent on the previous method mark.**	dM1
	$3x-10y-4z=-13$ *	Correct equation	A1*
			(5)
(b)	$\begin{pmatrix}-3\\10\\4\end{pmatrix}\bullet\begin{pmatrix}\alpha\\1\\1\end{pmatrix}=14-3\alpha$ or $\left(\begin{pmatrix}\alpha\\1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\-2\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-10\\-4\end{pmatrix}=3\alpha-1$	Attempt scalar product between $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}$ and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$ or $\begin{pmatrix}\alpha\\1\\1\end{pmatrix}-$ (a point in the plane Π_1) and their $\begin{pmatrix}-3\\10\\4\end{pmatrix}$	M1
	$\therefore d=\left\lvert\frac{3\alpha-1}{\sqrt{3^2+10^2+4^2}}\right\rvert$ or $\therefore d=\left\lvert\frac{13}{\sqrt{3^2+10^2+4^2}}-\frac{14-3\alpha}{\sqrt{3^2+10^2+4^2}}\right\rvert$	Use of correct distance method. **Dependent on the previous method mark.** (Modulus not needed here)	dM1
	Note: $d=\left\lvert\frac{3\alpha-10-4\pm 13}{\sqrt{3^2+10^2+4^2}}\right\rvert$ could score the first 2 method marks		
	$\left\lvert\frac{3\alpha-1}{5\sqrt{5}}\right\rvert=\frac{1}{\sqrt{5}}$	Set their distance = $\frac{1}{\sqrt{5}}$. **Dependent on both previous method marks**	ddM1
	$3\alpha-1=\pm 5$	Correct equations (must see ± for this mark but may still score one of the final A marks if ± is missing). May be implied and allow un-simplified.	A1
	$\alpha=2,\ -\frac{4}{3}$	cso	A1, A1
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{3}{4}\cosh u$	Correct expression	B1
	$\int\frac{x^2}{\sqrt{16x^2+9}}\mathrm{d}x=\int\frac{\frac{9}{16}\sinh^2 u}{\sqrt{16.\frac{9}{16}\sinh^2 u+9}}.\frac{3}{4}\cosh u\,\mathrm{d}u$ M1: A **complete** substitution attempt. A1: Correct expression		M1A1
	$=k\int\sinh^2 u\,\mathrm{d}u$		A1
	$\sinh^2 u=\frac{\cosh 2u-1}{2}$	Use of $\sinh^2 u=\pm\frac{1}{2}\cosh 2u\pm\frac{1}{2}$ May be implied by their integration	M1
	$=\frac{9}{128}\int(\cosh 2u-1)\mathrm{d}u$		A1
			(6)
(b)	$x=0\Rightarrow u=0,\ x=1\Rightarrow u=\text{arsinh}\left(\frac{4}{3}\right)$	Correct limits	B1
	$\int(\cosh 2u-1)\mathrm{d}u=\left[\frac{1}{2}\sinh 2u-u\right]$	Attempt integration of the form $\alpha\sinh 2u+\beta u$	M1
	$\int_0^1\frac{64x^2}{\sqrt{16x^2+9}}\mathrm{d}x=\frac{9}{2}\left[\frac{1}{2}\sinh 2u-u\right]_0^{\text{arsinh}\left(\frac{4}{3}\right)}$		
	$=\frac{9}{2}\left\{\left(\frac{4}{3}\sqrt{1+\frac{16}{9}}\right)-\ln\left(\frac{4}{3}+\sqrt{1+\frac{16}{9}}\right)(-0)\right\}$	Substitute u limits and subtract the right way round. **Dependent on the previous M** (Condone omission of "– 0")	dM1
	$=10,-\frac{9}{2}\ln 3$	cao	A1, A1
			(5)
			Total 11
	Alternative – changes back to x		
	$I=\frac{9}{2}\left[\sinh u\cosh u-u\right]_0^{\text{arsinh}\left(\frac{4}{3}\right)}$		
	$I=\frac{9}{2}\left[\frac{4x}{3}\sqrt{\frac{16x^2}{9}+1}-\text{arsinh}\left(\frac{4x}{3}\right)\right]_0^1$	Use of $\frac{4x}{3}$	B1
	$=\frac{9}{2}\left\{\left(\frac{4}{3}\sqrt{1+\frac{16}{9}}\right)-\ln\left(\frac{4}{3}+\sqrt{1+\frac{16}{9}}\right)(-0)\right\}$	Substitute x limits and subtract the right way round. **Dependent on the previous M** (Condone omission of "– 0")	dM1
	$=10,-\frac{9}{2}\ln 3$		A1, A1

Alternatives to 5(a)

<table>
<tr><td rowspan="4">Way 2</td><td rowspan="2">Tangent at $\left(5\cos\theta, 3\sin\theta\right)$ is
$y = -\frac{3\cos\theta}{5\sin\theta}x + \frac{3}{\sin\theta}$</td><td>M1: Full attempt at a general tangent</td><td rowspan="2">M1A1</td></tr>
<tr><td>A1: Correct tangent</td></tr>
<tr><td>$c^2 - 25m^2 = \frac{9}{\sin^2\theta} - 25\frac{9\cos^2\theta}{25\sin^2\theta}$</td><td>Substitutes their c and m into $c^2 - 25m^2$</td><td>M1</td></tr>
<tr><td>$c^2 - 25m^2 = 9$ *</td><td>Achieves printed answer with no errors</td><td>A1*</td></tr>
</table>

<table>
<tr><td rowspan="6">Way 3</td><td>$\frac{x^2}{25} + \frac{y^2}{9} = 1 \Rightarrow \frac{2x}{25} + \frac{2y}{9}\frac{dy}{dx} = 0$</td><td></td><td></td></tr>
<tr><td>$\frac{dy}{dx} = -\frac{9x}{25y} \Rightarrow m = -\frac{9x}{25y}$</td><td></td><td></td></tr>
<tr><td colspan="2">$y = -\frac{9x}{25m} = mx + c \Rightarrow x = -\frac{25mc}{9 + 25m^2}$ and $y = \frac{9c}{9 + 25m^2}$</td><td></td></tr>
<tr><td colspan="2">M1: Differentiates implicitly, uses $\frac{dy}{dx} = m$ and $y = mx + c$ to obtain x and y in terms of m and c
A1: $x = -\frac{25mc}{9 + 25m^2}$ and $y = \frac{9c}{9 + 25m^2}$</td><td>M1A1</td></tr>
<tr><td>$\frac{x^2}{25} + \frac{y^2}{9} = 1 \Rightarrow \frac{25m^2c^2}{\left(9 + 25m^2\right)^2} + \frac{9c^2}{\left(9 + 25m^2\right)^2} = 1$</td><td>Substitutes their x and y in terms of m and c into E</td><td>M1</td></tr>
<tr><td>$25m^2c^2 + 9c^2 = \left(9 + 25m^2\right)^2 \Rightarrow c^2 - 25m^2 = 9$ *</td><td>Achieves printed answer with no errors</td><td>A1*</td></tr>
</table>

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