Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level in Further Pure Mathematics F3 (WFM03/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2014
Publications Code IA038891

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where} |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required..

Question Number	Scheme		Marks
1.(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\left(\frac{2}{3}\right)\frac{1}{1+\frac{4x^2}{9}} = \frac{6}{9+4x^2}$	M1: Use formula for derivative of arctan: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ \frac{p}{1+(qx)^2},\ q\neq 1$ Condone missing brackets around qx but must be $1+(qx)^2$ not $1-(qx)^2$ and p may be 1 A1: Answer **as shown**	M1A1
	Allow correct answer only		
			(2)
	Alternative		
	$y=\arctan\left(\frac{2x}{3}\right)\Rightarrow \tan y=\frac{2x}{3}\Rightarrow \sec^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3\sec^2 y}=\frac{2}{3\left(1+\tan^2 y\right)}$		
	$=\frac{2}{3\left(1+\left(\frac{2}{3}x\right)^2\right)}$	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ \frac{p}{1+(qx)^2},\ q\neq 1$ Condone missing brackets around qx but must be $1+(qx)^2$ not $1-(qx)^2$ and p may be 1	M1
	$=\frac{6}{9+4x^2}$	Answer **as shown**	A1
(b)	$\therefore \int \arctan\left(\frac{2x}{3}\right)\mathrm{d}x = \left[x\arctan\left(\frac{2x}{3}\right)\right]-\int \frac{6x}{9+4x^2}\mathrm{d}x$		M1A1ft
	M1: Use of parts in correct direction Allow e.g. $x\arctan\left(\frac{2x}{3}\right)-\int x\,\mathrm{d}\left(\arctan\left(\frac{2x}{3}\right)\right)$ **for M1** A1ft: Follow through their answer from part (a)		
	$=\left[x\arctan\left(\frac{2x}{3}\right)\right]-\left[\frac{3}{4}\ln(9+4x^2)\right](+c)$		M1A1
	M1: Use of ln correctly for their fraction A1: Cao (+ c not required) Allow $x\arctan\left(\frac{2x}{3}\right)\times x$ and $-\frac{3}{4}\ln k(9+4x^2)$		
			(4)
			Total 6

Question Number	Scheme		Marks
2.	$\pm\frac{a}{e}=\pm 9$ **and** $a^2(1-e^2)=8$	Both equations correct	B1
	$a^4-81a^2+648=0$ or $81e^4-81e^2+8=0$	M1: Eliminates an unknown to produce a quadratic in a^2 or e^2 A1: Correct three term quadratic in any form with terms collected	M1A1
	$\left(a^2-72\right)\left(a^2-9\right)=0 \Rightarrow a^2=...$ or $\left(9e^2-8\right)\left(9e^2-1\right)=0 \Rightarrow e^2=...$	Uses a standard method (see notes) to solve quadratic as far as $a^2=...$ or $e^2=...$ (Must be $a^2=...$ or $e^2=...$ at this stage not $a=...$ or $e=....$ but this may be implied by later work) **May be implied by correct answers only.**	M1
	$a=3$ and $a=6\sqrt{2}$	M1: Complete method to find a. Either square roots from $a^2=\ldots$ or square roots from $e^2=\ldots.$ and uses $a=9e$ at least once A1: cao (both answers correct). Do not accept ± for either of the answers unless the negative is rejected later.	M1A1
			(6)
			Total 6

Question Number	Scheme		Marks
3.(a)	$\{\frac{1}{2}(e^{x}+e^{-x})\}^{2}-\{\frac{1}{2}(e^{x}-e^{-x})\}^{2}=\{\frac{1}{4}(e^{2x}+2+e^{-2x})\}-\{\frac{1}{4}(e^{2x}-2+e^{-2x})\}$		M1
	M1: Uses the correct exponential forms for cosh and sinh and squares both brackets obtaining 3 terms each time		
	$\frac{1}{2}+\frac{1}{2}=1$	**At least one line of intermediate working** (e.g. combines fractions with a common denominator) with no errors seen and concludes = 1	A1
			(2)
(b)	$(e^{x}-e^{-x})+7\times\frac{1}{2}(e^{x}+e^{-x})=9$ $\Rightarrow \frac{9}{2}e^{x}+\frac{5}{2}e^{-x}-9=0$	M1: Uses exponential forms **and** collects terms A1: Any correct form with terms collected	M1A1
	$\Rightarrow 9e^{2x}-18e^{x}+5=0$ so $e^{x}=\dots$	Solves their three term quadratic in e^x as far as $e^x =$	M1
	$e^{x}=\frac{1}{3}$ or $\frac{5}{3}$	Both values correct	A1
	$x=\ln\frac{1}{3}$ and $\ln\frac{5}{3}$	Both values correct (accept equivalents)	A1
			(5)
			Total 7
	Alternatives for (b) – Special Cases		
Way 2	$2\sinh x=9-7\cosh x\Rightarrow 45\cosh^{2}x-126\cosh x+85=0$		M1A1
	M1: Attempt to square both sides A1: Correct quadratic in cosh$x$		
	$(15\cosh x-17)(3\cosh x-5)=0\Rightarrow \cosh x=\frac{17}{15}$ or $\cosh x=\frac{5}{3}$		
	$\frac{e^{x}+e^{-x}}{2}=\frac{17}{15}\Rightarrow 15e^{2x}-34e^{x}+15=0,\ \frac{e^{x}+e^{-x}}{2}=\frac{5}{3}\Rightarrow 3e^{2x}-10e^{x}+3=0$		
	$(5e^{x}-3)(3e^{x}-5)=0\Rightarrow e^{x}=\frac{3}{5},\ e^{x}=\frac{5}{3}$ $(3e^{x}-1)(e^{x}-3)=0\Rightarrow e^{x}=\frac{1}{3},\ e^{x}=3$ $e^{x}=\frac{5}{3}$ and $e^{x}=\frac{1}{3}$	M1: Solves at least one of their three term quadratics in e^x as far as $e^x =$…., having used the correct exponential form for cosh$x$ A1: $e^{x}=\frac{5}{3}$ and $e^{x}=\frac{1}{3}$ seen	M1A1
	$x=\ln\frac{1}{3}$ and $\ln\frac{5}{3}$	These values only with $\ln 3$ and $\ln\frac{3}{5}$ rejected	A1
Way 3	$7\cosh x=9-2\sinh x\Rightarrow 45\sinh^{2}x+36\sinh x-32=0$		
	M1: Attempt to square both sides A1: Correct quadratic in sinh$x$		M1A1
	$(15\sinh x-8)(3\sinh x+4)=0\Rightarrow \sinh x=\frac{8}{15}$ or $\sinh x=-\frac{4}{3}$		
	$\frac{e^{x}-e^{-x}}{2}=\frac{8}{15}\Rightarrow 15e^{2x}-16e^{x}-15=0,\ \frac{e^{x}-e^{-x}}{2}=-\frac{4}{3}\Rightarrow 3e^{2x}+8e^{x}-3=0$		
	$(3e^{x}-5)(5e^{x}+3)=0\Rightarrow e^{x}=\frac{5}{3},\ e^{x}=-\frac{3}{5}$ $(3e^{x}-1)(e^{x}+3)=0\Rightarrow e^{x}=\frac{1}{3},\ e^{x}=-3$ $e^{x}=\frac{5}{3}$ and $e^{x}=\frac{1}{3}$	M1: Solves at least one of their three term quadratics in e^x as far as $e^x =$…., having used the correct exponential form for sinh$x$ A1: $e^{x}=\frac{5}{3}$ and $e^{x}=\frac{1}{3}$ seen	M1A1
	$x=\ln\frac{1}{3}$ and $\ln\frac{5}{3}$	These values only	A1
	Note: For these special cases, if they use the ln form of arcosh or arsinh from their cosh = … or sinh = … then only the first 2 marks are available as they are not using exponentials.		

Question Number	Scheme		Marks
4. (a)	$\det \mathbf{M} = 6 - k^2$	A correct (possibly un-simplified) determinant	B1
	$\mathbf{M}^T = \begin{pmatrix} 3 & k & k \\ k & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **or** minors $\begin{pmatrix} 2 & k & -2k \\ k & 3 & -k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$ **or** cofactors $\begin{pmatrix} 2 & -k & -2k \\ -k & 3 & k^2 \\ 0 & 0 & 6-k^2 \end{pmatrix}$		B1
	$\dfrac{1}{6-k^2}\begin{pmatrix} 2 & -k & 0 \\ -k & 3 & 0 \\ -2k & k^2 & 6-k^2 \end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1A1A1
			(5)
(b)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ -2 & 1 & 5 \end{pmatrix}\begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$ $\Rightarrow a = \ldots \text{or}\, b = \ldots \text{or}\, c = \ldots$	Uses $k = 1$ in the inverse and attempts to multiply to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao
			(3)
			Total 8
	Alternative for (b)		
	$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix} \Rightarrow \begin{matrix} 3a+b=-5 \\ a+2b=10 \\ a+c=7 \end{matrix}$ $\Rightarrow a = \ldots \text{or}\, b = \ldots \text{or}\, c = \ldots$	Multiplies to give 3 equations and attempts to obtain a numerical value for at least one of a, b or c	M1
	$x = -4,\ y = 7,\ z = 11$	M1: Obtains values for all three coordinates A1: Correct coordinates	M1A1cao

Question	Scheme		Marks
5(a)	$I_n = \left[\cos^{n-1}\theta\sin\theta\right]_0^{\frac{\pi}{4}} - (-)\int_0^{\frac{\pi}{4}}(n-1)\cos^{n-2}\theta\sin^2\theta\,\mathrm{d}\theta$		M1A1
	M1: Attempt parts the correct way round A1: Correct expression		
	so $I_n = \left(\frac{1}{\sqrt{2}}\right)^n +$	Uses limits to obtain $\left(\frac{1}{\sqrt{2}}\right)^n$	B1
	i.e. $I_n = ...+\int_0^{\frac{\pi}{4}}(n-1)\cos^{n-2}\theta(1-\cos^2\theta)\mathrm{d}\theta$		**d**M1
	M1: Replaces $\sin^2\theta$ by 1 - $\cos^2\theta$ **Dependent on the previous method mark**		
	So $I_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2} - (n-1)I_n$, and $nI_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2}$ *		**dd**M1A1cso
	M1: Replaces expressions for I_n and I_{n-1} **Dependent on both previous method marks** A1: Achieves printed answer with no errors seen		
			(6)
	Alternative		
	$I_n = \int_0^{\frac{\pi}{4}} \cos^{n-2}\theta\cos^2\theta\,\mathrm{d}\theta = \int_0^{\frac{\pi}{4}} \cos^{n-2}\theta(1-\sin^2\theta)\mathrm{d}\theta$		2^{nd} M1
	Writes $\cos^n\theta$ as $\cos^{n-2}\theta\cos^2\theta$ and replaces $\cos^2\theta$ by 1 - $\sin^2\theta$		
	$I_n = I_{n-2} + \left[\frac{1}{n-1}\cos^{n-1}\theta\sin\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\frac{1}{(n-1)}\cos^n\theta\,\mathrm{d}\theta$		**d**M1A1
	dM1: Attempt parts the correct way round A1: Correct expression		
	$I_n = I_{n-2} + \frac{1}{n-1}\left(\frac{1}{\sqrt{2}}\right)^n - \frac{1}{n-1}I_n$	B1: Uses limits to obtain $\frac{1}{n-1}\left(\frac{1}{\sqrt{2}}\right)^n$ **dd**M1: Replaces expressions for I_n and I_{n-1}	B1**dd**M1
	$nI_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2}$	Achieves printed answer with no errors seen	A1
(b)	$I_1 = \int_0^{\frac{\pi}{4}}\cos\theta\,\mathrm{d}\theta = [\sin\theta]_0^{\frac{\pi}{4}} = \frac{1}{\sqrt{2}}$	M1: Attempt I_1 A1: $\frac{1}{\sqrt{2}}$	M1A1
	$I_3 = \frac{1}{3}(\frac{1}{2\sqrt{2}} + 2I_1)$, $I_5 = \frac{1}{5}(\frac{1}{4\sqrt{2}} + 4I_3)$ or $3I_3 = \frac{1}{2\sqrt{2}} + 2I_1$, $5I_5 = \frac{1}{4\sqrt{2}} + 4I_3$	M1: Uses reduction formula first time (allow slips providing the reduction formula is being used) M1: Uses reduction formula second time (allow slips providing the reduction formula is being used)	M1M1
	$I_5 = \frac{43\sqrt{2}}{120}$ or $\frac{43}{60\sqrt{2}}$		A1
			(5)
			Total 11

Question	Scheme		Marks
6(a)	$\frac{dx}{d\theta}=4\sinh\alpha$ and $\frac{dy}{d\theta}=2\cosh\alpha$ so $\frac{dy}{dx}=\frac{2\cosh\alpha}{4\sinh\alpha}$		M1A1
	M1: Differentiates x and y and divides correctly A1: Correct derivative in terms of α		
	OR $\frac{2x}{16}-\frac{2yy'}{4}=0\Rightarrow y'=\frac{x}{4y}=\frac{4\cosh\alpha}{8\sinh\alpha}$		
	M1: Differentiates implicitly to obtain $px-qyy'=0$ and makes y' the subject A1: Correct derivative in terms of α		
	OR $y=\frac{\sqrt{x^2-16}}{2}\Rightarrow y'=\frac{x}{2\sqrt{x^2-16}}=\frac{4\cosh\alpha}{2\sqrt{16\cosh^2\alpha-16}}\left(=\frac{4\cosh\alpha}{8\sinh\alpha}\right)$		
	M1: Differentiates explicitly to obtain $y'=\frac{kx}{\sqrt{x^2-16}}$ A1: Correct derivative in terms of α		
	Equation of tangent is $(y-2\sinh\alpha)=\frac{\cosh\alpha}{2\sinh\alpha}(x-4\cosh\alpha)$ **(I)**		M1
	Correct straight line method using their gradient in terms of α		
	$2y\sinh\alpha-4\sinh^2\alpha=x\cosh\alpha-4\cosh^2\alpha$ **(II)**		
	$2y\sinh\alpha+4\left(\cosh^2\alpha-\sinh^2\alpha\right)-x\cosh\alpha=0\Rightarrow 2y\sinh\alpha-x\cosh\alpha+4=0*$		A1*
	See use of $\cosh^2\alpha-\sinh^2\alpha=1$ to give printed answer – **there must be some working to establish the printed answer: (I) to * is A0, (II) to * is A1**		
			(4)
(b)	Puts $x=0$ to give A is $\left(0,\ \frac{-2}{\sinh\alpha}\right)$	M1: Uses $x=0$ in the given equation to find y	M1A1
		A1: $y=\frac{-2}{\sinh\alpha}$ or $y=\frac{-4}{2\sinh\alpha}$	
			(2)
(c)	$b^2=a^2\left(e^2-1\right)\Rightarrow a^2e^2=20$	Uses the **correct** eccentricity formula to obtain a value for a^2e^2 or ae Or finds a value for e and multiplies by a. Or finds a value for e^2 and multiplies by a^2.	M1
	$ae=\sqrt{20}$ or $2\sqrt{5}$	Correct value for ae **Allow correct answer only**	A1
	Gradient $AS=\frac{\frac{2}{\sinh\alpha}}{2\sqrt{5}}$ **or** Gradient $BS=-\frac{10\sinh\alpha}{2\sqrt{5}}$ Or $\overrightarrow{AS}=\begin{pmatrix}2\sqrt{5}\\ \frac{\frac{2}{\sinh\alpha}}{2\sqrt{5}}\end{pmatrix}$ or $\overrightarrow{BS}=\begin{pmatrix}2\sqrt{5}\\ -10\sinh\alpha\end{pmatrix}$		B1
	At least one correct gradient or vector (allow as "coordinates") in terms of sinhα (allow if also in terms of a and or e) E.g Gradient $AS=\frac{\frac{2}{\sinh\alpha}}{ae \text{ or } 4e \text{ or } a\frac{\sqrt{5}}{2}}$ **or** Gradient $BS=-\frac{10\sinh\alpha}{ae \text{ or } 4e \text{ or } a\frac{\sqrt{5}}{2}}$		
	$\frac{\frac{2}{\sinh\alpha}}{2\sqrt{5}}\times-\frac{10\sinh\alpha}{2\sqrt{5}}=-1$ so AS and BS are perpendicular	M1: Multiplies their AS and BS gradients or uses scalar product e.g. $\overrightarrow{\mathbf{SB}}.\overrightarrow{\mathbf{SA}}$ in terms of sinhα only and must be seen explicitly.	M1A1
		A1: Product = -1 or scalar product = 0 with no errors and conclusion	
			(5)
			Total 11

Question Number	Scheme		Marks
7.(a)	$\frac{dx}{dt}=6t,\ \frac{dy}{dt}=12$	Both derivatives correct	B1
	$S=(2\pi)\int 12t\sqrt{(6t)^2+12^2}\ dt$	M1: Use of a correct surface area formula with their derivatives **(2π not needed for this mark)** A1: Correct expression **including 2π** which may be implied by later work)	M1A1
	$=\frac{2\pi}{9}[(36t^2+144)^{\frac{3}{2}}]$	Recognisable attempt at integration e.g $t=2\tan\theta$. **Dependent on the first M.**	dM1
	$=\frac{2\pi}{9}\left\{720^{\frac{3}{2}}-144^{\frac{3}{2}}\right\}$	Uses the limits 0 and 4 and subtracts. **Dependent on the first M.**	dM1
	$=\pi(1920\sqrt{5}-384)$	Cao (Allow equivalent fractions for 1920 and or 384)	A1
			(6)
(b)	$L=\int_0^4\sqrt{(6t)^2+12^2}\ dt=6\int_0^4\sqrt{t^2+4}\ dt$	Use of a correct arc length formula and obtains $k=6$	B1
(c)	$t=2\sinh\theta\Rightarrow\frac{dt}{d\theta}=2\cosh\theta$	Correct derivative	B1
	$L=6\int\sqrt{4\sinh^2\theta+4}\times 2\cosh\theta\, d\theta$	Complete substitution	M1
	$=24\int\cosh^2\theta\, d\theta=12\int(\cosh 2\theta+1)\, d\theta$	Uses $\cosh^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2\theta$	M1
	$6\sinh 2\theta+12\theta$	Correct integration	A1
	$L=6\sinh 2(\text{arsinh}2)+12\text{arsinh}2(-0)$	Use limits arsinh 2 (and 0)	M1
	$=24\sqrt{5}+12\ln(2+\sqrt{5})$ *	Correct solution with no errors	A1*
			(7)
			Total 13
	Alternative - integration using exponentials (last 4 marks)		
	$24\int\cosh^2\theta\, d\theta=12\int(\frac{e^\theta+e^{-\theta}}{2})^2 d\theta=6\int(e^{2\theta}+e^{-2\theta}+2)\, d\theta$		M1
	Substitutes the correct exponential form of $\cosh\theta$ and squares		
	$3e^{2\theta}-3e^{-2\theta}+12\theta$	Correct integration	A1
	$L=3e^{2\text{arsinh}2}-3e^{-2\text{arsinh}2}+12\text{arsinh}2(-0)$	Use limits arsinh 2 (and 0)	M1
	$=24\sqrt{5}+12\ln(2+\sqrt{5})$ *	Correct solution with no errors	A1*

Question Number	Scheme		Marks
8(a)	$((2+3\lambda)\mathbf{i}+(1+2\lambda)\mathbf{j}+(-2+\lambda)\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$ $\Rightarrow 2+1+4+3\lambda+2\lambda-2\lambda=19 \Rightarrow \lambda=...$		M1
	Correct dot product leading to value for λ		
	$\lambda=4$	Correct λ	A1
	$\left(2+3\times"4",1+2\times"4",-2+"4"\right)$	Substitutes their λ to give coordinates	M1
	(14, 9, 2)	Correct coordinates (allow as vector)	A1
			(4)
(b)	$\overrightarrow{AB}=2\mathbf{i}+2\mathbf{j}-4\mathbf{k}=2(\mathbf{i}+\mathbf{j}-2\mathbf{k})$ so is perpendicular to plane		M1
	Correct $\overrightarrow{AB}$ and conclusion		
	Also B lies on the plane as $(4\mathbf{i}+3\mathbf{j}-6\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$		M1
	Substitutes B into the plane equation and conclusion		
	So coordinates of B are (4, 3, -6)*	Both M's scored with final conclusion	A1*
			(3)
	Alternative		
	$((2+\lambda)\mathbf{i}+(1+\lambda)\mathbf{j}+(-2-2\lambda)\mathbf{k}).(\mathbf{i}+\mathbf{j}-2\mathbf{k})=19$ $\Rightarrow 2+1+4+\lambda+\lambda+4\lambda=19 \Rightarrow \lambda=...$		M1
	Correct dot product leading to value for λ (= 2)		
	$\left(2+"2",1+"2",-2-2\times"2"\right)$	Substitutes their λ to give coordinates	M1
	So coordinates of B are (4, 3, -6)*	Both M's scored with final conclusion	A1
(c)	$\overrightarrow{OA}'=\overrightarrow{OA}+2\overrightarrow{AB}$ or $\overrightarrow{OB}+\overrightarrow{AB}$ (2+4, 1+4, -2-8) or (4+2, 3+2, -6-4)	Correct strategy for finding $A^{/}$	M1
	(6, 5, -10)	Correct coordinates	A1
			(2)
(d)	**NB require line through their (14, 9, 2) and their (6, 5, -10)**		
	±(14**i** + 9**j** + 2**k** – (6**i** + 5**j** -10**k**))	Correct attempt at the direction	M1
	a = 8**i** + 4**j** + 12**k**	μ (8**i** + 4**j** + 12**k**)	A1
	$\mathbf{b}=(6\mathbf{i}+5\mathbf{j}-10\mathbf{k})\times(8\mathbf{i}+4\mathbf{j}+12\mathbf{k})$ or $(14\mathbf{i}+9\mathbf{j}+2\mathbf{k})\times(8\mathbf{i}+4\mathbf{j}+12\mathbf{k})$ $=\left(=100\mathbf{i}-152\mathbf{j}-16\mathbf{k}\right)$		**d**M1
	Attempt vector product of their $6\mathbf{i}+5\mathbf{j}-10\mathbf{k}$ with their $8\mathbf{i}+4\mathbf{j}+12\mathbf{k}$ **Dependent on the previous M1**		
	$\mathbf{r}\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=25\mathbf{i}-38\mathbf{j}-4\mathbf{k}$	$\lambda\left(\mathbf{r}\times(2\mathbf{i}+\mathbf{j}+3\mathbf{k})=25\mathbf{i}-38\mathbf{j}-4\mathbf{k}\right)$	A1
	Must be in this form for A1 and not just stating **a** and **b**		
			(4)
			Total 13

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE