Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 4 June 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Pearson

1. The complex number $z = x + \mathrm{i}y$ satisfies the equation

$$|z - 3 - 4\mathrm{i}| = |z + 1 + \mathrm{i}|$$

(a) Determine an equation for the locus of z giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(b) Shade, on an Argand diagram, the region defined by

$$|z - 3 - 4\mathrm{i}| \leqslant |z + 1 + \mathrm{i}|$$

You do **not** need to determine the coordinates of any intercepts on the coordinate axes.

(1)

Question 1 continued

(Total for Question 1 is 4 marks)

2.

$$x\frac{dy}{dx} - y^3 = 4$$

(a) Show that

$$x\frac{d^3y}{dx^3} = ay\left(\frac{dy}{dx}\right)^2 + \left(by^2 + c\right)\frac{d^2y}{dx^2}$$

where a, b and c are integers to be determined.

(4)

Given that $y = 1$ at $x = 2$

(b) determine the Taylor series expansion for y in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(3)

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Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) Express

$$\frac{1}{(n+3)(n+5)}$$

in partial fractions.

(2)

(b) Hence, using the method of differences, show that for all positive integer values of n,

$$\sum_{r=1}^{n} \frac{1}{(r+3)(r+5)} = \frac{n(pn+q)}{40(n+4)(n+5)}$$

where p and q are integers to be determined.

(4)

(c) Use the answer to part (b) to determine, as a simplified fraction, the value of

$$\frac{1}{9\times11} + \frac{1}{10\times12} + \ldots + \frac{1}{24\times26}$$

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

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4. (a) Show that the substitution $y^2 = \frac{1}{t}$ transforms the differential equation

$$\frac{dy}{dx} + y = xy^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dt}{dx} - 2t = -2x \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) and determine y^2 in terms of x.

(6)

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Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the values of x for which

$$\frac{x+1}{(x-3)(x+2)} \leqslant 1-\frac{2}{x-3}$$

(6)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 6 marks)

6. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - \mathrm{i}}{z + 1} \qquad z \neq -1$$

Given that T maps the imaginary axis in the z-plane to the circle C in the w-plane, determine

(i) the coordinates of the centre of C

(ii) the radius of C

(7)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7. Given that $y = e^x \sin x$

(a) show that

$$\frac{d^6 y}{dx^6} = k\frac{d^2 y}{dx^2}$$

where k is a constant to be determined.

(4)

(b) Hence determine the first 5 non-zero terms in the Maclaurin series expansion for y, giving each coefficient in simplest form.

(3)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. (a) Given that $t = \ln x$, where $x > 0$, show that

$$\frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

(3)

(b) Hence show that the transformation $t = \ln x$, where $x > 0$, transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2y = 1 + 4\ln x - 2(\ln x)^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 1 + 4t - 2t^2 \qquad \text{(II)}$$

(1)

(c) Solve differential equation (II) to determine y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Use De Moivre's theorem to show that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(4)

(b) Hence determine the smallest positive root of the equation

$$48x^6 - 72x^4 + 27x^2 - 1 = 0$$

giving your answer to 3 decimal places.

(4)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10.

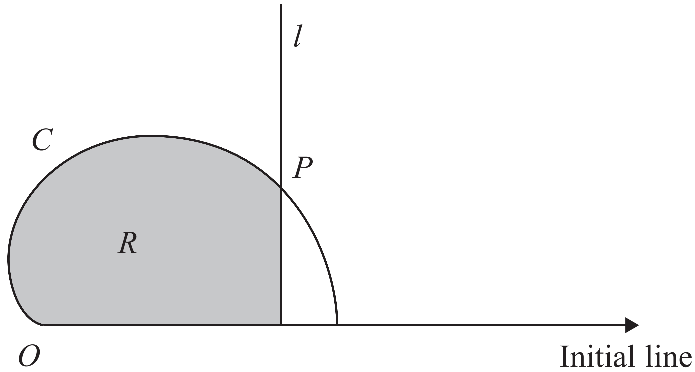

Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 1 + \cos\theta \qquad 0 \leqslant \theta \leqslant \pi$$

and the line l with polar equation

$$r = k\sec\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

where k is a positive constant.

Given that

- C and l intersect at the point P
- $OP = 1 + \frac{\sqrt{3}}{2}$

(a) determine the exact value of k.

(2)

The finite region R, shown shaded in Figure 1, is bounded by C, the initial line and l.

(b) Use algebraic integration to show that the area of R is

$$p\pi + q\sqrt{3} + r$$

where p, q and r are simplified rational numbers to be determined.

(7)

Question 10 continued

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Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS