Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 15 January 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. Using algebra, solve the inequality

$$\frac{1}{x+2} > 2x+3$$

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

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Turn over

2.

$$z = 6 - 6\sqrt{3}\,\mathrm{i}$$

(a) (i) Determine the modulus of z

(ii) Show that the argument of z is $-\dfrac{\pi}{3}$

(3)

Using de Moivre's theorem, and making your method clear,

(b) determine, in simplest form, z^4

(2)

(c) Determine the values of w such that $w^2 = z$, giving your answers in the form $a + \mathrm{i}b$, where a and b are real numbers.

(3)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. (a) Show that for $r \geqslant 1$

$$\frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}} \equiv A\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)$$

where A is a constant to be determined.

(2)

(b) Hence use the method of differences to determine a simplified expression for

$$\sum_{r=1}^{n} \frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}$$

(3)

(c) Determine, as a surd in simplest form, the constant k such that

$$\sum_{r=1}^{n} \frac{kr}{\sqrt{r(r+1)}+\sqrt{r(r-1)}} = \sqrt{\sum_{r=1}^{n} r}$$

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Determine, in ascending powers of $\left(x-\frac{\pi}{6}\right)$ up to and including the term in $\left(x-\frac{\pi}{6}\right)^3$, the Taylor series expansion about $\frac{\pi}{6}$ of

$$y=\tan\left(\frac{3x}{2}\right)$$

giving each coefficient in simplest form.

(7)

(b) Hence show that

$$\tan\frac{3\pi}{8}\approx 1+\frac{\pi}{4}+\frac{\pi^2}{A}+\frac{\pi^3}{B}$$

where A and B are integers to be determined.

(2)

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Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

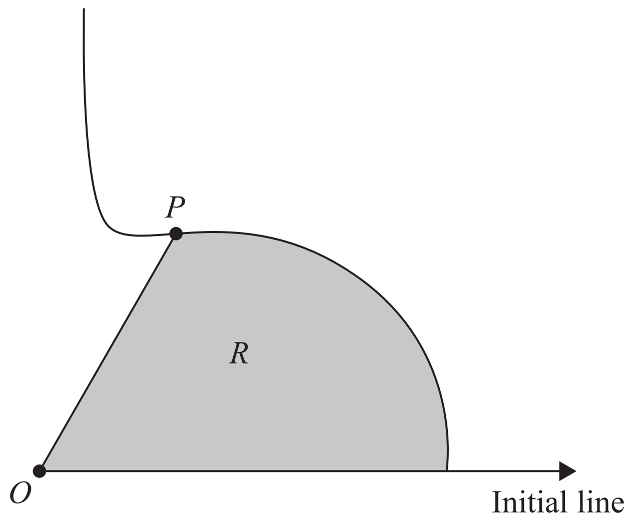

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = 10\cos\theta + \tan\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on the curve where $\theta = \dfrac{\pi}{3}$

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line OP, where O is the pole.

Use algebraic integration to show that the exact area of R is

$$\frac{1}{12}\left(a\pi + b\sqrt{3} + c\right)$$

where a, b and c are integers to be determined.

(9)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 8e^{-3t} \qquad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Determine the general solution of this differential equation.

(6)

Given that the motion of the particle satisfies $x = \frac{1}{2}$ and $\frac{dx}{dt} = \frac{1}{2}$ when $t = 0$

(b) determine the particular solution for the motion of the particle.

(4)

On the graph of the particular solution found in part (b), the first turning point for $t > 0$ occurs at $x = a$.

(c) Determine, to 3 significant figures, the value of a.

[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 14 marks)

7. A transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$ is given by

$$w = \frac{z - 3}{2\mathrm{i} - z} \qquad z \neq 2\mathrm{i}$$

The line in the z-plane with equation $y = x + 3$ is mapped by T onto a circle C in the w-plane.

(a) Determine

(i) the coordinates of the centre of C

(ii) the exact radius of C

(8)

The region $y > x + 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) On a single Argand diagram

(i) sketch the circle C

(ii) shade and label the region R

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. (a) For all the values of x where the identity is defined, prove that

$$\cot 2x + \tan x \equiv \operatorname{cosec} 2x$$

(3)

(b) Show that the substitution $y^2 = w\sin 2x$, where w is a function of x, transforms the differential equation

$$y\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 \tan x = \sin x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}x} + 2w\operatorname{cosec} 2x = \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(4)

(c) By solving differential equation (II), determine a general solution of differential equation (I) in the form $y^2 = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a function in terms of $\cos x$

$$\left[\text{You may use without proof } \int \operatorname{cosec} 2x \,\mathrm{d}x = \frac{1}{2}\ln\left|\tan x\right| \ (+\text{constant})\right]$$

(6)

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Question 8 continued

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS