Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number

Candidate Number

# Pearson Edexcel International Advanced Level

## Thursday 8 June 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F2

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

***Turn over***



©2023 Pearson Education Ltd.
N:1/1/1/1/



Pearson

**1.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) Show that, for $r \geqslant 2$

$$\frac{2}{\sqrt{r}+\sqrt{r-2}} = \sqrt{r} - \sqrt{r-2}$$

**(2)**

(b) Hence use the method of differences to determine

$$\sum_{r=2}^{n} \frac{2}{\sqrt{r}+\sqrt{r-2}}$$

giving your answer in simplest form.

**(3)**

(c) Hence show that

$$\sum_{r=4}^{50} \frac{2}{\sqrt{r}+\sqrt{r-2}} = A + B\sqrt{2} + C\sqrt{3}$$

where $A$, $B$ and $C$ are integers to be determined.

**(2)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 1 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 1 continued**

**(Total for Question 1 is 7 marks)**



DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

**2.** The complex number $z_1$ is defined as

$$z_1 = \frac{\left(\cos\frac{5\pi}{12} + \mathrm{i}\sin\frac{5\pi}{12}\right)^4}{\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)^3}$$

(a) Without using your calculator show that

$$z_1 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$$

**(4)**

(b) Shade, on a single Argand diagram, the region $R$ defined by

$$|z - z_1| \leqslant 1 \qquad \textbf{and} \qquad 0 \leqslant \arg(z - z_1) \leqslant \frac{3\pi}{4}$$

**(4)**

Given that the complex number $z$ lies in $R$

(c) determine the smallest possible positive value of $\arg z$

**(2)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

**Question 2 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 2 continued**

**(Total for Question 2 is 10 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**3.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given that

$$\frac{x+2}{x+4} \leqslant \frac{x}{k(x-1)}$$

where $k$ is a positive constant,

(a) show that

$$(x+4)(x-1)(px^2+qx+r) \leqslant 0$$

where $p$, $q$ and $r$ are expressions in terms of $k$ to be determined.

**(3)**

(b) Hence, or otherwise, determine the values for $x$ for which

$$\frac{x+2}{x+4} \leqslant \frac{x}{3(x-1)}$$

**(4)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

**Question 3 continued**

**(Total for Question 3 is 7 marks)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**4.** (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 48x^2 - 34$$

**(5)**

Given that $y = 4$ and $\frac{dy}{dx} = 21$ at $x = 0$

(b) determine the particular solution of the differential equation.

**(4)**

(c) Hence find the value of $y$ at $x = -2$, giving your answer in the form $pe^q + r$ where $p$, $q$ and $r$ are integers to be determined.

**(2)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 5 4 A 0 1 2 3 2

**Question 4 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 4 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 11 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

5. The transformation $T$ from the $z$-plane, where $z = x + \mathrm{i}y$, to the $w$-plane, where $w = u + \mathrm{i}v$ is given by

$$w = \frac{z+1}{z-3} \qquad z \neq 3$$

The straight line in the $z$-plane with equation $y = 4x$ is mapped by $T$ onto the circle $C$ in the $w$-plane.

(a) Show that $C$ has equation

$$3u^2 + 3v^2 - 2u + v + k = 0$$

where $k$ is a constant to be determined.

**(5)**

(b) Hence determine

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(2)**



P 7 2 8 5 4 A 0 1 6 3 2

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 5 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA



P 7 2 8 5 4 A 0 1 8 3 2

**Question 5 continued**

**(Total for Question 5 is 7 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**6.** Given that $y = \sec x$

(a) show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \sec x \tan x \left(p \sec^2 x + q\right)$$

where $p$ and $q$ are integers to be determined.

**(4)**

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\sec x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$, up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

**(3)**

(c) Use the answer to part (b) to determine, to four significant figures, an approximate value of $\sec\left(\frac{7\pi}{24}\right)$

**(2)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

**Question 6 continued**





Turn over

**Question 6 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

**(Total for Question 6 is 9 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**7.** (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$x\frac{dy}{dx} + y + 4x^2y^3 \ln x = 0 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{2z}{x} = 8x \ln x \qquad x > 0 \qquad \text{(II)}$$

**(5)**

(b) By solving differential equation (II), determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

**(6)**

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA



P 7 2 8 5 4 A 0 2 4 3 2

**Question 7 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 7 continued**

**(Total for Question 7 is 11 marks)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**8.**

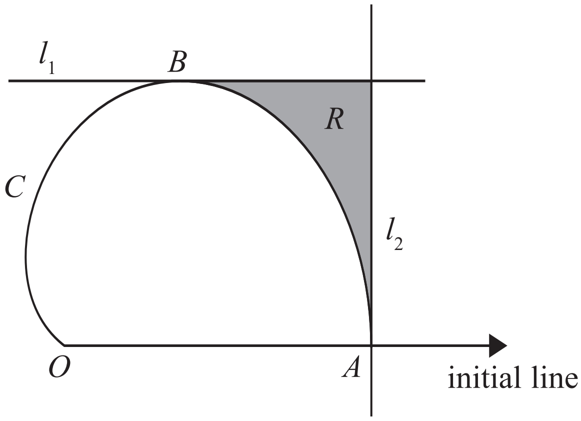


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation

$$r = 6(1 + \cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$$

Given that $C$ meets the initial line at the point $A$, as shown in Figure 1,

(a) write down the polar coordinates of $A$.

**(1)**

The line $l_1$, also shown in Figure 1, is the tangent to $C$ at the point $B$ and is parallel to the initial line.

(b) Use calculus to determine the polar coordinates of $B$.

**(4)**

The line $l_2$, also shown in Figure 1, is the tangent to $C$ at $A$ and is perpendicular to the initial line.

The region $R$, shown shaded in Figure 1, is bounded by $C$, $l_1$ and $l_2$

(c) Use algebraic integration to find the exact area of $R$, giving your answer in the form $p\sqrt{3} + q\pi$ where $p$ and $q$ are constants to be determined.

**(8)**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA



P 7 2 8 5 4 A 0 2 8 3 2

**Question 8 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

**Question 8 continued**

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA





Turn over

**Question 8 continued**

**(Total for Question 8 is 13 marks)**

**TOTAL FOR PAPER IS 75 MARKS**



P 7 2 8 5 4 A 0 3 2 3 2

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA