Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. Given that $y = \ln(5 + 3x)$

(a) determine, in simplest form, $\dfrac{d^3y}{dx^3}$

(3)

(b) Hence determine the Maclaurin series expansion of $\ln(5 + 3x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in simplest form.

(2)

(c) Hence write down the Maclaurin series expansion of $\ln(5 - 3x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in simplest form.

(1)

(d) Use the answers to parts (b) and (c) to determine the first 2 non-zero terms, in ascending powers of x, of the Maclaurin series expansion of

$$\ln\left(\frac{5 + 3x}{5 - 3x}\right)$$

(2)

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Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. (a) Express

$$\frac{1}{(2n-1)(2n+1)(2n+3)}$$

in partial fractions.

(2)

(b) Hence, using the method of differences, show that for all integer values of n,

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)(2r+3)} = \frac{n(n+2)}{a(2n+b)(2n+c)}$$

where a, b and c are integers to be determined.

(4)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 6 marks)

3. (a) Show that the transformation $y = \frac{1}{z}$ transforms the differential equation

$$x^2 \frac{dy}{dx} + xy = 2y^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{2}{x^2} \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) to determine z in terms of x.

(4)

(c) Hence determine the particular solution of differential equation (I) for which $y = -\frac{3}{8}$ at $x = 3$

Give your answer in the form $y = f(x)$.

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4.

$$\frac{dy}{dx} = y^2 - x$$

(a) Show that

$$\frac{d^4 y}{dx^4} = Ay\frac{d^3 y}{dx^3} + B\frac{dy}{dx}\frac{d^2 y}{dx^2}$$

where A and B are integers to be determined.

(4)

Given that $y = 1$ at $x = -1$

(b) determine the Taylor series solution for y, in ascending powers of $(x + 1)$ up to and including the term in $(x + 1)^4$, giving each coefficient in simplest form.

(3)

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Question 4 continued

(Total for Question 4 is 7 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the set of values of x for which

$$\frac{x^2 - 9}{|x+8|} > 6 - 2x$$

(6)

P 7 2 4 6 7 A 0 1 6 3 6

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 6 marks)

6. A complex number z is represented by the point P in an Argand diagram.

Given that

$$|z - 2\mathrm{i}| = |z - 3|$$

(a) sketch the locus of P. You do **not** need to find the coordinates of any intercepts.

(2)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{\mathrm{i}z}{z - 2\mathrm{i}} \qquad z \neq 2\mathrm{i}$$

Given that T maps $|z - 2\mathrm{i}| = |z - 3|$ to a circle C in the w-plane,

(b) find the equation of C, giving your answer in the form

$$|w - (p + q\mathrm{i})| = r$$

where p, q and r are real numbers to be determined.

(6)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Use de Moivre's theorem to show that

$$\cos 5x \equiv \cos x\left(a\sin^4 x + b\sin^2 x + c\right)$$

where a, b and c are integers to be determined.

(4)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\cos 5\theta = \sin 2\theta \sin\theta - \cos\theta$$

giving your answers to 3 decimal places.

(4)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

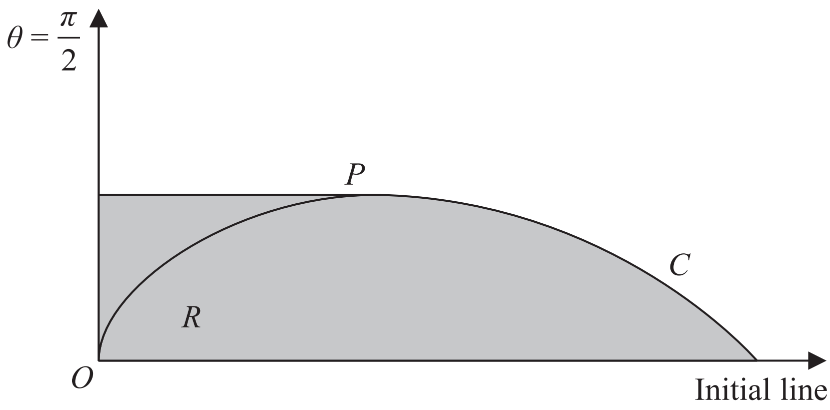

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 - \sin\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C, such that the tangent to C at P is parallel to the initial line.

(a) Use calculus to determine the polar coordinates of P

(4)

The finite region R, shown shaded in Figure 1, is bounded by

- the line with equation $\theta = \frac{\pi}{2}$
- the tangent to C at P
- part of the curve C
- the initial line

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}\left(a\pi + b\sqrt{3} + c\right)$$

where a, b and c are integers to be determined.

(6)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9. (a) Given that $x = t^{\frac{1}{2}}$, determine, in terms of y and t,

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Hence show that the transformation $x = t^{\frac{1}{2}}$, where $t > 0$, transforms the differential equation

$$x\frac{d^2y}{dx^2} - \left(6x^2 + 1\right)\frac{dy}{dx} + 9x^3y = x^5 \qquad \text{(I)}$$

into the differential equation

$$4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = t \qquad \text{(II)}$$

(2)

(c) Solve differential equation (II) to determine a general solution for y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS