Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Further Pure Mathematics F2 (WFM02)
Paper : WFM02/01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2022
Question Paper Log Number P71101A
Publications Code WFM02_01_2201_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- o.e. – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1(a)	$r=\sqrt{(-4)^2+\left(-4\sqrt{3}\right)^2}=\ldots$	M1
	$\tan\theta=\dfrac{-4\sqrt{3}}{-4}\Rightarrow\theta=\tan^{-1}\left(\sqrt{3}\right)\pm\pi$	M1
	$8\left(\cos\left(-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$	A1
		(3)
(b)	$z=r\mathrm{e}^{\mathrm{i}\theta}\Rightarrow\left(r\mathrm{e}^{\mathrm{i}\theta}\right)^3=-4-4\sqrt{3}\Rightarrow r^3\left(\mathrm{e}^{3\mathrm{i}\theta}\right)=8\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}$	
	$\Rightarrow r=\sqrt[3]{8}=2$	M1
	$3\theta=-\dfrac{2\pi}{3}(+2k\pi)\Rightarrow\theta=-\dfrac{2\pi}{9}+\left(\dfrac{2k\pi}{3}\right)$	M1
	So $z=2\mathrm{e}^{-\frac{8\pi}{9}\mathrm{i}},2\mathrm{e}^{-\frac{2\pi}{9}\mathrm{i}},2\mathrm{e}^{\frac{4\pi}{9}\mathrm{i}}$	A1ft A1
		(4)
		(7 marks)

Notes:

(a)

M1: For a correct attempt at the modulus, implied by a correct modulus if no method seen and allow recovery if correct answer follows a minor slip in notation.

M1: For an attempt to find a value of θ in the correct quadrant. Accept $\tan^{-1}\left(\sqrt{3}\right)\pm\pi$ or $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\pm\pi$

May be implied by sight of an of $-\dfrac{2}{3}\pi,\dfrac{4}{3}\pi,-\dfrac{5}{6}\pi,\dfrac{7}{6}\pi$.

A1: cao as in scheme, no other solution.

(b)

M1: Applies De Moivre's Theorem and proceeds to find a value for r ie $\left(\text{their }8\right)^{\frac{1}{3}}$

M1: Proceeds to find at least one value for θ – ie their argument/3.

A1ft: At least two roots correct for their r and θ. (Must come from correct method, watch for correct roots coming from an incorrect angle due to errors.)

A1: All three correct roots and no others. Accept e.g $2\mathrm{e}^{\mathrm{i}-\frac{8\pi}{9}}$ as a slip in notation, so allow marks.

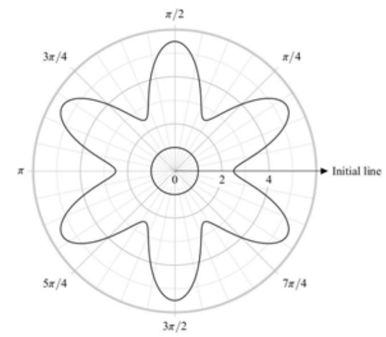
Question	Scheme	Marks
2	$2m^2-5m-3=0 \Rightarrow (2m+1)(m-3)=0 \Rightarrow m=...$	M1
	So C.F. is $(y_{CF}=)\,A\mathrm{e}^{-\frac{1}{2}x}+B\mathrm{e}^{3x}$	A1
	P.I. is $y_{PI}=ax\mathrm{e}^{3x}$	B1
	$\frac{\mathrm{d}y_{PI}}{\mathrm{d}x}=3ax\mathrm{e}^{3x}+a\mathrm{e}^{3x},\ \frac{\mathrm{d}^2y_{PI}}{\mathrm{d}x^2}=9ax\mathrm{e}^{3x}+3a\mathrm{e}^{3x}+3a\mathrm{e}^{3x}$ $\Rightarrow 2(9ax+6a)\mathrm{e}^{3x}-5(3ax+a)\mathrm{e}^{3x}-3ax\mathrm{e}^{3x}=2\mathrm{e}^{3x}\Rightarrow a=...$	M1
	$a=\frac{2}{7}$	A1
	General solution is $y=A\mathrm{e}^{-\frac{1}{2}x}+B\mathrm{e}^{3x}+\frac{2}{7}x\mathrm{e}^{3x}$	B1ft
		(6)
		(6 marks)

Notes:

M1: Forms and solves the auxiliary equation.
A1: Correct complementary function (no need for $y=\ldots$)
B1: Correct form for the particular integral. Accept any PI that includes $ax\mathrm{e}^{3x}$, so e.g. $(ax+b)\mathrm{e}^{3x}$ is fine.
M1: Attempts to differentiate their PI twice and substitutes into the left hand side of the equation. The derivatives must be changed functions. There is no need to reach a value for the unknown(s) but their PI must contain an unknown constant.
A1: Correct value of a (and any other coefficients as zero). Must have had a suitable PI
B1ft: For $y=$ their CF + their PI. Must include the $y=$. The PI must be a function of x that matches their initial choice of PI, with their constants substituted.

Question	Scheme	Marks
3(a)	Meet when $x^2-8x=\frac{4x}{4-x} \Rightarrow \left(x^2-8x\right)\left(4-x\right)=4x \Rightarrow x\left(4x-32-x^2+8x-4\right)=0$	M1
	(so $x=0$ or) $x^2-12x+36=0$	A1
	$\Rightarrow x\left(x-6\right)^2=0 \Rightarrow x=\ldots$	M1
	Meet at (6,−12)	A1
	e.g. touch at (6,−12) as repeated root.	B1
		(5)
Alt	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2-8x\right)=2x-8$ and $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4x}{4-x}\right)=\frac{4(4-x)-4x(-1)}{(4-x)^2}=\frac{16}{(4-x)^2}$	M1A1
	$2x-8=\frac{16}{(4-x)^2} \Rightarrow (x-4)^3=8 \Rightarrow x=\ldots$	M1
	Meet at (6,−12)	A1
	e.g. $6^2-6\times 9=-12$ and $\frac{4\times 6}{4-6}=-12$, so curves meet at tangent at (6,−12)	B1
		(5)
(b)	$x^2-8x=\frac{4x}{4+x} \Rightarrow x\left(x-8\right)\left(4+x\right)-4x=0 \Rightarrow x\left(x^2-4x-36\right)=0 \Rightarrow x=..$	M1
	$x=(0), 2\pm 2\sqrt{10} \Rightarrow$ critical value is (0 and) $2-2\sqrt{10}$	A1
	Other C.V.'s are 0, ± 4	B1
	E.g. extremes are $x<2-2\sqrt{10}$ and $x>6$ or any two suitable ranges.	M1
	Solution is $x<2-2\sqrt{10}, -4<x<0, 4<x<6, x>6$	A1A1
		(6)
		(11 marks)

Notes:
(a) **M1:** Attempts to find intersection by setting equations equal and cross multiplies and factorises the x out or cancels. **A1:** Correct quadratic reached. May be implied by solutions of 0,6 seen from the cubic (by calculator) **M1:** Solves the quadratic to find roots. **A1:** Obtains the correct point where the curves meet. **B1:** Correct reason given for why the curves touch. Accepted "repeated root" as reason. As a minimum, accept "$(x-6)^2=0$ therefore touches". Alternatively, accept discriminant = 0 shown with conclusion, or may find gradient at both points and show equal, with conclusion. Alt: **M1:** Attempts derivatives of both curves **A1:** Both derivatives correct. **M1:** Sets derivatives equal and solves to find x value where gradients agree. **A1:** Obtains the correct point where the curves meet. **B1:** Correct value checked in both curves with conclusion that they meet at a tangent or equivalent working as per main scheme. **(b)** **M1:** Attempts to find the intersection of the other branch of $\frac{4x}{4-\lvert x\rvert}$ with x^2-8x. Allow for any attempt at solving $\frac{4x}{4+x}=x^2-8x$ that reaches a value for x **A1:** Correct value of $2-2\sqrt{10}$ identified. (No need to see the second root rejected for this mark.) **B1:** Both 0 and ± 4 identified as critical values for the ranges needed at some stage in working. **M1:** Forms at least two suitable ranges from their critical values (allow if e.g. $\leqslant$ is used instead of $<$). Likely to be the extreme ranges, so look for $x <$ their $2-2\sqrt{10}$ and $x >$ their 6. However, allow if this latter is included as part of the range $x > 4$ for this mark. **A1:** At least two correct ranges. **A1:** Fully correct answer as in scheme.

Question	Scheme	Marks
4(a)	 Completes to a closed loop with "petals" containing circle of radius 1 (whether the circle is drawn or not)	M1
	Fully correct – 6 petals in roughly the right places, but allow if curvature is not quite smooth.	A1
	Circle centre O radius 1.	B1
		(3)
(b)	$\left(\frac{1}{2}\right)\int r^2\ \mathrm{d}\theta = \left(\frac{1}{2}\right)\int\left(\underline{16-12\cos 6\theta+\frac{9}{4}\cos^2 6\theta}\right)\mathrm{d}\theta$	M1
	$=\frac{1}{2}\int_0^{2\pi}\left(16-12\cos 6\theta+\underline{\frac{9}{8}(1+\cos 12\theta)}\right)\mathrm{d}\theta$	M1
	$=\frac{1}{2}\left[16\theta-2\sin 6\theta+\frac{9}{8}\left(\theta+\frac{1}{12}\sin 12\theta\right)\right]_0^{2\pi}$	M1 A1
	$A_{outer}=\frac{1}{2}\int_0^{2\pi} r^2\ \mathrm{d}\theta=\frac{1}{2}\int_0^{2\pi}\left(16-12\cos 6\theta+\frac{9}{4}\cos^2 6\theta\right)\mathrm{d}\theta$ $=\frac{1}{2}\left(32\pi-0+\frac{9}{8}(2\pi+0)-(0)\right)$	dM1
	So Area required is $\frac{1}{2}\left(32\pi+\frac{9\pi}{4}\right)-\pi(1^2)=\ldots$	B1
	$=\frac{129}{8}\pi$	A1
		(7)
		(10 marks)

Notes:

(a)

M1: Allow for any closed loop that oscillates, though may not have the correct number of "petals" but require at least 4 . Need not have correct places of maximum radius.

A1: Fully correct sketch, 6 "petals" in the right places, with maximum radius between the 5 and 6 radius lines, minimum between the 2 and 3 radius lines.

B1: For a circle of radius 1 and centre O drawn.

(b)

M1: Attempts to square r as part of an integral for the outer curve, achieving a 3 term quadratic in $\cos 6\theta$

M1: Applies the double angle formula to the $\cos^2$ term from their expansion (not dependent on the first M, but must have a $\cos^2$ term). Accept $\cos^2 6\theta \rightarrow \frac{1}{2}(\pm 1 \pm \cos 12\theta)$

M1: Attempts to integrate, achieving the form $\alpha\theta+\beta\sin 6\theta+\gamma\sin 12\theta$ where $\alpha,\beta,\gamma\neq 0$

A1: Correct integration – limits and the $\frac{1}{2}$ not needed. Look for $16\theta - 2\sin 6\theta + \frac{9}{8}\left(\theta + \frac{1}{12}\sin 12\theta\right)$ oe.

dM1: Depends on at least two of the previous M's being scored. For a correct overall strategy for the area contained in the outer loop, with an attempt at the r^2 (should be 3 term expansion). Correct appropriate limits and the $\frac{1}{2}$ should be present or implied by working, but note variations on the scheme are possible, e.g. $2\times\frac{1}{2}\int_0^{\pi} r^2 \,\mathrm{d}\theta$, in which the $2\times\frac{1}{2}$ may be implied rather than seen.

B1: Subtracts correct area of π for inner circle

A1: cso. Check carefully the integration was correct as the sin terms disappear with the limits.

Question	Scheme	Marks
5(a)	$\frac{dy}{dx}=\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}\times\frac{1}{x}$	M1 A1
	$\frac{d^2y}{dx^2}=\frac{1}{2}\frac{0-\left(\sqrt{4+\ln x}+x\times\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}\times\frac{1}{x}\right)}{x^2(4+\ln x)}$ or $\frac{d^2y}{dx^2}=-\frac{1}{4x}(4+\ln x)^{-\frac{3}{2}}\times\frac{1}{x}-\frac{1}{x^2}\times\frac{1}{2}(4+\ln x)^{-\frac{1}{2}}$ oe	M1
	$=\frac{\ldots}{4x^2(4+\ln x)^{\frac{3}{2}}}=-\frac{9+2\ln x}{4x^2(4+\ln x)^{\frac{3}{2}}}$ *	M1 A1*
		(5)
Alt(a)	$y^2=4+\ln x\Rightarrow 2y\frac{dy}{dx}=\frac{1}{x}$	M1 A1
	$\Rightarrow 2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=-\frac{1}{x^2}$	M1
	$\Rightarrow\frac{d^2y}{dx^2}=-\frac{1}{2yx^2}-\frac{2}{8x^2y^3}=\frac{-2y^2-1}{4x^2y^3}$	M1
	$=-\frac{9+2\ln x}{4x^2(4+\ln x)^{\frac{3}{2}}}$ *	A1*
		(5)
(b)	$y_{x=1}=2,\ \left.\frac{dy}{dx}\right\|_{x=1}=\frac{1}{4},\ \left.\frac{d^2y}{dx^2}\right\|_{x=1}=-\frac{9}{32}$	M1
	So $y=2+\frac{1}{4}(x-1)-\frac{1}{2!}\times\frac{9}{32}(x-1)^2+\ldots$	M1
	$=2+\frac{1}{4}(x-1)-\frac{9}{64}(x-1)^2+\ldots$	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Attempts the derivative of y using the chain rule, look for $\frac{K}{x}(4+\ln x)^{-\frac{1}{2}}$ oe

A1: Correct derivative.

M1: Attempts the second derivative of y using the product or quotient rule and chain rule. Look for the correct form for their $\frac{dy}{dx}$ for the answer up to slips in coefficients.

M1: Attempts to simplify to get correct denominator. Must be correct work for their second derivative, but may have been errors in differentiating.

A1*: For a correct unsimplified second derivative, with no errors before reaching the given answer.

Note it is a given answer so needs a suitable intermediate line with at least the formation of the correct common denominator between two fractions before reaching the answer.

Alt:

M1: Squares and uses implicit differentiation to achieve $\alpha y\dfrac{dy}{dx}=\dfrac{\beta}{x}$

A1: Correct derivative.

M1: Differentiates again using implicit differentiation and product rule. Look for $\gamma y\dfrac{d^2y}{dx^2}+\delta\left(\dfrac{dy}{dx}\right)^2=\dfrac{\nu}{x^2}$

M1: Makes $\dfrac{d^2y}{dx^2}$ the subject and forms single fraction with denominator kx^2y^3

A1*: Obtains the correct second derivative, with no errors seen in working.

(b)

M1: Evaluates y, $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $x = 1$, if substitution is not seen, accept stated values for all three following attempts at the first and second derivatives as an attempt to find these.

M1: Applies Taylor's theorem with their values.

A1: Correct expression (don't be concerned if the $y =$ is missing.)

5(b) Alt	$y=\sqrt{4+\ln\left(1+(x-1)\right)}=\sqrt{4+\left((x-1)-\dfrac{(x-1)^2}{2}+...\right)}$	**M1**
	$=4^{\frac{1}{2}}+\dfrac{1}{2}\times 4^{-\frac{1}{2}}\times\left((x-1)-\dfrac{(x-1)^2}{2}\right)+\dfrac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times 4^{-\frac{3}{2}}\times\left((x-1)-...\right)^2+...$	**M1**
	$=2+\dfrac{1}{4}(x-1)-\dfrac{1}{8}(x-1)^2-\dfrac{1}{64}(x-1)^2+...=2+\dfrac{1}{4}(x-1)-\dfrac{9}{64}(x-1)^2+...$	**A1**
		(3)

Notes:

M1: Writes the x as $1+(x-1)$ and attempts to expand using the Maclaurin series for $\ln(1+x)$ with correct expansion of $\ln(1+(x-1))$.

M1: Attempts a binomial expansion using their ln expansion. Alternatively, may gain this before the first M if they expand using ln's, e.g. $4^{\frac{1}{2}}+\dfrac{1}{2}4^{-\frac{1}{2}}\ln x+\dfrac{\frac{1}{2}\times\frac{-1}{2}}{2!}(\ln x)^2$

A1: Fully correct expression (don't be concerned if the $y =$ is missing.)

Question	Scheme	Marks
6(a)	Let $x = \arctan A$ and $y = \arctan B$ then $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$ Or $\tan(\arctan A - \arctan B) = \dfrac{\tan\arctan A - \tan\arctan B}{1+\tan\arctan A\tan\arctan B}$	B1
	$\tan(x-y) = \dfrac{A-B}{1+AB} \Rightarrow x-y = \arctan\left(\dfrac{A-B}{1+AB}\right)$	M1
	So $\arctan A - \arctan B = x - y = \arctan\left(\dfrac{A-B}{1+AB}\right)$ *	A1*
		(3)
(b)	$A = r+2, B = r \Rightarrow \left(\dfrac{A-B}{1+AB} =\right)\dfrac{r+2-r}{1+(r+2)r} = \dfrac{2}{\ldots}$	M1
	$= \dfrac{2}{r^2+2r+1} = \dfrac{2}{(1+r)^2}$ *	A1*
		(2)
(c)	$\sum_{r=1}^{n} \arctan\left(\dfrac{2}{(1+r)^2}\right) = \sum_{r=1}^{n}\left(\arctan(r+2) - \arctan(r)\right) = \ldots$	M1
	$= \left(\cancel{\arctan 3} - \arctan 1\right) + \left(\cancel{\arctan 4} - \arctan 2\right) + \left(\cancel{\arctan 5} - \cancel{\arctan 3}\right) + \ldots$ $+\left(\arctan(n+1) - \cancel{\arctan(n-1)}\right) + \left(\arctan(n+2) - \cancel{\arctan n}\right)$	A1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \arctan 1$	M1
	$= \arctan(n+2) + \arctan(n+1) - \arctan 2 - \dfrac{\pi}{4}$	A1
		(4)
(d)	As $n \to \infty$, $\arctan n \to \dfrac{\pi}{2}$	M1
	So $\sum_{r=1}^{\infty} \arctan\left(\dfrac{2}{(1+r)^2}\right) = \dfrac{\pi}{2} + \dfrac{\pi}{2} - \arctan 2 - \dfrac{\pi}{4} = \dfrac{3\pi}{4} - \arctan 2$	A1
		(2)
		(11 marks)

Notes:

(a)

B1: For any correct statement or use of the compound angle formula with **consistent variables** of x and y or arctan A and arctan B. Can be either way round (may be working in reverse).

M1: Attempts to apply tan or arctan on an appropriate identity with either x and y or arctan A and arctan B.

Should have $\dfrac{\tan x \pm \tan y}{1 \pm \tan x \tan y}$ (oe with arctans or A's and B's) as part of the identity, and allow if they change between x,y and arctan's during the step.

A1*: Must have scored the B and M marks. Replaces tan x and tan y by A and B respectively if appropriate with fully correct work leading to the given result and conclusion made and no erroneous statements.

Note: for working in reverse e.g.

Let $x = \arctan A$ and $y = \arctan B$ then

$\arctan A - \arctan B = \arctan\left(\dfrac{A-B}{1+AB}\right) \Leftrightarrow x - y = \arctan\left(\dfrac{A-B}{1+AB}\right) \Leftrightarrow \tan(x-y) = \dfrac{A-B}{1+AB}$ Scores M1

$\Leftrightarrow \tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$ Scores B1 – but enter as the first mark.

Which is the correct identity for $\tan(x-y)$ hence the result is true. Score A1

The conclusion here must include reference to the identity being true, e.g. with a tick, or statement, before deducing the final result.

(b)

M1: Substitutes in $A = r + 2$ and $B = r$ and simplifies the numerator to 2 (may be implied)

A1*: Expands the denominator (must be seen) and then factorises to the given result, no errors seen.

(c)

M1: Applies the result of (a) to the series – allow if they have a different A and B due to error.

A1: At least first three and final two brackets of terms correctly written out – must be clear enough to show cancelling.

M1: Extracts the non-cancelling terms.

A1: Correct result with no errors seen – must see the arctan 1 before reaching $\dfrac{\pi}{4}$.

Note: Insufficient terms to gain the first A is not an error, so M1A0M1A1 is possible if e.g. only the first two terms are shown. Condone missing brackets on arctan $n + 1$ etc.

(d)

M1: Identifies the value arctan tends towards as n increase. Need not see limits, as long as the value is identified.

A1: Correct answer.

Question	Scheme	Marks
7(a)	$z=(0+)\mathrm{i}y \Rightarrow w=\dfrac{(1+\mathrm{i})\mathrm{i}y+2(1-\mathrm{i})}{\mathrm{i}y-\mathrm{i}}=\dfrac{-y+2+\mathrm{i}(y-2)}{\mathrm{i}(y-1)}=\dfrac{y-2+\mathrm{i}(y-2)}{y-1}$	M1
	$\Rightarrow u=v$ or $\operatorname{Im} w=\operatorname{Re} w$	A1
		(2)
(b)	$w=\dfrac{(1+\mathrm{i})z+2(1-\mathrm{i})}{z-\mathrm{i}} \Rightarrow z=\dfrac{2(1-\mathrm{i})+\mathrm{i}w}{w-1-\mathrm{i}}=\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}$	M1
	$\dfrac{2-v+\mathrm{i}(u-2)}{u-1+\mathrm{i}(v-1)}\times\dfrac{u-1-\mathrm{i}(v-1)}{u-1-\mathrm{i}(v-1)}$ $=\dfrac{(2-v)(u-1)+(u-2)(v-1)+\mathrm{i}\left((u-1)(u-2)-(2-v)(v-1)\right)}{\ldots}$ $\operatorname{Im} z=0 \Rightarrow (u-1)(u-2)-(2-v)(v-1)=0$	M1
	$\Rightarrow (u-1)(u-2)-(2-v)(v-1)=0 \Rightarrow u^2-3u+2+v^2-3v+2=0$	A1
	$\Rightarrow \left(u-\dfrac{3}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2=\dfrac{1}{2}$	M1
	Centre is $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ and radius is $\dfrac{\sqrt{2}}{2}$	A1A1
		(6)
		(8 marks)

Notes:

(a)

M1: Correct method to find the equation of the image line – e.g. substitutes in $z=\mathrm{i}y$ and rearranges to Cartesian form. May use $x+\mathrm{i}y$ and later set $x=0$. Alternatively, may start as in (b) and then set $(2-v)(u-1)+(u-2)(v-1)=0 \Rightarrow 2u-v-uv-2+uv+2-2v-u=0$ etc.

Another alternative is to find the image points of two points on the imaginary axis and to find the line from these.

A1: For $u=v$ oe equation. Accept Im w = Re w, or $x=y$ if they have set $w=x+\mathrm{i}y$.

(b)

Note: Accept work done in part (a) that is relevant to part (b) for credit if appropriate.

M1: Makes z the subject, substitutes $w=u+\mathrm{i}v$ into the equation.

M1: Multiplies the numerator by the complex conjugate of denominator **and** extracts the imaginary part and sets it equal to zero to form an equation in u and v. Do not be concerned about the denominator.

A1: Correct equation in u and v for the circle.

M1: Completes the square on their equation to extract centre and radius. Not dependent, so allow as long as a suitable equation in u and v has been reached.

A1: Correct centre or correct radius. Accept either $\dfrac{3}{2}+\dfrac{3}{2}\mathrm{i}$ or $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$ for the centre.

A1: Correct centre and correct radius. As above. Accept equivalent forms (need not be simplified)

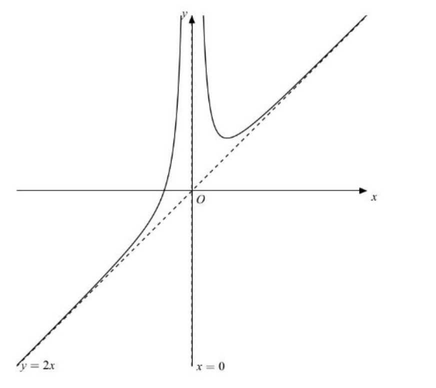
Allow the final two A marks if all that is wrong is an error in the denominator. (M1M0A0M1A1A1 is possible.)

7(b) **Alt1**	Real axis is $z = x(+0\mathrm{i})$, so $u+\mathrm{i}v = \frac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} = \frac{(1+\mathrm{i})x+2(1-\mathrm{i})}{x-\mathrm{i}} \times \frac{x+\mathrm{i}}{x+\mathrm{i}} =$ $\frac{(1+\mathrm{i})x^2+2x(1-\mathrm{i})+(\mathrm{i}-1)x+2(\mathrm{i}+1)}{x^2+1} = \frac{x^2+x+2+\mathrm{i}\left(x^2-x+2\right)}{x^2+1}$	**M1**
	$u = \frac{x^2+x+2}{x^2+1} = 1+\frac{x+1}{x^2+1}; v = \frac{x^2-x+2}{x^2+1} = 1-\frac{x-1}{x^2+1} \Rightarrow u+v = 2+\frac{2}{x^2+1}$ $\Rightarrow (u-1)^2+(v-1)^2 = \frac{(x+1)^2+(x-1)^2}{\left(x^2+1\right)^2} = \frac{2x^2+2}{\left(x^2+1\right)^2} = \frac{2}{x^2+1} = u+v-2$	**M1** **A1**
	$\Rightarrow \left(u-\frac{3}{2}\right)^2+\left(v-\frac{3}{2}\right)^2 = \frac{1}{2}$	**M1**
	Centre is $\frac{3}{2}+\frac{3}{2}\mathrm{i}$ and radius is $\frac{\sqrt{2}}{2}$	**A1A1**
		(6)

Notes

M1: Sets $z = x$ in the equation (or uses $x + \mathrm{i}y$ and later sets $y = 0$) and multiplies by complex conjugate.
M1: Eliminates x from the equations (one suitable method is shown, others are possible).
A1: Correct equation in u and v for the circle.
M1: Completes the square on their equation to extract centre and radius
A1: Correct centre or correct radius. Accept either $\frac{3}{2}+\frac{3}{2}\mathrm{i}$ or $\left(\frac{3}{2},\frac{3}{2}\right)$ for the centre.
A1: Correct centre and correct radius. As above.

7(b) **Alt 2**	Unlikely to be seen As i and $-\mathrm{i}$ are inverse points of the line, so their images are inverse points of the circle. $\mathrm{i} \to \infty,\ -\mathrm{i} \to \frac{-\mathrm{i}+1+2-2\mathrm{i}}{-2\mathrm{i}} = \frac{3}{2}+\frac{3}{2}\mathrm{i}$ Hence (as ∞ is the other point) the centre is $\frac{3}{2}+\frac{3}{2}\mathrm{i}$ $0 \to \frac{2-2\mathrm{i}}{-\mathrm{i}} = 2+2\mathrm{i}$ So radius is $\left\lvert\frac{3}{2}+\frac{3}{2}\mathrm{i}-2-2\mathrm{i}\right\rvert = \ldots$ $= \frac{\sqrt{2}}{2}$	**M1** **M1** **A1** **M1** **A1** **A1**
(b) Alt 3	M1: Attempt to find images of three different points on the real axis. M1: Correct method to find centre from three points – e.g. intersection of two perpendicular bisectors. A1: Correct equation for the centre. M1: Uses centre and one point to find radius. A1: Correct centre A1: Correct radius	

Question	Scheme	Marks
8(a)	$\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}x}-2$	**B1**
	$\frac{\mathrm{d}y}{\mathrm{d}x}+2yx(y-4x)=2-8x^3 \to \frac{\mathrm{d}v}{\mathrm{d}x}+2+2\left(v+2x\right)x\left(v+2x-4x\right)=2-8x^3$	
	$\to \frac{\mathrm{d}v}{\mathrm{d}x}+2+2x\left(v^2-4x^2\right)=2-8x^3$	**M1**
	$\to \frac{\mathrm{d}v}{\mathrm{d}x}=-2xv^2\,*$	**A1***
		(4)
(b)	$\frac{1}{v^2}\frac{\mathrm{d}v}{\mathrm{d}x}=-2x \Rightarrow \int v^{-2}\,\mathrm{d}v=-2\int x\,\mathrm{d}x$	**B1**
	$\Rightarrow \frac{v^{-1}}{-1}=-2\frac{x^2}{2}(+c)$	**M1**
	$\Rightarrow \frac{1}{v}=x^2+c$	**A1**
	$\Rightarrow v=\frac{1}{x^2+c}$	**A1**
		(4)
(c)	$y=2x+\frac{1}{x^2+c}$	**B1ft**
		(1)
(d)	$-1=2\times -1+\frac{1}{1+c}\Rightarrow c=...$	**M1**
	$y=2x+\frac{1}{x^2}$	**A1**
	 Attempts the sketch for their equation, with at least one of - One branch correct - Vertical asymptote for their equation - Long term behaviour tends to infinity - Minimum in quadrant 1	**M1**
	Fully correct shape, two branches tending to infinity as x tends to infinity both directions, with minimum in first quadrant No need for oblique asymptote marked.	**A1**
	y-axis a vertical asymptote labelled	**B1ft**
		(5)
		(14 marks)

Notes:

(a)

B1: Correct differentiation of the given transformation. Allow any correct connecting derivative, e.g. $\frac{\mathrm{d}y}{\mathrm{d}v}=1+2\frac{\mathrm{d}x}{\mathrm{d}v}$ or $\frac{\mathrm{d}v}{\mathrm{d}y}=1-2\frac{\mathrm{d}x}{\mathrm{d}y}$

M1: For a complete substitution into the equation (I).

M1: Applies difference of squares, or completely expands brackets of the left hand side. Alternatively, may rearrange and factorise to give $8x^2y-2xy^2-8x^3=-2x\left(y^2-4xy+4x^2\right)=-2x(y-2x)^2$ before substituting.

A1*: Reaches the given answer with no errors seen.

(b)

B1: Correct separation of the variables.

M1: Attempts the integration, usual rule, power increased by 1 on at least one term. No need for + c for the method.

A1: Correct integration including the + c

A1: Correct expression for v.

(c)

B1: Follow through their answer to (b), so $y=2x+$ their v from (b)

(d)

M1: Uses the point $(-1,-1)$ to find a value for the constant in their equation. Must have had a constant of integration in their equation to score this mark.

A1: Correct equation for y following a correct general solution. Withhold this mark for $y=2x+\frac{1}{x^2}+c$ leading to the correct equation.

Note: the following three marks may be scored from a correct equation that arose from having no constant in (b) or from $y=2x+\frac{1}{x^2}+c$ (which gives the same equation).

M1: Attempts a sketch for their curve. See scheme. Look for at least one of the key features for their equation shown.

A1: Correct shape, two branches tending to infinity as x tends to infinity both directions with a minimum in first quadrant. Not a follow through mark, so must be the correct curve.

B1ft: Correct vertical asymptote at $x = 0$. Need not be labelled if it is clearly the y-axis. Follow through their equation as long as there is at least one vertical asymptote (ie for a negative c they need a pair of asymptotes symmetric about the y-axis).