Pearson Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\lfloor\;$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

WFM02 Further Pure Mathematics F2 Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z^5 - 32\text{i} = 0 \Rightarrow r^5 = 32 \Rightarrow r = 2$	Correct value for r. May be shown explicitly or used correctly.	B1
	$5\theta = \frac{\pi}{2} + 2n\pi \Rightarrow \theta = \frac{\pi}{10} + \frac{2n\pi}{5}$	Applies a correct strategy for establishing at least 2 values of θ. This can be awarded if if the initial angle $\left(\frac{\pi}{2} \text{ or } \frac{\pi}{10}\right)$ is incorrect but otherwise their strategy is correct.	M1
	$z = 2\text{e}^{\text{i}\frac{\pi}{10}}, 2\text{e}^{\text{i}\frac{\pi}{2}}, 2\text{e}^{\text{i}\frac{9\pi}{10}}, 2\text{e}^{\text{i}\frac{13\pi}{10}}, 2\text{e}^{\text{i}\frac{17\pi}{10}}$ or	At least 2 correct, follow through their r	A1ft
	$z = 2\text{e}^{\left(\frac{\pi}{10}+\frac{2n\pi}{5}\right)\text{i}}, \quad n = 0,1,2,3,4$	All correct. Must have $r = 2$	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x}$		
Way 1	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x} \Rightarrow \dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0$	Collects to one side	M1
	$\dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0 \Rightarrow \dfrac{x^2-(2-x)(x+3)}{x(2-x)} \leqslant 0$ M1: Attempt common denominator A1: Correct fraction		M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x^2-(2-x)(x+3)=0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = \ldots$	Solves the 3TQ in the numerator	M1
	$x = \dfrac{3}{2}, -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$		A1A1
			(8)
			Total 8
	Alternative 1: $\times x^2(2-x)^2$		
	$x^3(2-x) \leqslant x(x+3)(2-x)^2$	Multiplies by a positive expression	M1
	$x^3(2-x) - x(x+3)(2-x)^2 \leqslant 0$	Collects to one side Correct inequality	M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x(2-x)\left[x^2 - (x+3)(2-x)\right] = 0$ $x^2 - (x+3)(2-x) = 0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = \ldots$	Attempts to factorise by taking out a factor of $x(2-x)$ and solves resulting 3TQ. May have quartic and apply the factor theorem.	M1
	$x = \dfrac{3}{2}, -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but not $\cap$		A1A1

Question Number	Scheme	Notes	Marks
3	$w=\dfrac{(2+\mathrm{i})z+4}{z-\mathrm{i}} \Rightarrow wz-wi=(2+\mathrm{i})z+4$ $\Rightarrow z=...$	Attempts to make z the subject	M1
	$z=\dfrac{w\mathrm{i}+4}{w-2-\mathrm{i}}$	Correct equation in any form	A1
	$z=\dfrac{(u+\mathrm{i}v)\mathrm{i}+4}{u+\mathrm{i}v-2-\mathrm{i}}$ $z=\dfrac{((u+\mathrm{i}v)\mathrm{i}+4)(u-2-(v-1)\mathrm{i})}{(u-2+(v-1)\mathrm{i})(u-2-(v-1)\mathrm{i})}$	Introduces $w = u + \mathrm{i}v$ and multiplies numerator and denominator by the conjugate of the denominator	M1
	$u(v-1)+(4-v)(u-2)=0$	Sets real part = 0 (with or without denominator) Depends on both M marks above	dM1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 2	$w=\dfrac{(2+\mathrm{i})z+4}{z-\mathrm{i}}, z=y\mathrm{i} \Rightarrow w=\dfrac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}$ $w=\dfrac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}\times\dfrac{\mathrm{i}}{\mathrm{i}}$	Solves simultaneously and multiplies numerator and denominator by i	M1
	$u=\dfrac{2y}{y-1}, v=\dfrac{y-4}{y-1}$	Correct real and imaginary parts	A1
	$u=\dfrac{2y}{y-1} \Rightarrow y=\dfrac{u}{u-2}$	Attempts y in terms of u or v	M1
	$y=\dfrac{u}{u-2} \Rightarrow v=\dfrac{\dfrac{u}{u-2}-4}{\dfrac{u}{u-2}-1}$	Obtains an equation connecting u and v	M1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 3	Apply the transformation to any point on the imaginary axis	Eg $(0,0)\to(0,4)\ \ (0,1)\to(4,-2)$	M1
	Apply the transformation to a second point on the imaginary axis	This is the second M mark on e-PEN	M1
	Both transformations correct	This is the first A mark on e-PEN	A1
	Complete method to obtain an equation for the line thro' their 2 points in the w-plane		M1
	Correct equation in any form		A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\dfrac{\mathrm{d}y}{\mathrm{d}x}-xy=\mathrm{e}^{3x} \qquad x>-1$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}-\dfrac{xy}{(x+1)}=\dfrac{\mathrm{e}^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I=\mathrm{e}^{\int\frac{-x}{x+1}\mathrm{d}x}=\mathrm{e}^{\int\left(-1+\frac{1}{x+1}\right)\mathrm{d}x}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$=\mathrm{e}^{-x+\ln(x+1)}$	For $-x+\ln(x+1)$	A1
	$=(x+1)\mathrm{e}^{-x}$	Correct integrating factor	A1
	$y(x+1)\mathrm{e}^{-x}=\int\dfrac{\mathrm{e}^{3x}}{x+1}\times(x+1)\mathrm{e}^{-x}\,\mathrm{d}x$	Uses their integrating factor to reach the form $yI=\int QI\,\mathrm{d}x$	M1
	$y(x+1)\mathrm{e}^{-x}=\dfrac{1}{2}\mathrm{e}^{2x}+c$	Correct equation (with or without $+\,c$)	A1
	$y=\dfrac{\mathrm{e}^{3x}}{2(x+1)}+\dfrac{c\mathrm{e}^{x}}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y=$...	A1
			(7)
(b)	$x=0, y=5\Rightarrow 5=\dfrac{1}{2}+c\Rightarrow c=\dfrac{9}{2}$	Substitutes $x=0$ and $y=5$ and attempts to find a value for c.	M1
	$y=\dfrac{\mathrm{e}^{3x}}{2(x+1)}+\dfrac{9\mathrm{e}^{x}}{2(x+1)}$	Cao (oe) Must have $y=$...	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y=\tan^2 x \Rightarrow \dfrac{dy}{dx}=2\tan x\sec^2 x$	Correct first derivative any correct form	B1
	$\dfrac{dy}{dx}=2\tan x\sec^2 x \Rightarrow \dfrac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\dfrac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x \Rightarrow \dfrac{d^3y}{dx^3}=8\sec^4 x\tan x+8\sec^2 x\tan^3 x+8\sec^4 x\tan x$ Or $\dfrac{d^2y}{dx^2}=6\sec^4 x-4\sec^2 x \Rightarrow \dfrac{d^3y}{dx^3}=24\sec^4 x\tan x-8\sec^2 x\tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$=8\sec^4 x\tan x+8\sec^2 x\tan x\left(\sec^2 x-1\right)+8\sec^4 x\tan x$ $=24\sec^4 x\tan x-8\sec^2 x\tan x=8\sec^2 x\tan x\left(3\sec^2 x-1\right)$ Fully correct expression		A1
			(5)
(b)	$(y)_{\frac{\pi}{3}}=3,\ (y')_{\frac{\pi}{3}}=8\sqrt{3},\ (y'')_{\frac{\pi}{3}}=80,\ (y''')_{\frac{\pi}{3}}=352\sqrt{3}$	Attempts the values up to the third derivative when $x=\dfrac{\pi}{3}$	M1
	$y=3+8\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+\dfrac{80}{2!}\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{352\sqrt{3}}{3!}\left(x-\dfrac{\pi}{3}\right)^3+\ldots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y=3+8\sqrt{3}\left(x-\dfrac{\pi}{3}\right)+40\left(x-\dfrac{\pi}{3}\right)^2+\dfrac{176\sqrt{3}}{3}\left(x-\dfrac{\pi}{3}\right)^3+\ldots$ Correct expansion Must start y = ... or $\tan^2 x$ = ... f(x) only accepted if f(x) has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$\lvert z+1-13\mathrm{i}\rvert = 3\lvert z-7-5\mathrm{i}\rvert \Rightarrow (x+1)^2+(y-13)^2 = 9\left\{(x-7)^2+(y-5)^2\right\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2+y^2-16x-8y+62=0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2 = 64+16-62=...$	Correct method for r or r^2	M1
	$r=\sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
			(5)
(b)	$\arg(z-8-6\mathrm{i}) = -\dfrac{3\pi}{4} \Rightarrow y-6=x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2+y^2-16x-8y+62=0$ $\Rightarrow x^2+(x-2)^2-16x-8(x-2)+62=0 \Rightarrow x=...$ or $\Rightarrow (y+2)^2+y^2-16x-8(y+2)+62=0 \Rightarrow y=...$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x=7-2\sqrt{2}$ or $y=5-2\sqrt{2}$	One correct "coordinate"	A1
	R is $7-2\sqrt{2}+(5-2\sqrt{2})\mathrm{i}$ **or** $x=7-2\sqrt{2}$ and $y=5-2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$x = t^2 \Rightarrow \frac{dx}{dy} = 2t\frac{dt}{dy}$ oe	Correct application of the chain rule	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2t}\frac{dy}{dt}\left(\text{or e.g. } \frac{1}{2\sqrt{x}}\frac{dy}{dt}\right)$	Any correct expression for $\frac{dy}{dx}$ or equivalent equation	A1
	$2\sqrt{x}\frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x^{-\frac{1}{2}}\frac{dy}{dx} + 2\sqrt{x}\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}\frac{dt}{dx}$ (NB $\frac{d^2y}{dt^2} = 2\frac{dy}{dx} + 4x\frac{d^2y}{dx^2}$)	Fully correct strategy to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ Chain rule used on at least one term. Depends on the first M mark	dM1
	$4x\frac{d^2y}{dx^2} + 2\left(1+2\sqrt{x}\right)\frac{dy}{dx} - 15y = 15x \Rightarrow 4x\frac{d^2y}{dx^2} + 4\sqrt{x}\frac{dy}{dx} + 2\frac{dy}{dx} - 15y = 15x$ $\Rightarrow \frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 15y = 15t^2$ * ddM1: Substitutes into the given differential equation. The full substitution must be seen. Depends on both M marks. A1*: Cso		ddM1 A1*
			(5)
(b)	$m^2 + 2m - 15 = 0 \Rightarrow m = 3, -5$	Attempts to solve $m^2 + 2m - 15 = 0$	M1
	$y = Ae^{-5t} + Be^{3t}$	Correct CF	A1
	$y = at^2 + bt + c \Rightarrow \frac{dy}{dt} = 2at + b \Rightarrow \frac{d^2y}{dt^2} = 2a$ $\Rightarrow 2a + 4at + 2b - 15at^2 - 15bt - 15c = 15t^2$ Starts with the correct PI form and differentiates twice and substitutes		M1
	$-15a = 15 \Rightarrow a = \ldots$ $4a - 15b = 0 \Rightarrow b = \ldots$ $2a + 2b - 15c = 0 \Rightarrow c = \ldots$	Complete method to find a, b and c by comparing coefficients. Values for all 3 needed. Depends on the second M mark.	dM1
	$y = Ae^{-5t} + Be^{3t} - t^2 - \frac{4}{15}t - \frac{38}{225}$	Correct GS. Must start $y = \ldots$	A1
			(5)
(c)	$y = Ae^{-5\sqrt{x}} + Be^{3\sqrt{x}} - x - \frac{4}{15}\sqrt{x} - \frac{38}{225}$	Correct equation (follow through their answer to (b)) Must start $y = \ldots$	B1ft
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$x = r\cos\theta = (1+\sin\theta)\cos\theta$ $\Rightarrow \dfrac{dx}{d\theta} = \cos^2\theta - (1+\sin\theta)\sin\theta$ or $\Rightarrow \dfrac{dx}{d\theta} = -\sin\theta + \cos 2\theta$	Differentiates $r\cos\theta$ using product rule or double angle formula	M1
		Correct derivative in any form	A1
	$\cos^2\theta - (1+\sin\theta)\sin\theta = 0 \Rightarrow 1-\sin^2\theta - \sin\theta - \sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ or $-\sin\theta + \cos 2\theta = 0 \Rightarrow -\sin\theta + 1 - 2\sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ Sets $\dfrac{dx}{d\theta} = 0$ and proceeds to a 3TQ in $\sin\theta$ Depends on the first M mark		dM1
	$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ $\Rightarrow \sin\theta = \dfrac{1}{2}, (-1) \Rightarrow \theta = ...$	Solves for θ. Depends o both M marks above.	ddM1
	$\left(\dfrac{3}{2}, \dfrac{\pi}{6}\right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1+\sin\theta)^2\, d\theta = \int (1 + 2\sin\theta + \sin^2\theta)\, d\theta$ $= \int \left(1 + 2\sin\theta + \dfrac{1}{2} - \dfrac{1}{2}\cos 2\theta\right) d\theta$	Attempts $\left(\dfrac{1}{2}\right)\int r^2\, d\theta$ and applies $\sin^2\theta = \pm\dfrac{1}{2} \pm \dfrac{1}{2}\cos 2\theta$ Ignore any limits shown	M1
	$\int (1+\sin\theta)^2\, d\theta = \dfrac{3}{2}\theta - 2\cos\theta - \dfrac{1}{4}\sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\dfrac{1}{2}\left[\dfrac{3}{2}\theta - 2\cos\theta - \dfrac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \dfrac{1}{2}\left[\dfrac{3\pi}{4} - \left(\dfrac{\pi}{4} - \sqrt{3} - \dfrac{\sqrt{3}}{8}\right)\right]\left(= \dfrac{\pi}{4} + \dfrac{9\sqrt{3}}{16}\right)$	Applies the limits of $\dfrac{\pi}{2}$ and their $\dfrac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	Trapezium: $\dfrac{1}{2}\left(2 + \left(2 - \dfrac{3}{2}\sin\dfrac{\pi}{6}\right)\right) \times \dfrac{3}{2}\cos\dfrac{\pi}{6}$ $\left(= \dfrac{39\sqrt{3}}{32}\right)$	Uses a correct strategy for the area of trapezium *OQSP*	M1
	Area of $R = \dfrac{39\sqrt{3}}{32} - \dfrac{\pi}{4} - \dfrac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\dfrac{1}{32}\left(21\sqrt{3} - 8\pi\right)$	Cao	A1
			(6)

		Total 11

Question Number	Scheme	Notes	Marks
9(a)	$n^5-(n-1)^5=n^5-\left(n^5-5n^4+10n^3-10n^2+5n-1\right)=...$ Starts the proof by expanding the bracket		M1
	$5n^4-10n^3+10n^2-5n+1*$	Correct proof with no errors. Full expansion of $(n-1)^5$ must be shown.	A1*
			(2)
(b)	$1^5-0^5=5(1)^4-10(1)^3+10(1)^2-5(1)+1$ $2^5-1^5=5(2)^4-10(2)^3+10(2)^2-5(2)+1$ .... $(n-1)^5-(n-2)^5=5(n-1)^4-10(n-1)^3+10(n-1)^2-5(n-1)+1$ $(n)^5-(n-1)^5=5(n)^4-10(n)^3+10(n)^2-5(n)+1$ $n^5=5\sum_{r=1}^{n}r^4-10\sum_{r=1}^{n}r^3+10\sum_{r=1}^{n}r^2-5\sum_{r=1}^{n}r+n$ M1: Applies the result from part (a) between 1 and n and sums **both** sides Min 3 lines shown A1: Correct equation If only the last line is seen, award M1A1 These marks can be implied by a correct following stage.		M1A1
	$n^5=5\sum_{r=1}^{n}r^4-10\times\frac{1}{4}n^2(n+1)^2+10\times\frac{1}{6}n(n+1)(2n+1)-5\times\frac{1}{2}n(n+1)+n$ M1: Introduces at least 2 correct summation formulae A1: Correct equation		M1A1
	$5\sum_{r=1}^{n}r^4=\frac{5}{2}n^2(n+1)^2-\frac{5}{3}n(n+1)(2n+1)+\frac{5}{2}n(n+1)+n^5-n=...$ $5\sum_{r=1}^{n}r^4=n(n+1)\left[\frac{5}{2}n(n+1)-\frac{5}{3}(2n+1)+\frac{5}{2}+n^3-n^2+n-1\right]$ Makes $5\sum_{r=1}^{n}r^4$ or $\sum_{r=1}^{n}r^4$ the subject and takes out a factor of $n(n+1)$		M1
	$\sum_{r=1}^{n}r^4=\frac{1}{30}n(n+1)\left[15n(n+1)-10(2n+1)+15+6\left(n^3-n^2+n-1\right)\right]$ $=\frac{1}{30}n(n+1)\left[6n^3+9n^2+n-1\right]=\frac{1}{30}n(n+1)(2n+1)(...)$ Takes out a factor of $n(n+1)(2n+1)$ Depends on all previous method marks		dM1
	$=\frac{1}{30}n(n+1)(2n+1)\left(3n^2+3n-1\right)$	cao	A1
			(7)
			Total 9

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