# Mark Scheme (Results)

## Summer 2019

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
(WFM02/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

**General Principles for Further Pure Mathematics Marking**

*(But note that specific mark schemes may sometimes override these general principles).*

**Method mark for solving 3 term quadratic:**

**1. Factorisation**

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = …

**2. Formula**

Attempt to use the correct formula (with values for a, b and c).

**3. Completing the square**

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = …

**Method marks for differentiation and integration:**

**1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \to x^{n-1}$ )

**2. Integration**

Power of at least one term increased by 1. ( $x^n \to x^{n+1}$ )

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **1.** **Way 1** | $x^2-6=x \Rightarrow x=...$ **or** $-x^2+6=x \Rightarrow x=...$ | Attempts to solve $x^2-6=x$ **or** $6-x^2=x$ or equivalent equations/inequalities e.g. $x^2-x-6>0$ so $x=\ldots$ | M1 |
| | $x^2-6=x \Rightarrow x=...$ **and** $-x^2+6=x \Rightarrow x=...$ | Attempts to solve $x^2-6=x$ **and** $6-x^2=x$ or equivalent equations/inequalities e.g. $x^2+x-6>0$ so $x=\ldots$ | M1 |
| | $(x-3)(x+2)=0 \Rightarrow (x=-2), x=3$ <br> $(x+3)(x-2)=0 \Rightarrow (x=-3), x=2$ | $x=2$ and $x=3$ seen as two roots (the other roots do not need to be seen and can be ignored if present) | A1 |
| | $x=2, x=3 \Rightarrow ...$ <br> One of $x=2$ **or** $x=3$, or $x=2$ **and** $x=3$ **only** used to form at least one inequality for their final answer **but if any other values of $x$ are used score M0**. <br> **Dependent on both previous method marks.** | | **d**M1 |
| | $x<2$ or $x>3$ | One correct region. Allow equivalent notation e.g. $(-\infty, 2)$, $(3, \infty)$. | A1 |
| | $x<2$ and $x>3$ | Both correct regions. Allow equivalent notation e.g. $(-\infty, 2)$, $(3, \infty)$. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but **not** $\cap$ | A1 |
| | | | **(6)** |
| **Way 2** | $\left(x^2-6\right)^2=x^2 \Rightarrow x^4-12x^2+36=x^2$ | Square both sides and attempts to expand to obtain a quartic equation | M1 |
| | $x^4-13x^2+36=0 \Rightarrow x^2=...$ <br> $\Rightarrow x=...$ | Correct attempt to solve quadratic in $x^2$ to obtain values for $x$ – the usual rules can be applied if necessary | M1 |
| | $x=2, (-2), 3, (-3)$ | $x=2$ and $x=3$ seen as two roots (the other roots do not need to be seen and can be ignored if present) | A1 |
| | $x=2, x=3 \Rightarrow ...$ <br> One of $x=2$ **or** $x=3$, or $x=2$ **and** $x=3$ **only** used to form at least one inequality for their final answer **but if any other values of $x$ are used score M0**. <br> **Dependent on both previous method marks.** | | **d**M1 |
| | $x<2$ or $x>3$ | One correct region. Allow equivalent notation e.g. $(-\infty, 2)$, $(3, \infty)$. | A1 |
| | $x<2$ and $x>3$ | Both correct regions. Allow equivalent notation e.g. $(-\infty, 2)$, $(3, \infty)$. Ignore what they have between their inequalities e.g. allow "or", "and", "," etc. but **not** $\cap$ | A1 |
| | | | **(6)** |
| | | | **Total 6** |

| Question Number | Scheme | Notes | Marks |
| --- | --- | --- | --- |
| **2** **Way 1** | $w=\dfrac{1}{z+1}\Rightarrow z=\dfrac{1-w}{w}$ | Makes $z$ the subject and obtains $z=\dfrac{\pm1\pm w}{w}$ | M1 |
| | $z=\dfrac{1-(u+\mathrm{i}v)}{u+\mathrm{i}v}\times\dfrac{u-\mathrm{i}v}{u-\mathrm{i}v}$ | Replaces $w$ with $u+\mathrm{i}v$ and multiplies top and bottom by complex conjugate of their denominator. This statement is sufficient. | M1 |
| | $x=0\Rightarrow\dfrac{u-(u^2+v^2)}{u^2+v^2}=0$ | Equates real part to zero | M1 |
| | $\Rightarrow u^2+v^2-u=0$ | Correct equation connecting $u$ and $v$ | A1 **M1 on ePEN** |
| | Centre $\left(\dfrac{1}{2},0\right)$ **or** radius $\dfrac{1}{2}$ | One correct **but must follow the use of a correct circle equation** | A1cso |
| | Centre $\left(\dfrac{1}{2},0\right)$ **and** radius $\dfrac{1}{2}$ | Both correct **but must follow the use of a correct circle equation** | A1cso |
| | | | **(6)** |
| **Way 2** | $z=\mathrm{i}y\Rightarrow w=\dfrac{1}{\mathrm{i}y+1}$ | Replaces $z$ with $\mathrm{i}y$ | M1 |
| | $u+iv=\dfrac{1}{\mathrm{i}y+1}\times\dfrac{1-\mathrm{i}y}{1-\mathrm{i}y}$ | Multiplies top and bottom by complex conjugate of denominator. This statement is sufficient. | M1 |
| | $u=\dfrac{1}{1+y^2}$ **or** $v=\dfrac{-y}{1+y^2}$ | $w=u+\mathrm{i}v$ and equates real or imaginary parts to obtain either $u$ or $v$ in terms of $y$ | M1 |
| | $\Rightarrow v^2+u^2=u$ | Correct equation connecting $u$ and $v$ | A1 **M1 on ePEN** |
| | Centre $\left(\dfrac{1}{2},0\right)$ **or** radius $\dfrac{1}{2}$ | One correct **but must follow the use of a <u>correct circle equation</u>** | A1cso |
| | Centre $\left(\dfrac{1}{2},0\right)$ **and** radius $\dfrac{1}{2}$ | One correct **but must follow the use of a <u>correct circle equation</u>** | A1cso |
| | | | **(6)** |
| | | | **Total 6** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3(a)** | $\frac{2}{(r-1)(r+1)} \equiv \frac{A}{(r-1)} + \frac{B}{(r+1)}$ | | |
| | $\frac{2}{(r-1)(r+1)} \equiv \frac{1}{(r-1)} - \frac{1}{(r+1)}$ | Oe e.g. allow $\frac{1}{(r-1)} + \frac{-1}{(r+1)}$ **Must be in terms of *r*.** | B1 |
| | **Do not allow this mark for just finding their constants e.g. *A* = 1, *B* = –1 but allow this mark to be recovered in (b) if the correct partial fractions are seen or used.** | | |
| | | | **(1)** |
| **(b)** | **To score in (b) they must be using partial fractions of the form** $\frac{A}{(r-1)} + \frac{B}{(r+1)}$ | | |
| | $\sum_{r=2}^{n} = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \ldots\ldots\ldots\ldots + \left(\frac{1}{n-2} - \frac{1}{n}\right) + \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$ Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their non-cancelling fractions identified below** Allow other letters for *n* (most likely to be *r*) except for the final mark – see below If terms are found beyond the limits of the summation e.g. *r* = 0, *r* = 1, these can be ignored for this mark as long as at least the terms for *r* = 2, 3, *n* – 1 and *n* are seen | | M1 |
| | $\sum_{r=2}^{n} \frac{2}{r^2-1} = 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{(n+1)}$ | $1, \frac{1}{2}\left(\text{or } \frac{3}{2}\right)$ identified as the only constant term(s). Follow through their partial fractions so allow $\frac{\textit{their } A}{1}, \frac{\textit{their } A}{2}$ | A1ft **M1 on ePEN** |
| | | $-\frac{1}{n}, -\frac{1}{(n+1)}$ identified as the only algebraic terms. Follow through their partial fractions so allow $\frac{\textit{their } B}{n}, \frac{\textit{their } B}{n+1}$ | A1ft |
| | $= \frac{3n(n+1) - 2(n+1) - 2n}{2n(n+1)}$ | Attempts common denominator from terms of the form $A, \frac{B}{n}, \frac{C}{n+1}$ only. Must see *n*(*n* + 1) in the denominator and an unsimplified quadratic expression in the numerator. **Dependent on the first method mark.** | **d**M1 |
| | $= \frac{3n^2 - n - 2}{2n(n+1)} = \frac{(3n+2)(n-1)}{2n(n+1)}$ * | Cso. No errors seen but see note below. | A1cso |
| | **Note: If extra terms were considered for the first M mark, (usually those for *r* = 1), allow a full recovery if the correct constant terms are 'extracted'. Some candidates attempt** $\sum_{r=1}^{n} \frac{2}{r^2-1} - \sum_{r=1}^{1} \frac{2}{r^2-1}$ **and the same ruling applies as the term for *r* = 1 effectively cancels out, leaving the correct non-cancelling terms.** | | |
| | | | |
| | | | **(5)** |

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|---|---|---|---|
| **3(c)** | $$S_{3n}=\sum_{r=2}^{3n}\frac{2}{(r-1)(r+1)}=\frac{(3\times 3n+2)(3n-1)}{2\times 3n(3n+1)}$$ Correct, possibly unsimplified, expression for $S_{3n}$ using the **given** result in (b) | | B1 |
| | $$S_{3n}-S_{n-1}=\frac{(3\times 3n+2)(3n-1)}{2\times 3n(3n+1)}-\frac{(3(n-1)+2)(n-2)}{2(n-1)n}$$ Attempts $S_{3n}-S_{n-1}$ using the **given** result in (b)<br>If there is any doubt about the "$S_{n-1}$", at least 3 of the $n$'s should be replaced by $n$ - 1 | | M1 |
| | $$=\frac{(9n+2)(3n-1)(n-1)-3(3n+1)(3n-1)(n-2)}{6n(3n+1)(n-1)}$$ $$=\frac{(3n-1)(9n^2-7n-2-3(3n^2-5n-2))}{6n(3n+1)(n-1)}$$ Attempts common denominator involving $n$, $3n+1$ and $n-1$ and attempts a factor of $3n-1$ in the numerator or vice versa. Note that the numerator may be expanded completely to give $24n^2+4n-4$ and the $3n-1$ then attempted as a factor.<br>**Dependent on the previous mark.** | | **d**M1 |
| | $$=\frac{2(3n-1)(2n+1)}{3n(3n+1)(n-1)}$$ | Cao | A1 |
| | | | **(4)** |
| | **If (c) is attempted using the method of differences (i.e. repeating the work in (b)) then this scores 0/4 in part (c).** | | |
| | | | **Total 10** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **4.** | $(\cos x)\dfrac{\mathrm{d}y}{\mathrm{d}x}+(\sin x)y=2\cos^3 x\sin x-3$ | | |
| **(a)** | $\dfrac{\mathrm{d}y}{\mathrm{d}x}+(\tan x)y=2\cos^2 x\sin x-3\sec x$ | Attempt to divide through by cos $x$. If the intention is not clear must see at least 2 terms divided by cos $x$. | M1 |
| | Integrating Factor: $I=\mathrm{e}^{\int \tan x\,\mathrm{d}x}$ | $I=\mathrm{e}^{\int \pm \text{their } \mathrm{P}(x)\,(\mathrm{d}x)}$ from $\dfrac{\mathrm{d}y}{\mathrm{d}x}+Py=...$ **Dependent on the first method mark.** May be implied by use of sec $x$ as the integrating factor. | **d**M1 |
| | $I=\sec x$ | $\dfrac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$ | A1 |
| | $y\sec x=\int \sec x\left(2\cos^2 x\sin x-3\sec x\right)(\mathrm{d}x)$<br>or<br>$\dfrac{\mathrm{d}}{\mathrm{d}x}(y\sec x)=\sec x\left(2\cos^2 x\sin x-3\sec x\right)$<br>$y\times \textit{their } I=\int Q(x)\times \textit{their } I\,(\mathrm{d}x)$ or $\dfrac{\mathrm{d}}{\mathrm{d}x}(y\times \textit{their } I)=Q(x)\times \textit{their } I$<br>If there is any doubt, must multiply at least one of their $Q(x)$ terms by $I$ | | M1 |
| | $\int 2\cos x\sin x\,\mathrm{d}x=\sin^2 x$ or $-\cos^2 x$ or $-\dfrac{1}{2}\cos 2x$<br>**Must follow the previous method mark** | | A1<br>**M1 on ePEN** |
| | $\int -3\sec^2 x\,\mathrm{d}x=-3\tan x$<br>**Must follow the previous method mark** | | A1 |
| | Examples of a correct answer:<br>$y=\cos x\sin^2 x-3\sin x+k\cos x$<br>or<br>$y=-\dfrac{1}{2}\cos x\cos 2x-3\sin x+k\cos x$<br>or<br>$y=-\cos^3 x-3\sin x+k\cos x$<br>**Follow through their integration and their integrating factor but must be $y=\ldots$ with the constant dealt with correctly and depends on the third method mark.** | | A1**ft** |
| | | | **(7)** |

| | | | |
|---|---|---|---|
| **(b)** | $$3\sqrt{3}=\frac{1}{2}.\left(\frac{\sqrt{3}}{2}\right)^2-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}-\frac{3}{4}\right)$$ $$3\sqrt{3}=-\frac{1}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}-\frac{1}{4}\right)$$ $$3\sqrt{3}=\left(-\frac{1}{2}\right)^3-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}+\frac{1}{4}\right)$$ Substitutes the given conditions into their $y=\mathrm{f}(x)$ and attempts to find their constant | | M1 |
| | $k=9\sqrt{3}-\frac{3}{4}$ or $9\sqrt{3}-\frac{1}{4}$ or $9\sqrt{3}+\frac{1}{4}$ | Correct constant for their method | A1 |
| | $$y=\cos x\sin^2 x-3\sin x+\left(9\sqrt{3}-\frac{3}{4}\right)\cos x$$ or $$y=-\frac{1}{2}\cos x\cos 2x-3\sin x+\left(9\sqrt{3}-\frac{1}{4}\right)\cos x$$ or $$y=-\cos^3 x-3\sin x+\left(9\sqrt{3}+\frac{1}{4}\right)\cos x$$ Or equivalent correct answer. Must be $y=\ldots$ | | A1 |
| | | | **(3)** |
| | | | **Total 10** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5** | $-8-8\mathrm{i}\sqrt{3}$ | | |
| **(a)** | $r=\left(\sqrt{(8)^2+\left(8\sqrt{3}\right)^2}\right)=16$ | 16 | B1 |
| | $\theta=-\pi+\tan^{-1}\left(\dfrac{8\sqrt{3}}{8}\right)$ or e.g. $\theta=-\dfrac{\pi}{2}-\tan^{-1}\left(\dfrac{8}{8\sqrt{3}}\right)$ | Correct strategy for the argument (may see correct value only from calculator) or can be implied by a correct argument not in range e.g. $\dfrac{4\pi}{3}$ | M1 |
| | $16\left(\cos\left(-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$ | Correct form and correct values. Condone careless use of brackets as long as the intention is clear. | A1 |
| | | | **(3)** |
| **(b)** | **Note that in (b) the candidate may legitimately work with e.g.** $16\left(\cos\left(\dfrac{2\pi}{3}\right)-\mathrm{i}\sin\left(\dfrac{2\pi}{3}\right)\right)$ | | |
| | $z^4=16\left(\cos\left(2k\pi-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(2k\pi-\dfrac{2\pi}{3}\right)\right)$ Correct use of $2k\pi$ seen or implied. This may be implied by the above expression seen or used with any non-zero integer value (positive or negative) for $k$ | | M1 |
| | $z=16^{\frac{1}{4}}\left(\cos\left(\dfrac{2k\pi}{4}-\dfrac{2\pi}{12}\right)+\mathrm{i}\sin\left(\dfrac{2k\pi}{4}-\dfrac{2\pi}{12}\right)\right)$ Divide their angle by 4 after $\pm 2k\pi$ and find 4th root of their 16 **Dependent on the previous mark.** | | **d**M1 |
| | $z=2\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right),\ 2\left(\cos\left(\dfrac{\pi}{6}\right)-\mathrm{i}\sin\left(\dfrac{\pi}{6}\right)\right),\ 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}},\ -\mathrm{i}+\sqrt{3}$ | | |
| | $z=2\left(\cos\left(\dfrac{\pi}{3}\right)+\mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right),\ 2\left(\cos\left(\dfrac{5\pi}{3}\right)-\mathrm{i}\sin\left(\dfrac{5\pi}{3}\right)\right),\ 2\mathrm{e}^{\frac{\pi}{3}\mathrm{i}},\ 1+\mathrm{i}\sqrt{3}$ | | |
| | $z=2\left(\cos\left(\dfrac{5\pi}{6}\right)+\mathrm{i}\sin\left(\dfrac{5\pi}{6}\right)\right),\ 2\left(\cos\left(\dfrac{7\pi}{6}\right)-\mathrm{i}\sin\left(\dfrac{7\pi}{6}\right)\right),\ 2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}},\ \mathrm{i}-\sqrt{3}$ | | |
| | $z=2\left(\cos\left(-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right),\ 2\left(\cos\left(\dfrac{2\pi}{3}\right)-\mathrm{i}\sin\left(\dfrac{2\pi}{3}\right)\right),\ 2\mathrm{e}^{-\frac{2\pi}{3}\mathrm{i}},\ -1-\mathrm{i}\sqrt{3}$ | | |
| | 1 correct root in any form | | A1 |
| | All 4 roots correct in any form (allow equivalent arguments) | | A1 |
| | All 4 roots in correct surd form or exact equivalent in required form. | | A1 |
| | **The A marks must follow correct work and should not be awarded if any correct roots are obtained fortuitously. So must follow a correct answer in part (a) although the argument may not be in the required range.** | | |
| | | | **(5)** |
| | | | **Total 8** |

**Special Case in (b):**

Candidates who do not consider $\pm 2k\pi$ **at any stage** and apply De Moivre correctly to get

$$z^4=16\left(\cos\left(-\frac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)\Rightarrow z=2\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right)\left(=-\mathrm{i}+\sqrt{3}\right)$$

Can score a B1 special case and this should be awarded as the first A mark on ePEN

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **6.** | $y=\dfrac{1}{\sqrt{1+x^2}}$ | | |
| | $\dfrac{dy}{dx}=-x(1+x^2)^{-\frac{3}{2}}$ | $\dfrac{dy}{dx}=kx\left(1+x^2\right)^{-\frac{3}{2}}$ | M1 |
| | | $\dfrac{dy}{dx}=-x\left(1+x^2\right)^{-\frac{3}{2}}$. Allow in any correct unsimplified form and isw if necessary. | A1 |
| | $\left(\dfrac{dy}{dx}\right)_{x=1}=-\dfrac{1}{2\sqrt{2}}$ <br> $\left(-(2)^{-\frac{3}{2}},\ -\dfrac{\sqrt{2}}{4}\ \text{are common}\right)$ | Correct value for $\dfrac{dy}{dx}$ at $x = 1$. Allow for any correct **exact** numerical possibly unsimplified expression and isw if necessary. | A1 |
| | $\dfrac{dy}{dx}=-x(1+x^2)^{-\frac{3}{2}}\Rightarrow\dfrac{d^2y}{dx^2}=-(1+x^2)^{-\frac{3}{2}}+\dfrac{3}{2}x.2x(1+x^2)^{-\frac{5}{2}}\left(=\dfrac{2x^2-1}{\left(1+x^2\right)^{\frac{5}{2}}}\right)$ <br> or <br> $\dfrac{dy}{dx}=-\dfrac{x}{(1+x^2)^{\frac{3}{2}}}\Rightarrow\dfrac{d^2y}{dx^2}=\dfrac{-(1+x^2)^{\frac{3}{2}}+\frac{3}{2}x.2x(1+x^2)^{\frac{1}{2}}}{(1+x^2)^3}\left(=\dfrac{2x^2-1}{\left(1+x^2\right)^{\frac{5}{2}}}\right)$ <br> **d**M1: Product rule: $\dfrac{d^2y}{dx^2}=\alpha(1+x^2)^{-\frac{3}{2}}+\beta x^2(1+x^2)^{-\frac{5}{2}}$ <br> Quotient rule: $\dfrac{d^2y}{dx^2}=\dfrac{\alpha(1+x^2)^{\frac{3}{2}}+\beta x^2(1+x^2)^{\frac{1}{2}}}{(1+x^2)^3}$ <br> **Dependent on the first method mark** <br> A1: Fully correct second derivative. <br> Allow in any correct unsimplified form and isw if necessary. | | **d**M1A1 |
| | $\left(\dfrac{d^2y}{dx^2}\right)_{x=1}=\dfrac{1}{4\sqrt{2}}$ <br> $\left(-(2)^{-\frac{3}{2}}+3(2)^{-\frac{5}{2}},\ \dfrac{\sqrt{2}}{8}\ \text{are common}\right)$ | Correct value for $\dfrac{d^2y}{dx^2}$ at $x = 1$. Allow for any correct **exact** numerical possibly unsimplified expression and isw if necessary. | A1 |
| | $f(x)=f(1)+(x-1)f'(1)+\dfrac{(x-1)^2}{2}f''(1)+...$ <br> $\left(\dfrac{1}{\sqrt{1+x^2}}\right)=\dfrac{1}{\sqrt{2}}-\dfrac{1}{2\sqrt{2}}(x-1)+\dfrac{1}{8\sqrt{2}}(x-1)^2$ <br> M1: Attempts f (1) and applies the correct Taylor series using their values. <br> Must see an attempt at the **first 3** terms. <br> **If the general series is not quoted and their series does not follow their values score M0** <br> A1: Correct expansion (allow equivalent simplified (single fraction) coefficients | | M1A1 |
| | | | **(8)** |
| | | | **Total 8** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7.** | $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}+(2-x^2)y=2x^3$ | | |
| **(a)** | $y=vx\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}v}{\mathrm{d}x}x+v$ | Correct expression | B1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2v}{\mathrm{d}x^2}x+2\frac{\mathrm{d}v}{\mathrm{d}x}$ | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\alpha x\frac{\mathrm{d}^2v}{\mathrm{d}x^2}+\beta\frac{\mathrm{d}v}{\mathrm{d}x}$ | M1 |
| | | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2v}{\mathrm{d}x^2}x+2\frac{\mathrm{d}v}{\mathrm{d}x}$ | A1 |
| | $x^2\left(\frac{\mathrm{d}^2v}{\mathrm{d}x^2}x+2\frac{\mathrm{d}v}{\mathrm{d}x}\right)-2x\left(\frac{\mathrm{d}v}{\mathrm{d}x}x+v\right)+\left(2-x^2\right)vx=2x^3$<br>Substitutes their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into the given equation to give an equation in $x$ and $v$ only | | M1 |
| | $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}-v=2$ * | Correct proof with no errors with at least one more line of working (usually $x^3\frac{\mathrm{d}^2v}{\mathrm{d}x^2}-x^3v=2x^3$) | A1 |
| | | | **(5)** |
| **(b)** | $m^2-1=0\Rightarrow m=\pm1$ | Attempts to solve $"m"^2-1=0$ | M1 |
| | $(v=)\,A\mathrm{e}^x+B\mathrm{e}^{-x}$ | Correct CF ($v=\ldots$ not required) | A1 |
| | PI is –2 | Correct PI | B1 |
| | $v=CF\ +\ PI\ =\ldots$ | Adds their CF and their non-zero PI to find $v$ in terms of $x$. **Must be $v=\ldots$ here unless this is implied by subsequent work.** | M1 |
| | $(y=)\,x(CF\ +\ PI)$ | Multiplies their $v$ by $x$ to find $y$ in terms of $x$. (Can be awarded if no PI is found or their PI = 0) | M1 |
| | $y=Ax\mathrm{e}^x+Bx\mathrm{e}^{-x}-2x$ | Correct expression. Must be $y=\ldots$ | A1 |
| | | | **(6)** |
| **(c)** | $\frac{\mathrm{d}y}{\mathrm{d}x}=Ax\mathrm{e}^x+A\mathrm{e}^x-Bx\mathrm{e}^{-x}+B\mathrm{e}^{-x}-2$ | Differentiate their GS wrt $x$ using the Product Rule | M1 |
| | $x=1,y=\mathrm{e}\Rightarrow\mathrm{e}=A\mathrm{e}+B\mathrm{e}^{-1}-2$<br>and<br>$x=1,\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}$<br>$\Rightarrow\mathrm{e}=A\mathrm{e}+A\mathrm{e}-B\mathrm{e}^{-1}+B\mathrm{e}^{-1}-2$ | Substitutes the given values in for $y$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain 2 equations | M1 |
| | $A=\frac{\mathrm{e}+2}{2\mathrm{e}},\ B=\frac{\mathrm{e}(\mathrm{e}+2)}{2}$ | Both values correct or exact equivalent | A1 |
| | $y=\left(\frac{\mathrm{e}+2}{2\mathrm{e}}\right)x\mathrm{e}^x+\left(\frac{\mathrm{e}(\mathrm{e}+2)}{2}\right)x\mathrm{e}^{-x}-2x$ | Correct expression (or equivalent). Must be $y=\ldots$ | A1 |
| | | | **(4)** |
| | | | **Total 15** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **8.** | $r = \sin\theta + \cos 2\theta$ | | |
| **(a)** | $y = r\sin\theta = \sin^2\theta + \sin\theta\cos 2\theta$ <br> or e.g. <br> $y = \sin\theta(\sin\theta + \cos 2\theta)$ | Correct expression | B1 |
| | $\frac{dy}{d\theta} = 2\sin\theta\cos\theta + \cos\theta\cos 2\theta - 2\sin\theta\sin 2\theta$ <br> M1: **Correct** use of Chain and Product Rules (allow sign errors only) **on a correct expression** <br> A1: Correct differentiation | | M1A1 |
| | $6\sin^2\theta - 2\sin\theta - 1 = 0$ | Correct quadratic | A1 |
| | $\sin\theta = \frac{2 \pm \sqrt{28}}{12} \Rightarrow$ at $P$, $\sin\theta = \frac{2+\sqrt{28}}{12} \Rightarrow \theta = \sin^{-1}\frac{2+\sqrt{28}}{12} = 0.653067...$ <br> $r = \sin\theta + \cos 2\theta = \frac{2+\sqrt{28}}{12} + \cos(2 \times "0.653...")$ <br> A complete method to find *OP*. Solves their 3TQ in sin *θ*, proceeds to obtain a value for *θ* and uses the given expression for *r* to find *OP*. <br> **or** <br> $\sin\theta = \frac{2 \pm \sqrt{28}}{12} \Rightarrow$ at $P$, $\sin\theta = \frac{2+\sqrt{28}}{12} \Rightarrow \cos 2\theta = 1 - 2\left(\frac{2+\sqrt{28}}{12}\right)^2$ <br> $r = \sin\theta + \cos 2\theta = \frac{1+\sqrt{7}}{6} + 1 - 2\left(\frac{1+\sqrt{7}}{6}\right)^2$ <br> A complete method to find *OP*. Solves their 3TQ in sin *θ*, proceeds to obtain a value for cos2*θ* using a correct identity and adds their sin *θ* value to find *OP*. <br> **Dependent on the first method mark.** | | **d**M1 |
| | $OP = r = 0.8692...$ | awrt 0.869 | A1 |
| | | | **(6)** |

| | | | |
|---|---|---|---|
| **(b)** | $\left(\sin\theta+\cos 2\theta\right)^2=\sin^2\theta+2\sin\theta\cos 2\theta+\cos^2 2\theta$<br>Attempt to find $r^2$.<br>Allow poor squaring as long as there is the intention to square the bracket. | | M1 |
| | $\int\sin^2\theta\ \mathrm{d}\theta=\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta$<br>$\int\cos^2 2\theta\ \mathrm{d}\theta=\frac{1}{2}\theta+\frac{1}{8}\sin 4\theta$ | Attempts to integrate $\sin^2\theta$ to obtain $\alpha\theta+\beta\sin 2\theta$ and attempts to integrate $\cos^2 2\theta$ to obtain $\alpha\theta+\beta\sin 4\theta$. This may be implied by an expression of the form $\alpha\theta+\beta\sin 2\theta+\gamma\sin 4\theta$<br>**Dependent on the first method mark.** | **d**M1 |
| | $\int 2\sin\theta\cos 2\theta\ \mathrm{d}\theta=\int 2\sin\theta\left(2\cos^2\theta-1\right)\ \mathrm{d}\theta$<br>$=\int\left(4\sin\theta\cos^2\theta-2\sin\theta\right)\ \mathrm{d}\theta=-\frac{4}{3}\cos^3\theta+2\cos\theta$<br>or<br>$\int 2\sin\theta\cos 2\theta\ \mathrm{d}\theta=\int\left(\sin 3\theta-\sin\theta\right)\mathrm{d}\theta=-\frac{1}{3}\cos 3\theta+\cos\theta$<br>Fully correct strategy for integrating $(2)\sin\theta\cos 2\theta$<br>**Dependent on the first method mark.** | | **d**M1 |
| | $\int r^2\mathrm{d}\theta=\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)+\cos\theta-\frac{1}{3}\cos 3\theta+\frac{1}{2}\left(\theta+\frac{1}{4}\sin 4\theta\right)$<br>or<br>$\int r^2\mathrm{d}\theta=\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)-\frac{4}{3}\cos^3\theta+2\cos\theta+\frac{1}{2}\left(\theta+\frac{1}{4}\sin 4\theta\right)$<br>Fully correct integration | | A1 |
| | $\frac{1}{2}\int_0^{\frac{\pi}{2}}r^2\mathrm{d}\theta=\ldots$ | Fully correct method using a correct formula and evidence of use of the limits $\frac{\pi}{2}$ and 0 with subtraction. **Dependent on the <u>first</u> and <u>at least one of the subsequent</u> method marks.** | **dd**M1 |
| | $=\frac{\pi}{4}-\frac{1}{3}$ | Correct exact area.<br>Allow equivalent exact expressions<br>e.g. $\frac{1}{2}\left(\frac{\pi}{2}-\frac{2}{3}\right)$ | A1 |
| | | | **(6)** |
| | | | **Total 12** |

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