Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 7 June 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. Solve the equation

$$z^5 = 32$$

Give your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$

(5)

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Question 1 continued

Q1

(Total 5 marks)

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2. Use algebra to find the set of values of x for which

$$\frac{x-4}{(x+3)} \leqslant \frac{5}{x(x+3)}$$

(9)

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Question 2 continued

Q2

(Total 9 marks)

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3. (a) Show that $r^3 - (r-1)^3 \equiv 3r^2 - 3r + 1$

(1)

(b) Hence prove by the method of differences that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

[*You may use* $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ *without proof.*]

(5)

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Question 3 continued

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Question 3 continued

Q3

(Total 6 marks)

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4. $$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$$

is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 40e^{-x}\sin 3x$$

where A is a constant.

(a) Find the value of A.

(5)

(b) Hence find the general solution of this differential equation.

(4)

(c) Find the particular solution of this differential equation for which both $y = 3$ and $\frac{dy}{dx} = 3$ at $x = 0$

(4)

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Question 4 continued

Q4

(Total 13 marks)

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5. $$y = e^{\cos^2 x}$$

(a) Show that

$$\frac{d^2 y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$$

(4)

(b) Hence find the Maclaurin series expansion of $e^{\cos^2 x}$ up to and including the term in x^2

(3)

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Question 5 continued

(Total 7 marks)

Q5

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6. Find the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = (\cos^2 x) \ln x, \qquad 0 < x < \frac{\pi}{2}$$

Give your answer in the form $y = f(x)$.

(8)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

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7.

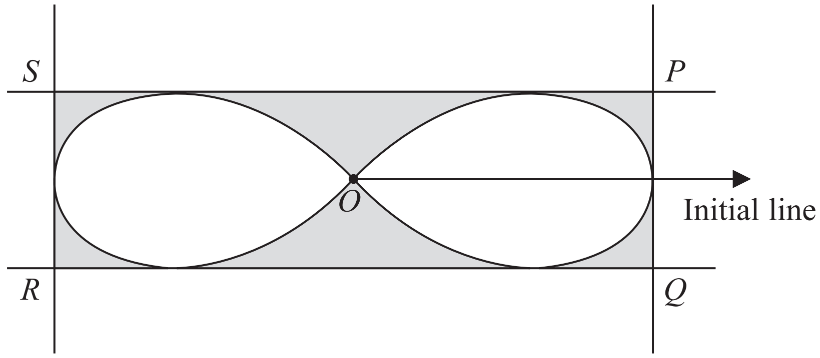

Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 4\cos 2\theta, \quad -\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4} \quad \text{and} \quad \frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, QR, RS and SP are tangents to C, where QR and SP are parallel to the initial line and PQ and RS are perpendicular to the initial line.

(a) Find the polar coordinates of the points where the tangent SP touches the curve. Give the values of θ to 3 significant figures.

(5)

(b) Find the exact area of the finite region bounded by the curve C, shown unshaded in Figure 1.

(5)

(c) Find the area enclosed by the rectangle $PQRS$ but outside the curve C, shown shaded in Figure 1.

(5)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 15 marks)

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8. (a) Use de Moivre's theorem to

(i) show that

$$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

(ii) find an expression for $\sin 5\theta$ in terms of $\cos\theta$ and $\sin\theta$

(4)

(b) Hence show that

$$\tan 5\theta = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$$

where $t = \tan\theta$ and $\cos 5\theta \neq 0$

(2)

(c) Hence find a quadratic equation whose roots are $\tan^2\dfrac{\pi}{5}$ and $\tan^2\dfrac{2\pi}{5}$

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers to be found.

(4)

(d) Deduce that $\tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = \sqrt{5}$

(2)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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