Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Wednesday 3 June 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Using algebra, find the set of values of x for which

$$\frac{x}{x+2} < \frac{2}{x+5}$$

(7)

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Question 1 continued

Q1

(Total 7 marks)

P 4 4 8 3 2 A 0 3 3 2

2. (a) Express $\dfrac{1}{(r+6)(r+8)}$ in partial fractions.

(1)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+6)(r+8)} = \frac{n(an+b)}{56(n+7)(n+8)}$$

where a and b are integers to be found.

(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total 5 marks)

P 4 4 8 3 2 A 0 7 3 2

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3. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 2xy = xe^{-x^2}y^3 \quad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4xz = -2xe^{-x^2} \quad \text{(II)}$$

(4)

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$.

(1)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 10 marks)

P 4 4 8 3 2 A 0 1 1 3 2

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4. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z-1}{z+1}, \quad z \neq -1$$

The line in the z-plane with equation $y = 2x$ is mapped by T onto the curve C in the w-plane.

(a) Show that C is a circle and find its centre and radius.

(7)

The region $y < 2x$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Sketch circle C on an Argand diagram and shade and label region R.

(2)

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Question 4 continued

P 4 4 8 3 2 A 0 1 3 3 2

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Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)

P 4 4 8 3 2 A 0 1 5 3 2

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5. Given that $y = \cot x$,

(a) show that

$$\frac{d^2y}{dx^2} = 2\cot x + 2\cot^3 x$$

(3)

(b) Hence show that

$$\frac{d^3y}{dx^3} = p\cot^4 x + q\cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of $\cot x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

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Question 5 continued

P 4 4 8 3 2 A 0 1 7 3 2

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Question 5 continued

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Question 5 continued

(Total 9 marks)

Q5

P 4 4 8 3 2 A 0 1 9 3 2

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \quad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

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Question 6 continued

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Question 6 continued

Question 6 continued

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Q6

(Total 13 marks)

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7.

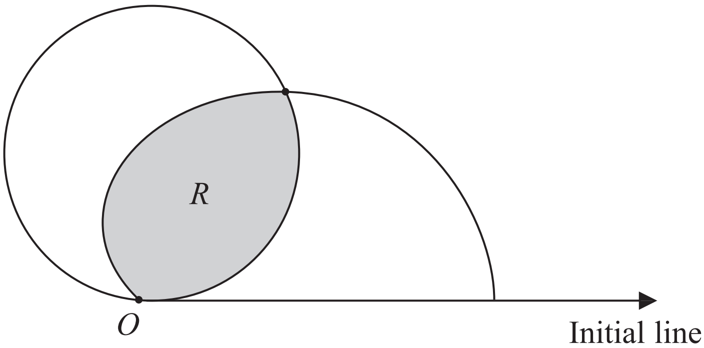

Figure 1

Figure 1 shows the two curves given by the polar equations

$$r = \sqrt{3}\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$$

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \pi$$

(a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$.

(2)

The region R, bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of R, giving your answer in the form $a\left(\pi - \sqrt{3}\right)$, where a is a constant to be found.

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 8 marks)

P 4 4 8 3 2 A 0 2 7 3 2

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8. (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,\mathrm{d}\theta$$

(4)

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Question 8 continued

P 4 4 8 3 2 A 0 2 9 3 2

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END