Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 23 May 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P 7 5 7 4 3 A 0 1 2 8

Pearson

1. (i) The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$$

where k is a constant.

(a) Determine the value of k for which **A** is singular.

(2)

Given that **A** is non-singular,

(b) determine $\mathbf{A}^{-1}$ in terms of k, giving your answer in simplest form.

(2)

(ii) The matrix **B** is defined by

$$\mathbf{B} = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$$

where p and q are integers.

State the value of p and the value of q when **B** represents

(a) an enlargement about the origin with scale factor –2

(b) a reflection in the y–axis.

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = z^3 - 13z^2 + 59z + p \qquad p \in \mathbb{Z}$$

Given that $z = 3$ is a root of the equation $f(z) = 0$

(a) show that $p = -87$

(2)

(b) Use algebra to determine the other roots of $f(z) = 0$, giving your answers in simplest form.

(4)

On an Argand diagram

- the root $z = 3$ is represented by the point P
- the other roots of $f(z) = 0$ are represented by the points Q and R
- the number $z = -9$ is represented by the point S

(c) Show on a single Argand diagram the positions of P, Q, R and S

(1)

(d) Determine the perimeter of the quadrilateral $PQSR$, giving your answer as a simplified surd.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. $$f(x) = x^3 - 5\sqrt{x} - 4x + 7 \qquad x \geqslant 0$$

The equation $f(x) = 0$ has a root α in the interval [0.25, 1]

(a) Use linear interpolation once on the interval [0.25, 1] to determine an approximation to α, giving your answer to 3 decimal places.

(3)

The equation $f(x) = 0$ has another root β in the interval [1.5, 2.5]

(b) Determine $f'(x)$

(2)

(c) Hence, using $x_0 = 1.75$ as a first approximation to β, apply the Newton–Raphson process once to $f(x)$ to determine a second approximation to β, giving your answer to 3 decimal places.

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The complex number z is defined by

$$z = -3 + 4\text{i}$$

(a) Determine $\left|z^2 - 3\right|$

(3)

(b) Express $\dfrac{50}{z^*}$ in the form kz, where k is a positive integer.

(3)

(c) Hence find the value of $\arg\left(\dfrac{50}{z^*}\right)$

Give your answer in radians to 3 significant figures.

(2)

Question 4 continued

(Total for Question 4 is 8 marks)

5. The equation $5x^2 - 4x + 2 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$

(a) Without solving the equation,

(i) show that $pq = \frac{5}{2}$

(ii) determine the value of $p + q$

(4)

(b) Hence, without finding the values of p and q, determine a quadratic equation with roots

$$\frac{p}{p^2 + 1} \text{ and } \frac{q}{q^2 + 1}$$

giving your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

(5)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)r \\ 0 & 2^n \end{pmatrix}$$

where r is a constant.

(4)

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4$$

The transformation represented by matrix **M** followed by the transformation represented by matrix **N** is represented by the matrix **B**

(b) (i) Determine **N** in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c and d are integers.

(ii) Determine **B**

(3)

Hexagon S is transformed onto hexagon S' by matrix **B**

(c) Given that the area of S' is 720 square units, determine the area of S

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question use the standard results for summations.**

(a) Show that for all positive integers n

$$\sum_{r=1}^{n}(12r^2 + 2r - 3) = An^3 + Bn^2$$

where A and B are integers to be determined.

(4)

(b) Hence determine the value of n for which

$$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n}(12r^2 + 2r - 3) = 270$$

(4)

Question 7 continued

(Total for Question 7 is 8 marks)

8. Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 7^{n-1} + 8^{2n+1}$$

is divisible by 57

(6)

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Question 8 continued

(Total for Question 8 is 6 marks)

9. The rectangular hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H

(a) Use calculus to show that an equation of the normal to H at P is

$$t^3x - ty = c(t^4 - 1)$$

(4)

The parabola C has equation $y^2 = 6x$

The normal to H at the point with coordinates (8, 2) meets C at the point Q where $y > 0$

(b) Determine the exact coordinates of Q

(4)

Given that

- the point R is the focus of C
- the line l is the directrix of C
- the line through Q and R meets l at the point S

(c) determine the exact length of QS

(5)

Question 9 continued

Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS