Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01/01)

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Summer 2019
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where } |pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where } |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Summer 2019
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\mathrm{f}(x) = 5 + 4x^2 - \frac{4}{3}x^3 - \frac{7}{2x}$; $x > 0$		
(a)	$\mathrm{f}'(x) = 8x - 4x^2 + \frac{7}{2}x^{-2}$	At least one of either $5 + 4x^2 \to \pm Ax$ or $-\frac{4}{3}x^3 \to \pm Bx^2$ or $-\frac{7}{2x} \to \pm Cx^{-2}$; $A, B, C \neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
			(2)
(b)	$\mathrm{f}(0.5) = -\frac{7}{6}$, $\mathrm{f}'(0.5) = 17$	Either $\mathrm{f}(0.5) = -\frac{7}{6}$ or awrt -1.17 or truncated -1.16 **or** $\mathrm{f}'(0.5) = 17$ **or a correct numerical expression** for either $\mathrm{f}(0.5)$ or $\mathrm{f}'(0.5)$ **Can be implied by later working**	B1
	$\left\{\alpha \simeq 0.5 - \frac{\mathrm{f}(0.5)}{\mathrm{f}'(0.5)}\right\} \Rightarrow \alpha \simeq 0.5 - \frac{-\frac{7}{6}}{17}$	Valid attempt at Newton-Raphson using their values of $\mathrm{f}(0.5)$ and $\mathrm{f}'(0.5)$	M1
	$\left\{\alpha = 0.56862745\ldots \text{ or } \frac{29}{51}\right\} \Rightarrow \alpha = 0.569$ (3 dp)	Correct $\mathrm{f}'(x)$ and 0.569 on first iteration (Ignore any subsequent iterations)	A1 **cso cao**
	Correct differentiation followed by 0.569 (with no working seen) scores full marks in part (b)		**(3)**
(c) **Way 1**	$\mathrm{f}(3) = \frac{23}{6} = 3.833333\ldots$ $\mathrm{f}(3.5) = -\frac{25}{6} = -4.166666\ldots$	Attempts to evaluate both $\mathrm{f}(3)$ and $\mathrm{f}(3.5)$ **and** either $\mathrm{f}(3) = \frac{23}{6}$ or awrt 4 or truncated 3 **or** $\mathrm{f}(3.5) = -\frac{25}{6}$ or awrt -4	M1
	Sign change {positive, negative} {and $\mathrm{f}(x)$ is continuous} therefore a root $\{\beta\}$ exists in the interval $\{[3, 3.5]\}$	Both values correct awrt (or truncated) to 1sf, sign change and conclusion.	A1
			(2)
(d) **Way 1**	$\frac{\beta - 3}{\text{"3.8333..."}} = \frac{3.5 - \beta}{\text{"4.1666..."}}$ **or** $\frac{\beta - 3}{3.5 - \beta} = \frac{\text{"3.8333..."}}{\text{"4.1666..."}}$ **or** $\frac{\beta - 3}{\text{"3.8333..."}} = \frac{3.5 - 3}{\text{"4.166..."} + \text{"3.8333..."}}$	A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.	M1
	• $\beta = \left(\frac{(3)(\text{"4.1666..."}) + (3.5)(\text{"3.8333..."})}{\text{"4.1666..."} + \text{"3.8333..."}}\right) = \left(\frac{12.5 + 13.4166\ldots}{8}\right)$ • $\beta = 3 + \left(\frac{\text{"3.8333..."}}{\text{"4.1666..."} + \text{"3.8333..."}}\right)(0.5)$ or $\beta = 3 + \left(\frac{\text{"}\frac{23}{6}\text{"}}{\text{"8"}}\right)(0.5)$ • $\beta = 3 + \left(\frac{\text{"}-3.8333\ldots\text{"}}{\text{"}-4.1666\ldots\text{"} + \text{"}-3.8333\ldots\text{"}}\right)(0.5)$	**dependent on the previous M mark** Rearranges to give $\beta = \ldots$	dM1
	$\left\{\beta = 3.239583\ldots \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96}\right\} \Rightarrow \beta = 3.24$ (2dp)	awrt 3.24 (Ignore any subsequent iterations)	A1
			(3)
			10

Question Number	Scheme	Notes	Marks
1. (d) **Way 2**	$\dfrac{x}{"3.8333..."} = \dfrac{0.5 - x}{"4.1666..."}$ $x = \dfrac{(0.5)("3.8333...")}{"3.8333..." + "4.1666..."} = 0.239583...$ $\Rightarrow \beta = 3 + 0.239583...$	Finds x using a correct method of similar triangles and applies "3 + their x"	M1 dM1
	$\left\{\beta = 3.239583... \text{ or } 3\dfrac{23}{96} \text{ or } \dfrac{311}{96}\right\} \Rightarrow \beta = 3.24 \ \ (2\,\text{dp})$	awrt 3.24	A1
			(3)
1. (d) **Way 3**	$\dfrac{0.5 - x}{"3.8333..."} = \dfrac{x}{"4.1666..."}$ $x = \dfrac{(0.5)("4.1666...")}{"3.8333..." + "4.1666..."} = 0.260416...$ $\Rightarrow \beta = 3.5 - 0.260416...$	Finds x using a correct method of similar triangles and applies "3.5 – their x"	M1 dM1
	$\left\{\beta = 3.239583... \text{ or } 3\dfrac{23}{96} \text{ or } \dfrac{311}{96}\right\} \Rightarrow \beta = 3.24 \ \ (2\,\text{dp})$	awrt 3.24	A1
			(3)

		Question 1 Notes
1. (b)	**Note**	Give full marks in part (b) for correct differentiation in (a) followed by the correct answer in (b) with no working.
	M1	This mark can be implied by applying at least one correct ***value*** of either $\text{f}(0.5)$ or their $\text{f}'(0.5)$ (where $\text{f}'(0.5)$ is found using their $\text{f}'(x)$) to 1 significant figure in $0.5 - \dfrac{\text{f}(0.5)}{\text{f}'(0.5)}$. So ***just writing*** $0.5 - \dfrac{\text{f}(0.5)}{\text{f}'(0.5)}$ with an incorrect ft answer on their $\text{f}'(0.5)$ scores B0M0A0.
	Note	Give B1M1A0 for a correct $\text{f}'(x)$ in (a) followed by only $\alpha \simeq 0.5 - \dfrac{\text{f}(0.5)}{\text{f}'(0.5)} = \dfrac{29}{51}$ in (b)
	Note	**Differentiating INCORRECTLY to give** $\text{f}'(x) = 8x - 4x^2 + 14x^{-2}$ leads to $\alpha \simeq 0.5 - \dfrac{-\frac{7}{6}}{59} = \dfrac{92}{177} = 0.5197740113... = 0.520$ (3 dp) **This response should be given B1 M1 A0**
	Note	**Differentiating INCORRECTLY to give** $\text{f}'(x) = 8x - 4x^2 + 14x^{-2}$ and $\alpha \simeq 0.5 - \dfrac{\text{f}(0.5)}{\text{f}'(0.5)} = 0.520$ or truncated 0.52 or 0.519 or awrt 0.520 is B1 M1 A0
(c)	**Note**	**Way 1: correct solution only** Required to state **both** values for $\text{f}(3)$ and $\text{f}(3.5)$ correct awrt (or truncated) to 1sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $\text{f}(3) \times \text{f}(3.5) < 0$ **or** $\text{f}(3) > 0 > \text{f}(3.5)$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a conclusion, e.g. $\{x \text{ or}\}\ \beta \in [3, 3.5]$ or $\{x \text{ or}\}\ \beta \in (3, 3.5)$ or root lies between 3 and 3.5. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\beta \in [3, 3.5]$" **or** "change of sign, so root is between 3 and 3.5" **or** "change of sign, so root"

	Question 1 Notes Continued	
1. (c)	**Note**	**Way 2** The root of $f(x)=0$ is $3.27491258...$, so they can choose x_1 which is less than $3.27491258...$ and choose x_2 which is greater than $3.27491258...$ with both x_1 and x_2 lying in the interval [3, 3.5]. **M1:** Finds $f(x_1)$ and $f(x_2)$ with one of these values correct awrt (or truncated) to 1sf **A1:** Both values correct awrt (or truncated) to 1sf, sign change and conclusion.
	Note	**Helpful Table**

x	$f(x)$
3	3.83333333...
3.1	2.58963440...
3.2	1.17558333...
3.3	−0.41660606...
3.4	−2.19474509...
3.5	−4.16666666...

1. (d)	**Note**	Condone writing the symbol α in place of β in part (d)
	Note	$\frac{\beta-3}{3.5-\beta}=\left\lvert\frac{"3.833..."}{"-4.1666..."}\right\rvert$ is a valid method for the first M mark
	Note	Give 1st M1 for either $\frac{f(3)}{-f(3.5)}=\frac{\beta-3}{3.5-\beta}$ or $\frac{f(1.2)}{\lvert f(1.3)\rvert}=\frac{\beta-3}{3.5-\beta}$ or $\frac{\lvert f(3)\rvert}{\lvert f(3.5)\rvert}=\frac{\beta-3}{3.5-\beta}$
	Note	Give M1 dM1 A1 for the correct statement $\frac{3\lvert f(3.5)\rvert+3.5f(3)}{\lvert f(3.5)\rvert+f(3)}=3.24$
	Note	Give M0 dM0 for $\frac{3\lvert f(3.5)\rvert+3.5f(3)}{\lvert f(3.5)\rvert+f(3)}=\frac{3("-4.166...")+3.5("3.8333...")}{("-4.166...")+("3.8333...")}$
	Note	Give M1 dM1 for the correct statement $\beta=\frac{3.5+3k}{k+1}$, where k is defined as $k=\frac{\lvert f(3.5)\rvert}{f(3)}=\frac{4.1666...}{3.8333...}=1.086957...$
	Note	Give M1 dM1 for the correct statement $\beta=\frac{3+3.5c}{c+1}$, where c is defined as $c=\frac{f(3)}{\lvert f(3.5)\rvert}=\frac{3.8333...}{4.1666...}=0.92$
	Note	$\frac{\beta-3}{3.5-\beta}=\frac{"3.8333..."}{"4.1666..."}\Rightarrow\beta=3.24$ with no intermediate working is M1 dM1 A1
	Note	$\frac{\beta-3}{3.8333...}=\frac{3.5-\beta}{-4.1666...}\Rightarrow\beta=-2.75$ is M0 dM0 A0
	Note	$\frac{\beta-3}{-3.8333...}=\frac{3.5-\beta}{-4.1666...}\Rightarrow\beta=3.24$ is M1 dM1 A1
	Note	$\frac{\beta-3}{3.5-\beta}=\frac{"4.1666..."}{"3.8333..."}\Rightarrow\beta=3.260416...$ is M0 dM0 A0

Question Number	Scheme		Notes	Marks
1. (d) **Way 4**	• $y-\frac{23}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3) \Rightarrow 0-\frac{23}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3)$ • $y--\frac{25}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3.5) \Rightarrow 0--\frac{25}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3.5)$		Complete method of finding a line joining the points $(3, \mathrm{f}(3))$, $(3.5, \mathrm{f}(3.5))$ followed by setting $y=0$	M1
	$\Rightarrow x=\dots$ or $\beta=\dots$	**dependent on the previous M mark** Rearranges to give $x=\dots$ or $\beta=\dots$		dM1
	$\left\{x \text{ or } \beta=3.239583\dots \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96}\right\} \Rightarrow \beta=3.24 \ \ (2\,\text{dp})$		awrt 3.24	A1
				(3)

Question Number	Scheme	Notes	Marks
2.	$\mathbf{M} = \begin{pmatrix} k-12 & 3 \\ 4 & k \end{pmatrix}$, where k is a real constant		
	$\left\{\det(\mathbf{M}) = (k-12)k - 4(3) \text{ and area ratio } = \frac{320}{20} = 16\right\}$		
	$20(k(k-12)-4(3)) = 320$ or $20(k(k-12)-4(3)) = -320$	$20(\text{applied } \det(\mathbf{M})) = \pm 320$, o.e. **Note:** Allow $320(\text{applied } \det(\mathbf{M})) = \pm 20$, o.e.	M1
	or $k(k-12)-4(3) = \frac{320}{20}$ or $k(k-12)-4(3) = -\frac{320}{20}$	At least one correct equation in k that can be simplified or un-simplified	A1
	$k^2 - 12k - 28 = 0$, $k^2 - 12k + 4 = 0$ $\{20k^2 - 240k - 560 = 0,\ 20k^2 - 240k + 80 = 0\}$		
	$(k-14)(k+2) = 0$ **or** $(k-6)^2 - 36 + 4 = 0$ to give $k = \ldots$	**dependent on the previous M mark** At least one correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $k = \ldots$	dM1
	$k = 14, -2, 6+4\sqrt{2}, 6-4\sqrt{2}$	At least two of either $k = 14$, $k = -2$, $k = 6+4\sqrt{2}$ or $k = 6-4\sqrt{2}$	A1
		All four correct values of k	A1
			(5)
			5

		Question 2 Notes
2.	**Note**	Allow 1st M1 for any of • $320(k(k-12)-4(3)) = 20$ • $k(k-12)-4(3) = \frac{20}{320}$ • $320(k(k-12)-4(3)) = -20$ • $k(k-12)-4(3) = -\frac{20}{320}$ which can be simplified or un-simplified.
	Note	Allow 1st M1 for any of • $20(k(k-12)+4(3)) = 320$ • $20(k(k-12)+4(3)) = -320$ • $320(k(k-12)+4(3)) = 20$ • $320(k(k-12)+4(3)) = 20$ or equivalent, which can be simplified or un-simplified.
	Note	Give 1st M0 for any of • $(k(k-12)-4(3)) = (20)(320)$ • $(k(k-12)+4(3)) = (20)(320)$
	Note	Give dM1 for using a calculator to write down at least one correct root for their 3TQ
	Note	For the 1st A1 mark • condone truncated 11.6 or awrt 11.7 in place of $k = 6+4\sqrt{2}$ • condone awrt 0.34 in place of $k = 6-4\sqrt{2}$
	Note	Allow $k = 6+\sqrt{32}$ instead of $k = 6+4\sqrt{2}$ and/or $k = 6-\sqrt{32}$ instead of $k = 6-4\sqrt{2}$ for any of the final two accuracy marks.
	Note	Allow final A1 (isw) for $k = 14, -2, 6+4\sqrt{2}$, awrt 11.6, $6-4\sqrt{2}$, awrt 0.34
	Note	Give 2nd A0 (i.e. the penultimate mark) for finding only one correct value for k as a result of rejecting (or ignoring) correct values for k
	Note	Give final A0 if any of $k = 14, -2, 6+4\sqrt{2}, 6-4\sqrt{2}$ are rejected
	Note	Give final A0 for extra solutions in addition to $k = 14, -2, 6+4\sqrt{2}, 6-4\sqrt{2}$

	Question 2 Notes Continued	
2.	**Note**	$320(k(k-12)-4(3))=20$ leads to $16k^2-192k-193=0$ and $k=12.9327..., -0.9327...$ $320(k(k-12)-4(3))=-20$ leads to $16k^2-192k-191=0$ and $k=12.9236..., -0.9236...$
	Note	$20(k(k-12)+4(3))=320$ leads to $k^2-12k-4=0$ and $k=12.3245..., -0.3245...$ $20(k(k-12)+4(3))=-320$ leads to $k^2-12k+28=0$ and $k=8.8284..., 3.1715...$
	Note	$320(k(k-12)+4(3))=20$ leads to $16k^2-192k+191=0$ and $k=10.9053..., 1.0946...$ $320(k(k-12)+4(3))=-20$ leads to $16k^2-192k+193=0$ and $k=10.8925..., 1.1074...$

Question Number	Scheme	Notes	Marks
3.	(i) $z^* - 3z = \dfrac{5\text{i}}{3-\text{i}}$; (ii) $w = -4+5\text{i}$, (b) $\arg(w+k) = \dfrac{\pi}{2}$, (c) $\lvert w + c\text{i}\rvert = 4\sqrt{5}$		
(i) **Way 1**	$\left\{z^* - 3z = \right\}\ (a - \text{i}b) - 3(a + \text{i}b)$	Left hand side $= (a-\text{i}b) - 3(a+\text{i}b)$ Can be implied by e.g. $-2a - 4b\text{i}$ **Note:** Can be seen (or implied) anywhere in their solution	B1
	$\ldots\ldots\ldots = \dfrac{5\text{i}}{(3-\text{i})}\dfrac{(3+\text{i})}{(3+\text{i})}$	Multiplies numerator **and** denominator of the right-hand side by $3+\text{i}$ or $-3-\text{i}$	M1
	$\ldots\ldots\ldots = \dfrac{15\text{i}-5}{10}$	Applies $\text{i}^2 = -1$ to give right-hand side $= \dfrac{15\text{i}-5}{10}$ or equivalent	A1
	So, $-2a - 4b\text{i} = -\dfrac{1}{2} + \dfrac{3}{2}\text{i}$	**dependent on the previous B and M marks** Equates either real parts or imaginary parts to give at least one of $a = \ldots$ or $b = \ldots$	ddM1
	$\Rightarrow a = \dfrac{1}{4}, b = -\dfrac{3}{8}\ \Rightarrow\ z = \dfrac{1}{4} - \dfrac{3}{8}\text{i}$	$z = \dfrac{1}{4} - \dfrac{3}{8}\text{i}$ **or** $z = 0.25 - 0.375\text{i}$ **or** $z = \dfrac{1}{4} + \left(-\dfrac{3}{8}\text{i}\right)$	A1
			(5)
(i) **Way 2**	$\left\{z^* - 3z = \right\}\ (a - \text{i}b) - 3(a + \text{i}b)$	Left hand side $= (a-\text{i}b) - 3(a+\text{i}b)$ Can be implied by e.g. $-2a - 4b\text{i}$ **Note:** Can be seen (or implied) anywhere in their solution	B1
	$(-2a - 4b\text{i})(3-\text{i}) = \ldots\ldots\ldots$	Multiplies their $(-2a - 4b\text{i})$ by $(3-\text{i})$	M1
	$-6a + 2a\text{i} - 12b\text{i} - 4b = \ldots\ldots\ldots$	Applies $\text{i}^2 = -1$ to give left-hand side $= -6a + 2a\text{i} - 12b\text{i} - 4b$ or equivalent	A1
	So, $(-6a - 4b) + (2a - 12b)\text{i} = 5\text{i}$ gives $-6a - 4b = 0,\ 2a - 12b = 5$	**dependent on the previous B and M marks** Equates **both** real parts and imaginary parts and solves simultaneously to give at least one of $a = \ldots$ or $b = \ldots$	ddM1
	$\Rightarrow a = \dfrac{1}{4}, b = -\dfrac{3}{8}\ \Rightarrow\ z = \dfrac{1}{4} - \dfrac{3}{8}\text{i}$	$z = \dfrac{1}{4} - \dfrac{3}{8}\text{i}$ **or** $z = 0.25 - 0.375\text{i}$ **or** $z = \dfrac{60}{240} - \dfrac{15}{40}\text{i}$	A1
			(5)
(ii)(a)	e.g. $\arg w = \pi - \tan^{-1}\left(\frac{5}{4}\right)$ or $= \frac{\pi}{2} + \tan^{-1}\left(\frac{4}{5}\right)$ or $= -\pi - \tan^{-1}\left(\frac{5}{4}\right)$	Uses trigonometry to find an expression for $\arg w$ so that $\arg w$ is in the range $(1.58\ldots, 3.14\ldots)$ or $(90°, 180°)$ or $(-4.71\ldots, -3.15\ldots)$ or $(-270°, -180°)$	M1
	$\arg w = \pi - 0.896055\ldots = 2.245537\ldots\ \{= 2.25\ (2\text{ dp})\}$ or $\arg w = -\pi - 0.896055\ldots = -4.037648\ldots\ \{= -4.04\ (2\text{ dp})\}$	awrt 2.25 **or** awrt -4.04 **or** awrt 8.53 **or** awrt -10.32	A1
	{**Note:** $\arg w = 128.6598\ldots°$ or $-231.3401\ldots°$ is M1 A0}		**(2)**
(b)	$\{\arg(-4+5\text{i}+k) = \frac{\pi}{2} \Rightarrow -4 + k = 0 \Rightarrow\}\ k = 4$	$k = 4$	B1
			(1)
(c)	$\lvert -4 + 5\text{i} + c\text{i}\rvert = 4\sqrt{5}$ $\Rightarrow \lvert -4 + (5+c)\text{i}\rvert = 4\sqrt{5}$	Squares and adds the real and imaginary parts of $w + c\text{i}$ and sets equal to either $(4\sqrt{5})^2$ or $4\sqrt{5}$	M1
	$\Rightarrow (-4)^2 + (5+c)^2 = (4\sqrt{5})^2$	$(-4)^2 + (5+c)^2 = (4\sqrt{5})^2$ o.e. Allow the equivalent result $\sqrt{(-4)^2 + (5+c)^2} = 4\sqrt{5}$	A1
	$16 + (5+c)^2 = 80 \Rightarrow (5+c)^2 = 64 \Rightarrow c = \ldots$ **or** $16 + (5+c)^2 = 80 \Rightarrow c^2 + 10c - 39 = 0$ $\Rightarrow (c+13)(c-3) = 0 \Rightarrow c = \ldots$	**dependent on the previous M mark** Solves their quadratic in c to give $c = \ldots$	dM1
	$c = -13, 3$	$c = -13, 3$	A1
			(4)
			12

	Question 3 Notes	
3. (i)	**Note**	Allow alternative ways of defining z. E.g. $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$ with $x \equiv a$ and $y \equiv b$
	Note	Give final A0 for defining $z = a + \mathrm{i}b$, finding $a = \frac{1}{4}, b = -\frac{3}{8}$ but not stating $z = \frac{1}{4} - \frac{3}{8}\mathrm{i}$
	Note	**<u>Alternative:</u>** Some may define $z = x - \mathrm{i}y$ and $z^* = x + \mathrm{i}y$ This gives $\{z^* - 3z = \}$ $(x + \mathrm{i}y) - 3(x - \mathrm{i}y) = -2x + 4y\mathrm{i}$ So, $-2x + 4y\mathrm{i} = -\frac{1}{2} + \frac{3}{2}\mathrm{i} \Rightarrow x = \frac{1}{4}, y = \frac{3}{8} \Rightarrow z = \frac{1}{4} - \frac{3}{8}\mathrm{i}$
(ii) (a)	**Note**	Allow M1 (implied) for awrt 2.2, awrt -3.8, truncated -4.0, awrt $129°$, truncated $128°$ or awrt $-231°$
(ii) (c)	**Note**	$\lvert -4 + (5 + c)\mathrm{i} \rvert = 4\sqrt{5} \Rightarrow (-4)^2 - (5 + c)^2 = (4\sqrt{5})^2$ unless recovered is 1st M0
	Note	$\lvert -4 + (5 + c)\mathrm{i} \rvert = 4\sqrt{5} \Rightarrow -16 + (5 + c)^2 = (4\sqrt{5})^2$ unless recovered is 1st M0
	Note	$\lvert -4 + 5\mathrm{i} + c\mathrm{i} \rvert = 4\sqrt{5} \Rightarrow (-4)^2 + (5)^2 + c^2 = (4\sqrt{5})^2$ unless recovered is 1st M0
	Note	If a 3TQ is formed in c then a correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ is required to give $c = ...$
	Note	Give dM1 for using a calculator to write down at least one correct root for their 3TQ
	Note	Having achieved a correct $16 + 25 + 10c + c^2 = 80$ give final dM1 A1 marks for writing down $c = -13, 3$ from no working.
	Note	Give final A0 for either • $c = -13, 3 \Rightarrow c = 3$ • $c = -13, 3 \Rightarrow c = -13$ • $c = 3, c = -13$ (reject) • $c = 3$ (reject), $c = -13$

Question Number	Scheme	Notes	Marks
4. (a) **Way 1**	$\sum_{r=1}^{3k}(4r+1) = 4.\frac{1}{2}(3k)(3k+1)+3k$	Either $\sum_{r=1}^{3k}4r \to 4.\frac{1}{2}(3k)(3k+1)$ or $\sum_{r=1}^{3k}1 \to 3k$	M1
		Correct expression, simplified or un-simplified	A1
	$= 6k(3k+1)+3k = 18k^2+9k$		
	$= 9k(2k+1) \quad \{p=9\}$	Obtains $9k(2k+1)$ with no errors	A1 **cso**
			(3)
(a) **Way 2**	$\sum_{r=1}^{k}(4r+1) = 4.\frac{1}{2}(k)(k+1)+k$	**Both** $\sum_{r=1}^{k}4r \to 4.\frac{1}{2}(k)(k+1)$ **and** $\sum_{r=1}^{k}1 \to k$	M1
	$= 2k(k+1)+k = 2k^2+3k$		
	$\sum_{r=1}^{3k}(4r+1) = 2(3k)(3k+1)+3k = 2(3k)^2+3(3k)$	Correct expression, simplified or un-simplified	A1
	$=18k^2+9k$		
	$= 9k(2k+1) \quad \{p=9\}$	Obtains $9k(2k+1)$ with no errors	A1 **cso**
			(3)
(b) **Way 1**	$\sum_{r=1}^{k}2r^2 = \sum_{r=1}^{3k}(4r+1)$		
	$2.\frac{1}{6}k(k+1)(2k+1)=9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9"$k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in *k* only	M1
	$\frac{1}{3}(k+1)=9 \Rightarrow k=26$	**dependent on the previous M mark** Cancels out two terms or factorises out two terms and solves a linear equation in *k* to give $k=...$	dM1
		$k=26$ **only**	A1
			(3)
(b) **Way 2**	$2.\frac{1}{6}k(k+1)(2k+1)=9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9"$k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in *k* only	M1
	$2k^3+3k^2+k=54k^2+27k$ $2k^3-51k^2-26k=0$ $k(2k^2-51k-26)=0$ $(2k+1)(k-26)=0 \Rightarrow k=26$	**dependent on the previous M mark** Cancels out or factorises *k* and a correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $k=...$	dM1
		$k=26$ **only**	A1
			(3)
(b) **Way 3**	$2.\frac{1}{6}k(k+1)(2k+1)=9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9"$k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in *k* only	M1
	$k(k+1)(2k+1)=27k(2k+1)$ $k(k+1)-27k=0 \Rightarrow k^2-26k=0$ $k(k-26) \Rightarrow k=26$	**dependent on the previous M mark** Cancels out two terms or factorises out two terms and solves a linear equation in *k* to give $k=...$	dM1
		$k=26$ **only**	A1
			(3)
			6

	Question 4 Notes	
4. (a)	**Note**	Give M1A1 for $\sum_{r=1}^{3n}(4r+1) = 4.\frac{1}{2}(3n)(3n+1)+3n$
	Note	Give M1A1A0 for $\sum_{r=1}^{3n}(4r+1) = 4.\frac{1}{2}(3n)(3n+1)+3n = 18n^2+9n = 9n(2n+1)$ without reference to $\sum_{r=1}^{3k}(4r+1) = 9k(2k+1)$
	Note	Give M1A1A1 for $\sum_{r=1}^{3n}(4r+1) = 4.\frac{1}{2}(3n)(3n+1)+3n = 18n^2+9n = 9n(2n+1) \Rightarrow \sum_{r=1}^{3k}(4r+1) = 9k(2k+1)$
	Note	**Way 2:** Give M1 for $\sum_{r=1}^{n}(4r+1) = 4.\frac{1}{2}(n)(n+1)+n$
	Note	Give final A0 for cancelling down their final answer $9k(2k+1)$ in part (a) E.g. $\sum_{r=1}^{3k}(4r+1) = 4.\frac{1}{2}(3k)(3k+1)+3k = 18k^2+9k = 9k(2k+1) = k(2k+1)$ gets M1 A1 A0
	Note	Give M0 A0 A0 for writing e.g. $k=1 \Rightarrow \sum_{1}^{3(1)}(4r+1) = p(1)((2(1)+1)) \Rightarrow 5+9+13=3p \Rightarrow p=9$ with no evidence of applying $\sum_{r=1}^{3k}4r \to 4.\frac{1}{2}(3k)(3k+1)$ or $\sum_{r=1}^{3k}1 \to 3k$
	Note	You can give M1 1st A1 marks in part (a) for work recovered for $\sum_{r=1}^{3k}(4r+1) = 4.\frac{1}{2}(3k)(3k+1)+3k$ in part (b)
(b)	**Note**	Condone giving 1st M1 for setting $\lambda k(k+1)(2k+1)$ equal to $"9"k(k+1)$ {slip}
	Note	Give A0 for giving more than one value of k as their final answer.
	Note	Where applicable, for A1, • $k=0$ and/or $k=-\frac{1}{2}$ needs to be rejected leaving $k=26$ as their final answer. • $k=26$ needs to be indicated as their final answer.
	Note	**Way 2:** Using fractions gives • $\frac{2}{3}k^3+k^2+\frac{1}{3}k=18k^2+9k \Rightarrow \frac{2}{3}k^3-17k^2-\frac{26}{3}k=0 \Rightarrow \frac{2}{3}k^2-17k-\frac{26}{3}=0$ $\Rightarrow k = \frac{17\pm\sqrt{(-17)^2-4(\frac{2}{3})(-\frac{26}{3})}}{2(\frac{2}{3})} = \frac{17\pm\sqrt{\frac{2809}{9}}}{\frac{4}{3}} = \frac{17\pm\frac{53}{3}}{\frac{4}{3}} \Rightarrow k=26$
	Note	**Way 3:** E.g. Give dM0 for $k^2+k-27k=0$ leading directly to $k=\frac{-1\pm\sqrt{(1)^2-4(1)(-27)}}{2(1)}$

Question Number	Scheme	Notes	Marks
5.	$\mathbf{A}=\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}2\sqrt{3} & -7\\ -4 & 5\sqrt{3}\end{pmatrix}$		
(a)	Rotation	Rotation or rotate (condone turn)	B1
	60 degrees {anti-clockwise}	60 degrees or $\frac{\pi}{3}$ or 300 degrees clockwise or $\frac{5\pi}{3}$ clockwise	B1 o.e.
	about $(0, 0)$	**This mark is dependent on at least one of the previous B marks being given.** about $(0, 0)$ or about O or about the origin	dB1
	Note: Give 2nd B0 for 60 degrees clockwise o.e.		**(3)**
(b)	$\{\mathbf{A}^6=\}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	Correct matrix	B1
			(1)
(c) **Way 1**	$\mathbf{B}^{-1}=\frac{1}{2}\begin{pmatrix}5\sqrt{3} & 7\\ 4 & 2\sqrt{3}\end{pmatrix}$	Correct matrix for $\mathbf{B}^{-1}$, which can be simplified or un-simplified	B1
	$\{\mathbf{C}=\mathbf{B}^{-1}\mathbf{A}\}=\frac{1}{2}\begin{pmatrix}5\sqrt{3} & 7\\ 4 & 2\sqrt{3}\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}=\dots$	Applies (their $\mathbf{B}^{-1}$)$\mathbf{A}$, where (their $\mathbf{B}^{-1}$) $\neq \mathbf{B}$, and finds at least one element (or at least one element calculation) of their matrix $\mathbf{C}$ **Note:** Allow one slip in copying down $\mathbf{A}$	M1
	$=\frac{1}{2}\begin{pmatrix}6\sqrt{3} & -4\\ 5 & -\sqrt{3}\end{pmatrix}$ **or** $=\begin{pmatrix}3\sqrt{3} & -2\\ \frac{5}{2} & -\frac{1}{2}\sqrt{3}\end{pmatrix}$	**dependent on the previous B1M1 marks** At least 2 elements in $\mathbf{C}$ are correct	A1
		All elements in $\mathbf{C}$ are correct	A1
			(4)
(c) **Way 2**	$\{\mathbf{BC}=\mathbf{A}\Rightarrow\}$ $\begin{pmatrix}2\sqrt{3} & -7\\ -4 & 5\sqrt{3}\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	Correct statement using 2×2 matrices. All 3 matrices must contain four elements. **Can be implied by the 4 correct equations that are below.**	B1
	$2\sqrt{3}a-7c=\frac{1}{2}$, $2\sqrt{3}b-7d=-\frac{\sqrt{3}}{2}$ $-4a+5\sqrt{3}c=\frac{\sqrt{3}}{2}$, $-4b+5\sqrt{3}d=\frac{1}{2}$ **and** finds at least one of either a, b, c or d	Applies $\mathbf{BC}=\mathbf{A}$ and attempts to solve simultaneous equations in a and c **or** b and d **and** finds at least one of either a, b, c or d	M1
	$=\frac{1}{2}\begin{pmatrix}6\sqrt{3} & -4\\ 5 & -\sqrt{3}\end{pmatrix}$ **or** $=\begin{pmatrix}3\sqrt{3} & -2\\ \frac{5}{2} & -\frac{1}{2}\sqrt{3}\end{pmatrix}$ **or** $a=3\sqrt{3}$, $b=-2$, $c=\frac{5}{2}$, $d=-\frac{1}{2}\sqrt{3}$	**dependent on the previous B1M1 marks** At least 2 elements in $\mathbf{C}$ are correct	A1
		All elements in $\mathbf{C}$ are correct	A1
			(4)
			8

		Question 5 Notes
5. (a)	**Note**	Writing “60 degrees” by itself implies by convention “60 degrees anti-clockwise”. So, • “Rotation 60 degrees about *O*” is B1 B1 B1 • “Rotation 60 degrees clockwise about *O*” is B1 B0 B1
	Note	Writing down “60 degrees anti-clockwise about *O*” with no reference to “rotation” or “turn” is B0 B1 B1
	Note	“original point” is not acceptable in place of the word “origin”.
	Note	Give B0 B0 B0 for a combination of 2 or more transformations.
(b)	**Note**	Give B0 for writing down $\mathbf{I}$ without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
	Note	Allow B1 for writing down $\mathbf{I}_2$ without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(c)	**Note**	Allow B1 for $\dfrac{1}{(2\sqrt{3})(5\sqrt{3})-(-7)(-4)}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$ or $\dfrac{1}{30-28}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$
	Note	Allow B1 for $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}\dfrac{1}{(2\sqrt{3})(5\sqrt{3})-(-7)(-4)}$ or $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}\dfrac{1}{30-28}$
	Note	You can ignore previous working prior to their finding $\mathbf{B}^{-1}\mathbf{A}$ (i.e. you can ignore an incorrect statement such as $\mathbf{A} = \mathbf{CB}$)

Question Number	Scheme	Notes	Marks
6.	$2x^2 + x + 4 = 0$ has roots α, β		
(a)	$\alpha + \beta = -\frac{1}{2},\ \alpha\beta = 2$	**Both** $\alpha + \beta = -\frac{1}{2}$ **and** $\alpha\beta = 2$	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \ldots$	**Use** of a **correct** identity for $\alpha^2 + \beta^2$ May be implied by their work	M1
	$= \left(-\frac{1}{2}\right)^2 - 2(2) = -\frac{15}{4}$	$-\frac{15}{4}$ or -3.75 or $-3\frac{3}{4}$ **from correct working**	A1 **cso**
(ii)	$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \ldots$ **or** $= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta) = \ldots$ **or** $= (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) = \ldots$	**Use** of a **correct** identity for $\alpha^3 + \beta^3$ May be implied by their work	M1
	$= \left(-\frac{1}{2}\right)^3 - 3(2)\left(-\frac{1}{2}\right) = \frac{23}{8}$ **or** $= \left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right)^2 - 3(2)\right) = \frac{23}{8}$ **or** $= \left(-\frac{1}{2}\right)\left(-\frac{15}{4} - 2\right) = \frac{23}{8}$	$\frac{23}{8}$ or 2.875 or $2\frac{7}{8}$ **from correct working**	A1 **cso**
			(4)
(c)	$\sum = \alpha^3 + \frac{1}{\beta} + \beta^3 + \frac{1}{\alpha}$ $= \alpha^3 + \beta^3 + \frac{\alpha+\beta}{\alpha\beta}$ $\sum = \frac{\alpha^3\beta+1}{\beta} + \frac{\alpha\beta^3+1}{\alpha}$ $= \frac{\alpha\beta(\alpha^3+\beta^3)+(\alpha+\beta)}{\alpha\beta}$ **e.g.** $= \frac{23}{8} + \frac{\left(-\frac{1}{2}\right)}{2} = \frac{21}{8}$ **or** $= \frac{2\left(\frac{23}{8}\right)+\left(-\frac{1}{2}\right)}{2}$	Simplifies $\frac{1}{\beta} + \frac{1}{\alpha}$ to give $\frac{\alpha+\beta}{\alpha\beta}$ **(can be implied)** **and** uses at least two of their $\alpha^3+\beta^3$, $\alpha+\beta$ or $\alpha\beta$ in an attempt to find a **numerical value** for the sum of $\left(\alpha^3 + \frac{1}{\beta}\right)$ and $\left(\beta^3 + \frac{1}{\alpha}\right)$	M1
	$\Pi = \left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ $= (\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta}$ $\Pi = \left(\frac{\alpha^3\beta+1}{\beta}\right)\left(\frac{\alpha\beta^3+1}{\alpha}\right)$ $= \frac{\alpha^4\beta^4 + \alpha^3\beta + \alpha\beta^3 + 1}{\alpha\beta}$ $= \frac{(\alpha\beta)^4 + \alpha\beta(\alpha^2+\beta^2)+1}{\alpha\beta}$ **e.g.** $= (2)^3 + \left(-\frac{15}{4}\right) + \frac{1}{2} = \frac{19}{4}$ **or** $= \frac{(2)^4 + 2\left(-\frac{15}{4}\right)+1}{2}$	Expands $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give 4 terms and uses at least one of their $\alpha\beta$ or $\alpha^2+\beta^2$ in an attempt to find a **numerical value** for the product	M1
	$x^2 - \frac{21}{8}x + \frac{19}{4} = 0$	**Applies** $x^2 - (\text{sum})x + \text{product}$ (can be implied), for their numerical values of the sum and product. **Note:** " $=0$ " is not required for this mark	M1
	$8x^2 - 21x + 38 = 0$	***Any integer multiple*** of $8x^2 - 21x + 38 = 0$, including the " $=0$ "	A1 **cso**
			(4)
			9

	Question 6 Notes	
6. (b)(i)	**Note**	Writing a correct $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ without attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^2-2\alpha\beta$ is M0
	Note	An incorrect $\alpha+\beta=\frac{1}{2}, \alpha\beta=2$ from (a) leading to $\alpha^2+\beta^2=\left(\frac{1}{2}\right)^2-2(2)=-\frac{15}{4}$ is M1 A0
	Note	Give M1 A1 for writing down $\alpha^2+\beta^2=-\frac{15}{4}$, if they give $\alpha+\beta=-\frac{1}{2}, \alpha\beta=2$ in (a)
(b)(ii)	**Note**	Allow M1 A1 for $\alpha^3+\beta^3=(\alpha^2+\beta^2)(\alpha+\beta)-\alpha\beta(\alpha+\beta)=\left(-\frac{15}{4}\right)\left(-\frac{1}{2}\right)-(2)\left(-\frac{1}{2}\right)=\frac{23}{8}$
	Note	E.g. writing a correct $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ ***without*** attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ is M0
	Note	E.g. writing a correct $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ ***without*** attempting to substitute at least one of either their $\alpha+\beta$, their $\alpha^2+\beta^2$ or their $\alpha\beta$ into $(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ is M0
	Note	Give M1 A1 for writing down $\alpha^3+\beta^3=\frac{23}{8}$, if they give $\alpha+\beta=-\frac{1}{2}, \alpha\beta=2$ in (a)
(b)	**ALT**	They can use the equation $2x^2+x+4=0$ with roots α, β to give $\left.\begin{cases}2\alpha^2+\alpha+4=0\\2\beta^2+\beta+4=0\end{cases}\right\} \Rightarrow 2\alpha^2+2\beta^2+\alpha+\beta+8=0$ So, $\alpha^2+\beta^2=\frac{1}{2}(-(\alpha+\beta)-8))=\frac{1}{2}\left(--\frac{1}{2}-8\right)=\frac{1}{2}\left(\frac{1}{2}-8\right)=-\frac{15}{4}$ $\left.\begin{cases}2\alpha^3+\alpha^2+4\alpha=0\\2\beta^3+\beta^2+4\beta=0\end{cases}\right\} \Rightarrow 2\alpha^3+2\beta^3+\alpha^2+\beta^2+4\alpha+4\beta=0$ So, $\alpha^3+\beta^3=\frac{1}{2}(-(\alpha^2+\beta^2)-4(\alpha+\beta)))=\frac{1}{2}\left(--\frac{15}{4}-4\left(-\frac{1}{2}\right)\right)=\frac{1}{2}\left(\frac{15}{4}+2\right)=\frac{23}{8}$
(a)	**Note**	Give B0 for $\alpha, \beta=\frac{-1+\sqrt{31}\,\mathrm{i}}{4}, \frac{-1-\sqrt{31}\,\mathrm{i}}{4}$ and then stating that $\alpha+\beta=-\frac{1}{2},\ \alpha\beta=2$
	Note	Give B0 for $\alpha+\beta=\frac{-1+\sqrt{31}\,\mathrm{i}}{4}+\frac{-1-\sqrt{31}\,\mathrm{i}}{4}=-\frac{1}{2}$ and $\alpha\beta=\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)=2$
(b)(i)	**Note**	Give M0 A0 for $\alpha^2+\beta^2=\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)^2+\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)^2=-\frac{15}{4}$
(b)(ii)	**Note**	Give M0 A0 for $\alpha^3+\beta^3=\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)^3+\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)^3=\frac{23}{8}$
(b)	**Note**	Using $\frac{-1+\sqrt{31}\,\mathrm{i}}{4}, \frac{-1-\sqrt{31}\,\mathrm{i}}{4}$ to find $\alpha+\beta=-\frac{1}{2},\ \alpha\beta=2$ followed by • $\alpha^2+\beta^2==\left(-\frac{1}{2}\right)^2-2(2)=-\frac{15}{4}$, scores M1 A0 in (b)(i) • e.g. $\alpha^3+\beta^3=\left(-\frac{1}{2}\right)^3-3(2)\left(-\frac{1}{2}\right)=\frac{23}{8}$, scores M1 A1 in (b)(ii)
(c)	**Note**	A correct method leading to $p=8, q=-21, r=38$ without writing a final answer of $8x^2-21x+38=0$ is final M1 A0

	Question 6 Notes Continued	
6. (c)	**Note**	Using $\frac{-1+\sqrt{31}\,\mathrm{i}}{4}, \frac{-1-\sqrt{31}\,\mathrm{i}}{4}$ explicitly to find the sum and product of $\alpha^3 + \frac{1}{\beta}$ and $\beta^3 + \frac{1}{\alpha}$ • i.e. sum $= \left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)^3 + \frac{1}{\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)} + \left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)^3 + \frac{1}{\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)} = \frac{21}{8}$ • ie. product $= \left(\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)^3 + \frac{1}{\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)}\right)\left(\left(\frac{-1-\sqrt{31}\,\mathrm{i}}{4}\right)^3 + \frac{1}{\left(\frac{-1+\sqrt{31}\,\mathrm{i}}{4}\right)}\right) = \frac{19}{4}$ • $x^2 - \frac{21}{8}x + \frac{19}{4} = 0 \Rightarrow 8x^2 - 21x + 38 = 0$ scores M0 M0 M1 A0 in part (c).
	Note	Using $\frac{-1+\sqrt{31}\,\mathrm{i}}{4}, \frac{-1-\sqrt{31}\,\mathrm{i}}{4}$ to find $\alpha + \beta = -\frac{1}{2},\ \alpha\beta = 2$ **and applying** $\alpha + \beta = -\frac{1}{2},\ \alpha\beta = 2$ can potentially score full marks in (c). E.g. • sum $= \alpha^3 + \beta^3 + \frac{\alpha+\beta}{\alpha\beta} = \frac{23}{8} + \frac{\left(-\frac{1}{2}\right)}{2} = \frac{21}{8}$ • product $= (\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta} = (2)^3 + \left(-\frac{15}{4}\right) + \frac{1}{2} = \frac{19}{4}$ • $x^2 - \frac{21}{8}x + \frac{19}{4} = 0 \Rightarrow 8x^2 - 21x + 38 = 0$
	Note	Give final M0 for $\sum = \frac{21}{8}$, $\Pi = \frac{19}{4}$ leading to $x^2 - \frac{21}{8} + \frac{19}{4} = 0$ (without recovery)
	Note	Allow final M1 for $\sum = \frac{21}{8}$, $\Pi = \frac{19}{4}$ with $x^2 - (\text{sum})x + (\text{product})$ leading to $x^2 - \frac{21}{8} + \frac{19}{4} = 0$
	Note	An alternative method uses a correct $\left(x - \alpha^3 - \frac{1}{\beta}\right)\left(x - \beta^3 - \frac{1}{\alpha}\right) = 0$
	Note	Allow 1st M1 and/or 2nd M1 for using an incorrect $\left(x - \alpha^3 + \frac{1}{\beta}\right)\left(x - \beta^3 + \frac{1}{\alpha}\right) = 0$
	Note	Give final M0 for an incorrect $\left(x - \alpha^3 + \frac{1}{\beta}\right)\left(x - \beta^3 + \frac{1}{\alpha}\right) = 0$ unless recovered
	Note	When expanding $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta}$, some will write $\frac{\alpha+\beta}{\alpha\beta}$ in place of $\frac{1}{\alpha\beta}$ So, allow 2nd M1 for expanding $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{\alpha+\beta}{\alpha\beta}$ and using at least one of their $\alpha\beta$ or $\alpha^2 + \beta^2$ in an attempt to find a **numerical value** for the product.

Question Number	Scheme		Notes	Marks
7.	$\mathrm{f}(z) = z^4 - 6z^3 + az^2 - 44z + b$; a, b are real constants. $z = -1 - 3\mathrm{i}$ is given.			
(a)	$-1 + 3\mathrm{i}$		$-1 + 3\mathrm{i}$	B1
				(1)
(b)	$z^2 + 2z + 10$		Attempt to expand $(z \pm (-1-3\mathrm{i}))(z \pm "(-1+3\mathrm{i})")$ or any valid method ***to establish a quadratic factor*** e.g. $z = -1 \pm 3\mathrm{i} \Rightarrow z + 1 = \pm 3\mathrm{i} \Rightarrow z^2 + 2z + 1 = -9$ or sum of roots $= -2$, product of roots $= 10$ to give $z^2 \pm (\text{their sum})z \pm (\text{their product})$	M1
			$z^2 + 2z + 10$	A1
	$\{\mathrm{f}(z) =\}\ (z^2 + 2z + 10)(z^2 - 8z + 18)$		Attempts to find the other quadratic factor e.g. using long division to obtain $z^2 + kz + ...,\ k = \text{value} \neq 0$ e.g. factorising/equating coefficients to obtain $\mathrm{f}(z) = (z^2 + 2z + 10)(z^2 \pm kz \pm c)$, $k = \text{value} \neq 0$, c can be 0	M1
			$z^2 - 8z + 18$ seen in their working	A1
	$\{z^2 - 8z + 18 = 0 \Rightarrow\}$			
	• $z = \dfrac{--8 \pm \sqrt{(-8)^2 - 4(1)(18)}}{2(1)}$ • $(z-4)^2 - 16 + 18 = 0 \Rightarrow z = ...$		**dependent on only the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their 2nd quadratic factor	dM1
	$\{z =\}\ 4 \pm \sqrt{2}\,\mathrm{i}$		$4 + \sqrt{2}\,\mathrm{i}$ **and** $4 - \sqrt{2}\,\mathrm{i}$	A1
				(6)
				7
	Question 7 Notes			
7. (a)	**Note**	Give B1 for either $4 + \sqrt{2}\,\mathrm{i}$ or $4 - \sqrt{2}\,\mathrm{i}$		
(b)	**Note**	The values of the constants, i.e. $a = 12, b = 180$ do not have to be found explicitly.		
	Note	You can assume $x \equiv z$ for solutions in this part		
	Note	Give final dM1A1 for $z^2 - 8z + 18 = 0 \Rightarrow z = 4 + \sqrt{2}\,\mathrm{i},\ 4 - \sqrt{2}\,\mathrm{i}$ with no intermediate working.		
	Note	They must be solving a 3TQ $"A"z^2 + "B"z + "C"$ where A, B, C ***are all numerical values*** $\neq 0$ for the final dM1 mark.		
	Note	<u>**Special Case:**</u> If ***their 2nd quadratic*** factor $z^2 + "B"z + "C"$ **can** be factorised then give Special Case 3rd dM1 for correct factorisation leading to $z = ...$ Otherwise, give 3rd dM0 for applying a method of factorisation to solve their 3TQ.		
	Note	<u>**Reminder: Method mark for solving a 3TQ**</u> **Formula:** $Az^2 + Bz + C = 0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C) **Completing the Square:** $z^2 + Bz + C = 0 \Rightarrow \left(z \pm \dfrac{B}{2}\right)^2 \pm q \pm C = 0,\ q \neq 0$, leading to $z = ...$		
	Note:	<u>**Comparing coefficients:**</u> $\mathrm{f}(z) = (z^2 + 2z + 10)(z^2 + \alpha z + \beta) \equiv z^4 - 6z^3 + az^2 - 44z + b$ $z^3: \alpha + 2 = -6 \Rightarrow \alpha = -8$; $z: 2\beta + 10\alpha = -44 \Rightarrow 2\beta - 80 = -44 \Rightarrow \beta = 18$ yielding 2nd quadratic factor = $z^2 - 8z + 18$ Also, constant: $10\beta = b \Rightarrow b = 180$; $z^2: \beta + 2\alpha + 10 = a \Rightarrow a = 18 - 16 + 10 = 12$		

	Question 7 Notes Continued	
7. (b)	**Note:**	**Long division:** $$\begin{array}{r} z^2 - 8z + 18 \\ z^2+2z+10 \,\big|\, \overline{z^4 - 6z^3 + az^2 - 44z + b} \\ \underline{z^4 + 2z^3 + 10z^2} \\ -8z^3 + (a-10)z^2 - 44z \\ \underline{-8z^3 - 16z^2 - 80z} \\ (a+6)z^2 + 36z + b \\ \underline{18z^2 + 36z + 180} \\ 0 \end{array}$$ Also, note $a = 12,\, b = 180$
	Note	Ignore errors in long division for the 2nd A1 mark and/or the 3rd A1 mark.
	Note	Ignore errors in stating $a = 12,\, b = 180$ for the 2nd A1 mark and/or the 3rd A1 mark.
	Note	The solutions $4 \pm \sqrt{2}\,\mathrm{i}$ need to follow on from a correct $z^2 - 8z + 18$ in order to gain the final A mark.
	Note	Give final A0 for writing $\dfrac{8 \pm 2\sqrt{2}\,\mathrm{i}}{2}$ followed by either $4 \pm 2\sqrt{2}\,\mathrm{i}$ or $8 \pm \sqrt{2}\,\mathrm{i}$

Question Number	Scheme	Notes	Marks
8.	$\mathrm{f}(n) = 3^{4n-2} + 2^{6n-3}$ is divisible by 17		
Way 1	$\mathrm{f}(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$\mathrm{f}(1) = 17$ is the minimum	B1
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - (3^{4k-2} + 2^{6k-3})$	Attempts $\mathrm{f}(k+1) - \mathrm{f}(k)$	M1
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 80(3^{4k-2}) + 63(2^{6k-3})$		
	$= 80(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ **or** $= 63(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$80(3^{4k-2} + 2^{6k-3})$ or $80\mathrm{f}(k)$; $-17(2^{6k-3})$ / $63(3^{4k-2} + 2^{6k-3})$ or $63\mathrm{f}(k)$; $+17(3^{4k-2})$	A1; A1
	$\mathrm{f}(k+1) = 80(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3}) + \mathrm{f}(k)$ **or** $\mathrm{f}(k+1) = 63(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2}) + \mathrm{f}(k)$ **or** $\mathrm{f}(k+1) = 80\mathrm{f}(k) - 17(2^{6k-3}) + \mathrm{f}(k)$ **or** $\mathrm{f}(k+1) = 63\mathrm{f}(k) + 17(3^{4k-2}) + \mathrm{f}(k)$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
Way 2	$\mathrm{f}(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$\mathrm{f}(1) = 17$ is the minimum	B1
	$\mathrm{f}(k+1) = 3^{4(k+1)-2} + 2^{6(k+1)-3}$	Attempts $\mathrm{f}(k+1)$	M1
	$\mathrm{f}(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$		
	$= 81(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ **or** $= 64(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$81(3^{4k-2} + 2^{6k-3})$ or $81\mathrm{f}(k)$; $-17(2^{6k-3})$ / $64(3^{4k-2} + 2^{6k-3})$ or $64\mathrm{f}(k)$; $+17(3^{4k-2})$	A1; A1
	$\mathrm{f}(k+1) = 81(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ **or** $\mathrm{f}(k+1) = 64(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$ **or** $\mathrm{f}(k+1) = 81\mathrm{f}(k) - 17(2^{6k-3})$ **or** $\mathrm{f}(k+1) = 64\mathrm{f}(k) + 17(3^{4k-2})$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
Way 3	**General Method:** Using $\mathrm{f}(k+1) - m\mathrm{f}(k),\ m \in \mathbb{Z}$		
	$\mathrm{f}(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$\mathrm{f}(1) = 17$ is the minimum	B1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - m(3^{4k-2} + 2^{6k-3})$	Attempts $\mathrm{f}(k+1) - m\mathrm{f}(k)$	M1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (81-m)(3^{4k-2}) + (64-m)(2^{6k-3})$		
	$= (81-m)(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ or $= (64-m)(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$(81-m)(3^{4k-2} + 2^{6k-3})$ or $(81-m)\mathrm{f}(k)$; $-17(2^{6k-3})$ / $(64-m)(3^{4k-2} + 2^{6k-3})$ or $(64-m)\mathrm{f}(k)$; $+17(3^{4k-2})$	A1; A1
	$\mathrm{f}(k+1) = (81-m)(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3}) + m\mathrm{f}(k)$ or $\mathrm{f}(k+1) = (64-m)(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2}) + m\mathrm{f}(k)$ or $\mathrm{f}(k+1) = (81-m)\mathrm{f}(k) - 17(2^{6k-3}) + m\mathrm{f}(k)$ **or** $\mathrm{f}(k+1) = (64-m)\mathrm{f}(k) + 17(3^{4k-2}) + m\mathrm{f}(k)$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
			6

Question Number	Scheme	Notes	Marks
8.	$\mathrm{f}(n) = 3^{4n-2} + 2^{6n-3}$ is divisible by 17		
Way 4	**General Method:** Using $\mathrm{f}(k+1) - m\mathrm{f}(k),\ m \in \mathbb{Z}$		
	$\mathrm{f}(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$\mathrm{f}(1) = 17$ is the minimum	B1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - m(3^{4k-2} + 2^{6k-3})$	Attempts $\mathrm{f}(k+1) - m\mathrm{f}(k)$	M1
	$\mathrm{f}(k+1) - m\mathrm{f}(k) = (81-m)(3^{4k-2}) + (64-m)(2^{6k-3})$		
	E.g. $m = 47 \Rightarrow \mathrm{f}(k+1) - 47\mathrm{f}(k) = 34(3^{4k-2}) + 17(2^{6k-3})$	$m = 47$ and $34(3^{4k-2})$	A1
		$m = 47$ and $17(2^{6k-3})$	A1
	$\mathrm{f}(k+1) = 34(3^{4k-2}) + 17(2^{6k-3}) + 47\mathrm{f}(k)$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	In Way 4 there are many alternatives. See below for examples of alternatives where $m = 30$ and $m = 13$		
	The A1A1dM1 marks for some alternatives using $\mathrm{f}(k+1) - m\mathrm{f}(k),\ m \in \mathbb{Z}$		
Way 4.1	$\mathrm{f}(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$		
	$= 30(3^{4k-2}) + 30(2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$		
	$= 30(3^{4k-2} + 2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$	$m = 30$ and $51(3^{4k-2})$	A1
		$m = 30$ and $34(2^{6k-3})$	A1
	$\mathrm{f}(k+1) = 30(3^{4k-2} + 2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$ **or** $\mathrm{f}(k+1) = 30\mathrm{f}(k) + 51(3^{4k-2}) + 34(2^{6k-3})$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1
Way 4.2	$\mathrm{f}(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$		
	$= 13(3^{4k-2}) + 13(2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$		
	$= 13(3^{4k-2} + 2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$	$m = 13$ and $68(3^{4k-2})$	A1
		$m = 13$ and $51(2^{6k-3})$	A1
	$\mathrm{f}(k+1) = 13(3^{4k-2} + 2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$ **or** $\mathrm{f}(k+1) = 13\mathrm{f}(k) + 68(3^{4k-2}) + 51(2^{6k-3})$	**dependent on at least one of the previous A marks being gained** Makes $\mathrm{f}(k+1)$ the subject and expresses it in terms of $\mathrm{f}(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1

	Question 8 Notes			
	Note	$f(n)=3^{4n-2}+2^{6n-3}$ can be written as $f(n)=3^{4n-2}+8^{2n-1}$		
Way 5	$f(n)=3^{4n-2}+2^{6n-3}=3^{4n-2}+8^{2n-1}$			
	$f(1)=3^2+8^1=17$ {is divisible by 17}	$f(1)=17$ is the minimum	B1	
	$f(k+1)-f(k)=3^{4(k+1)-2}+8^{2(k+1)-1}-(3^{4k-2}+8^{2k-1})$	Attempts $f(k+1)-f(k)$	M1	
	$f(k+1)-f(k)=80(3^{4k-2})+63(8^{2k-1})$			
	$=80(3^{4k-2}+8^{2k-1})-17(8^{2k-1})$ **or** $=63(3^{4k-2}+8^{2k-1})+17(3^{4k-2})$	$80(3^{4k-2}+8^{2k-1})$ or $80f(k)$; $-17(8^{2k-1})$ / $63(3^{4k-2}+8^{2k-1})$ or $63f(k)$; $+17(3^{4k-2})$	A1;	A1
	$f(k+1)=80(3^{4k-2}+8^{2k-1})-17(8^{2k-1})+f(k)$ **or** $f(k+1)=63(3^{4k-2}+8^{2k-1})+17(3^{4k-2})+f(k)$ **or** $f(k+1)=80f(k)-17(8^{2k-1})+f(k)$ **or** $f(k+1)=63f(k)+17(3^{4k-2})+f(k)$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+8^{2k-1})$	dM1	
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso	
			(6)	

	Note	Some students may set $f(k)=17M$ and so may prove the following general results • $\{f(k+1)=81f(k)-17(2^{6k-3})\}\Rightarrow f(k+1)=1377M-17(2^{6k-3})$ or $=17(3^4M-2^{6k-3})$ • $\{f(k+1)=64f(k)+17(3^{4k-2})\}\Rightarrow f(k+1)=1088M+17(3^{4k-2})$ or $=17(2^6M+3^{4k-2})$
	Note	**Final A1 mark is dependent on all previous marks being scored in Q8**
	Note	**Final A1:** There must be a correct final expression for $f(k+1)$ **and** a correct conclusion. The conclusion must convey the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
	Note	Allow as part of their conclusion "true for all positive values of *n*"
	Note	Allow as part of their conclusion "true for all values of *n*"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Referring to *n* as a real number in their conclusion (e.g. true for all $n\in\mathbb{R}$) is final A0
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1 mark
	Note	Allow $f(k+1)=3^4f(k)-17(2^{6k-3})$ as a correct alternative to $f(k+1)=81f(k)-17(2^{6k-3})$
	Note	Allow $f(k+1)=2^6f(k)+17(3^{4k-2})$ as a correct alternative to $f(k+1)=64f(k)+17(3^{4k-2})$

Question Number	Scheme	Notes	Marks
9.	$C: y^2 = 4ax$; $P(ap^2, 2ap)$ lies on C; circle: $(x-10a)^2 + y^2 = \frac{9}{4}a^2$		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}} = \frac{\sqrt{a}}{\sqrt{x}}$	$\frac{dy}{dx} = \pm k x^{-\frac{1}{2}}$; $k \neq 0$	M1
	$2y\frac{dy}{dx} = 4a$	$ky\frac{dy}{dx} = c$; $k, c \neq 0$	
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2a\left(\frac{1}{2ap}\right)$	their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dx}{dt}}$; Condone $t \equiv p$	
	{At P, $x = ap^2$, $y = 2ap \Rightarrow$} $\frac{dy}{dx} = \frac{1}{p}$	Correct calculus work leading to $m_T = \frac{1}{p}$	A1
	So, at P, $m_N = -p$	Applies $m_N = \frac{-1}{m_T}$, to find m_N in terms of p, where m_T is found using calculus. Can be implied by later working.	M1
	either $y - 2ap = -p(x - ap^2)$ **or** $y - 0 = -p(x - 10a)$	Correct straight line method for an equation of a normal, where m_N $(\neq m_T)$ is found using calculus.	M1
	$0 - 2ap = -p(10a - ap^2)$ | $2ap - 0 = -p(ap^2 - 10a)$ $\Rightarrow p = \ldots \Rightarrow x = \ldots$ or $y = \ldots$ **or** $2ap - 0 = -p(x - 10a) \Rightarrow x = \ldots$	**dependent on the previous M mark** Complete method to find either the x or y coordinate of P	dM1
	either $x = 8a$, $y = 4\sqrt{2}\,a$ **or** $P(8a, 4\sqrt{2}\,a)$	**Either** $x = 8a$ **or** $y = 4\sqrt{2}\,a$ **or** $y =$ awrt $5.66a$	A1
		$P(8a, 4\sqrt{2}\,a)$ or both $x = 8a$ and $y = 4\sqrt{2}\,a$	A1
	Note: $p = 2\sqrt{2}$ or $\sqrt{8}$. **Note:** Ignore the additional solution $P(8a, -4\sqrt{2}\,a)$		**(7)**
(b) **Way 1**	Area $SBP = \frac{1}{2}(10a - a)(4\sqrt{2}a)$	$\frac{1}{2}(10a - a)$(their y_P from (a))	M1
	$= 18\sqrt{2}\,a^2$	$18\sqrt{2}\,a^2$	A1
			(2)
(c) **Way 1**	$PB = \sqrt{(10a - "8a")^2 + ("4\sqrt{2}a")^2}$ $\{= 6a\}$	Complete Pythagoras method for finding length PB	M1
	$PR = 6a - 1.5a$	**dependent on the previous M mark** $PR =$ "their $6a$" $- 1.5a$	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)
(c) **Way 2**	$p = 2\sqrt{2} \Rightarrow l: y = -2\sqrt{2}x + 20\sqrt{2}a$ $(x - 10a)^2 + (-2\sqrt{2}x + 20\sqrt{2}a)^2 = \frac{9}{4}a^2$ $\Rightarrow 36x^2 - 720ax + 3591a^2 = 0$ $\Rightarrow 9(2x - 21a)(2x - 19a) = 0 \Rightarrow x = \ldots$	Substitutes their equation of l into the circle equation followed by a correct method for solving their 3TQ to give $x = \ldots$ or $y = \ldots$	M1
	$\Rightarrow R(9.5a, \sqrt{2}\,a)$		
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}\,a)^2}$	**dependent on the previous M mark** Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)

Question Number	Scheme	Notes	Marks
9. (b) **Way 2**	Area $SBP = \frac{1}{2}\begin{vmatrix} a & 10a & "8a" & a \\ 0 & 0 & "4\sqrt{2}a" & 0 \end{vmatrix}$ $= \frac{1}{2}\left\|0 - 0 + 40\sqrt{2}\,a^2 - 0 + 0 - 4\sqrt{2}\,a^2\right\|$	Complete applied method for finding area SBP using $S(a, 0)$, $B(10a, 0)$ and their P from (a)	M1
	$= 18\sqrt{2}\,a^2$	$18\sqrt{2}\,a^2$	A1
			(2)
9. (c) **Way 3**	$x_R = 10a - 1.5\cos\left(\tan^{-1}\left(\frac{"4\sqrt{2}\,a"}{10a - "8a"}\right)\right)$ $y_R = 1.5\sin\left(\tan^{-1}\left(\frac{"4\sqrt{2}\,a"}{10a - "8a"}\right)\right)$	Uses their P from (a) in a correct method for writing down either x_R or y_R	M1
	$\Rightarrow R(9.5a, \sqrt{2}\,a)$		
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}\,a)^2}$	**dependent on the previous M mark** Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$	$PR = 4.5a$	A1
			(3)

	Question 9 Notes	
9. (a)	**Note**	Allow 1st M1 1st A1 (sufficient use of calculus) for $\{m_T =\}\ \frac{4a}{2y}$ which leads to $\{m_T =\}\frac{1}{p}$
	Note	Allow 1st M1 1st A1 (sufficient use of calculus) for $\{m_T =\}\ \sqrt{\frac{a}{x}}$ which leads to $\{m_T =\}\frac{1}{p}$
	Note	Give 3rd M1 for either • $2ap = "(-p)"(ap^2) + c \Rightarrow y = "(-p)"x + \text{their } c$ **or** • $0 = "(-p)"(10a) + c \Rightarrow y = "(-p)"x + \text{their } c$
	Note	Writing coordinates the wrong way around E.g. finding $x = 8a,\ y = 4\sqrt{2}\,a$ followed by $(4\sqrt{2}\,a, 8a)$ is final A0
	Note	Give final A0 for $(8a, 5.65685...a)$ without reference to $y = 4\sqrt{2}\,a$ or $2\sqrt{8}\,a$
	Note	Accept $y_P = 2\sqrt{8}\,a$ written in place of $y_P = 4\sqrt{2}\,a$ for the final A1 A1 marks
	Note	**Special Case** If they write down either $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{p},\ m_T = \frac{1}{p}$ or $m_N = -p$ with no evidence of using calculus then they can gain any of or all the final 4 marks in part (a).
	ALT	**Alternative Method for the 3rd M mark and 4th M mark**

Scheme	Notes	Marks
$\{B(10a, 0), P(ap^2, 2ap) \Rightarrow\}$ $m_{BP} = \frac{2ap - 0}{ap^2 - 10a} = -p$	Finds gradient of BP and sets the result equal to the gradient of their normal	3rd M1
$\Rightarrow p = ... \Rightarrow x = ...$ or $y = ...$	**dependent on the previous M mark** Complete method to find either the x or y coordinate of P	4th M1

	Question 9 Notes Continued	
9. (b)	**Note**	Give A0 $25.4558...a^2$ without reference to $18\sqrt{2}\,a^2$
	Note	Condone one slip of either writing 9 for $10a-a$ or writing $"4\sqrt{2}"$ instead of $"4\sqrt{2}\,a"$ for the M mark in (b)
(c)	**Note**	**Way 2:** For reference, $(x-10a)^2+(-2\sqrt{2}x+20\sqrt{2}a)^2=\frac{9}{4}a^2$ $x^2-20ax+100a^2+8x^2-160ax+800a^2=\frac{9}{4}a^2$ $9x^2-180ax+900a^2=\frac{9}{4}a^2$ $9x^2-180ax+\frac{3591}{4}a^2=0$ **or** $9x^2-180ax+897.75a^2=0$ **or** $x^2-20ax+99.75a^2=0$ **or** $4x^2-80ax+399a^2=0$ $x=\frac{180a\pm\sqrt{(180a)^2-4(9)(\frac{3591}{4})a^2}}{2(9)}=\frac{180a\pm 9a}{2(9)}$ $x=\frac{189a}{18},\frac{171a}{18}=10.5a,\,9.5a$
	Note	The method $PB=\sqrt{(10a-"8a")^2+("4\sqrt{2}a")^2}$ needs to be referred to in part (c) or the result of $PB=\sqrt{(10a-"8a")^2+("4\sqrt{2}a")^2}$ needs to be used in part (c) to gain the M mark in part (c)

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