Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Wednesday 13 January 2016 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 4 6 9 4 8 A 0 1 3 2

PEARSON

1. $$z = 3 + 2\mathrm{i}, \qquad w = 1 - \mathrm{i}$$

Find in the form $a + b\mathrm{i}$, where a and b are real constants,

(a) zw

(2)

(b) $\dfrac{z}{w^*}$, showing clearly how you obtained your answer.

(3)

Given that

$$|z + k| = \sqrt{53}, \text{ where } k \text{ is a real constant}$$

(c) find the possible values of k.

(4)

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Question 1 continued

(Total 9 marks)

Q1

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2.

$$f(x) = x^2 - \frac{3}{\sqrt{x}} - \frac{4}{3x^2}, \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.6, 1.7]

(2)

(b) Taking 1.6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α. Give your answer to 3 decimal places.

(5)

Question 2 continued

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Q2

(Total 7 marks)

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3. The quadratic equation

$$x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) (i) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(ii) show that $\alpha^2 + \beta^2 = -2$

(iii) find the value of $\alpha^3 + \beta^3$

(5)

(b) (i) show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

(ii) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(6)

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Question 3 continued

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Question 3 continued

(Total 11 marks)

Q3

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4.

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation represented by the matrix **A**.

(3)

(b) Hence find the smallest positive integer value of n for which

$$\mathbf{A}^n = \mathbf{I}$$

where **I** is the 2×2 identity matrix.

(1)

The transformation represented by the matrix **A** followed by the transformation represented by the matrix **B** is equivalent to the transformation represented by the matrix **C**.

Given that $\mathbf{C} = \begin{pmatrix} 2 & 4 \\ -3 & -5 \end{pmatrix}$,

(c) find the matrix **B**.

(4)

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(8r^3 - 3r) = \frac{1}{2}n(n+1)(2n+3)(an+b)$$

where a and b are integers to be found.

(4)

Given that

$$\sum_{r=5}^{10}(8r^3 - 3r + kr^2) = 22\,768$$

(b) find the exact value of the constant k.

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 8 marks)

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6. The rectangular hyperbola H has equation $xy = c^2$, where c is a non-zero constant.

The point $P\left(cp, \dfrac{c}{p}\right)$, where $p \neq 0$, lies on H.

(a) Show that the normal to H at P has equation

$$yp - p^3x = c(1 - p^4)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Find, in terms of c and p, the coordinates of Q.

(4)

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Question 6 continued

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Question 6 continued

(Total 9 marks)

Q6

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7.

$$f(x) = x^4 - 3x^3 - 15x^2 + 99x - 130$$

(a) Given that $x = 3 + 2i$ is a root of the equation $f(x) = 0$, use algebra to find the three other roots of the equation $f(x) = 0$

(7)

(b) Show the four roots of $f(x) = 0$ on a single Argand diagram.

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

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8. The parabola P has equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of P.

The point B, which does not lie on the parabola, has coordinates (q, r) where q and r are positive constants and $q > a$. The line l passes through B and S.

(a) Show that an equation of the line l is

$$(q - a)\,y = r(x - a)$$

(3)

The line l intersects the directrix of P at the point C.

Given that the area of triangle OCS is three times the area of triangle OBS, where O is the origin,

(b) show that the area of triangle OBC is $\dfrac{6}{5}qr$

(5)

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total 8 marks)

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Q8

9. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)

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Question 9 continued

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Question 9 continued

Q9

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END