Mark Scheme

October 2019

Pearson Edexcel IAL Mathematics
C34 Paper WMA02/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.
6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1	$3\sin x - \cos x$		
(a)	$R = \sqrt{10}$	Must be exact. Condone $R = \pm\sqrt{10}$	B1
	$\tan\alpha = \frac{1}{3} \Rightarrow \alpha = ...$	Condone $\tan\alpha = \pm\frac{1}{3}$ or $\tan\alpha = \pm\frac{3}{1}$ $\sin\alpha = \pm\frac{1}{"\sqrt{10}"}$ or $\sin\alpha = \pm\frac{3}{"\sqrt{10}"}$ or $\cos\alpha = \pm\frac{1}{"\sqrt{10}"}$ or $\cos\alpha = \pm\frac{3}{"\sqrt{10}"}$ $\Rightarrow \alpha = ...$ Implied by 0.32 or 18.4°	M1
	$\alpha = 0.322$	Awrt 0.322 following a correct statement	A1
			(3)
(b)(i)	$19 - \sqrt{10}$	$19 - "\sqrt{10}"$ or awrt 15.8 (ft on their R)	B1ft
(ii)	$\frac{\pi t}{12} + 4 - "0.322" = \frac{3\pi}{2} \Rightarrow t = ...$	Condone $\frac{\pi t}{12} + 4 \pm \text{their } \alpha = \frac{3\pi}{2} \Rightarrow t = ...$ Don't be too concerned by the mechanics of their attempt to solve the equation. Note that 15.95 is evidence that $\frac{5\pi}{2}$ has been selected and scores M0	M1
	$t = 3.95$	Awrt 3.95 Condone 3hrs 57 mins or 3:57 am If multiple answers are given (and not rejected) withhold the final A1	A1
			(3)
			Total 6

Extra Note 1:
Although highly unlikely, it is possible to do (b)(ii) in degrees. In such an attempt the 4 would also need to be changed to degrees.

M1: $\frac{\pi t}{12} + 4 - "0.322" = \frac{3\pi}{2} \Leftrightarrow \frac{180t}{12} + 4 \times \frac{180}{\pi} - "18.4" = 270$

Extra Note 2:
You may see attempts that rely on differentiation. This is essentially the same and would require candidates selecting the second zero

M1: $A\cos\left(\frac{\pi t}{12} + 4 \pm "0.322"\right) = 0 \Rightarrow \frac{\pi t}{12} + 4 \pm "0.322" = \frac{3\pi}{2} \Rightarrow t = ...$

Extra Note 3: Answers without working can score all marks (even though for (b) they were asked to use part (a))

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x)=\left(\frac{1}{3}-x\right)^{-2}$		
(a)	$\left(\frac{1}{3}-x\right)^{-2}=\left(\frac{1}{3}\right)^{-2}(1-3x)^{-2}$	$\left(\frac{1}{3}\right)^{-2}$ or $\frac{1}{3^{-2}}$ or 3^2 or 9 oe seen before the bracket	B1
	$\left(\frac{1}{3}\right)^{-2}\left(\underline{\underline{1+(-2)(-3x)+\frac{(-2)(-3)}{2!}(-3x)^2+\frac{(-2)(-3)(-4)}{3!}(-3x)^3}}+...\right)$ M1: Correct **form** for the 3rd or 4th term with index -2 and the correct power of x. Either $\frac{(-2)(-3)}{2!}(...x)^2$ or $\frac{(-2)(-3)(-4)}{3!}(...x)^3$ but condone missing brackets. Do not accept vector or 'C' notation for the coefficients. A1: Correct underlined expression oe. FYI $=1+6x+27x^2+108x^3+...$		M1A1
	$=9+54x+243x^2+972x^3+...$	All 4 terms correct and simplified. Isw after a correct answer	A1
			(4)
(b)	$(a+bx)(9+'54'x+'243'x^2+'972'x^3+...)=...+3x+27x^2$ $"54"a+"9"b=3$ or $"243"a+"54"b=27$	Expands $(a+bx)\times$ their part (a) and sets their x coefficient = 3 **or** their x^2 coefficient = 27	M1
	$"54"a+"9"b=3,\ "243"a+"54"b=27$ $\Rightarrow a=...,b=...$	Sets their x coefficient = 3 **and** their x^2 coefficient = 27 **and** attempts to solve. Don't be concerned by the process of their attempt to solve. For example calculators may be used.	dM1
	$a=-\frac{1}{9},\ b=1$	Correct values or exact equivalent. Condone $a=-0.\dot{1}$	A1
			(3)
(c)	$"972"a+"243"b=....$	Attempts their $a\times$their 972 + their $b\times$their 243	M1
	$=135$	CSO. It must follow a correct (a) and (b) Note that $135x^2$ is A0...the question demands the coefficient,	A1
			(2)
			Total 9

Extra note: You may see an attempt in (a) as follows

$$\mathrm{f}(x)=\left(\frac{1}{3}-x\right)^{-2}=\left(\frac{1}{3}\right)^{-2}+(-2)\left(\frac{1}{3}\right)^{-3}(-x)^1+\frac{(-2)(-3)}{2}\left(\frac{1}{3}\right)^{-4}(-x)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{1}{3}\right)^{-5}(-x)^3$$

It can be marked in the same way with B1 being $\left(\frac{1}{3}\right)^{-2}+\ldots$.

Question Number	Scheme	Notes	Marks
3(a)	$g(x)\geqslant -1$	Allow equivalents e.g. $y\geq -1$, $y\in[-1,\infty)$, $g\geqslant -1$, Range $\geqslant -1$ etc but not $f(x)\geqslant -1$ or $x\geqslant -1$	B1
			(1)
(b)	$fg(x)=\dfrac{5(2x^2-1)+2}{2x^2-1-3}$	Correct attempt at fg(x), condoning slips on the bracket.	M1
	$=\dfrac{10x^2-3}{2x^2-4}\qquad (x\neq \pm\sqrt{2})$	Allow other simplified forms. (Note that the domain is not required). ISW after a correct answer	A1
			(2)
(c)	$x=\dfrac{5y+2}{y-3}\Rightarrow xy-\ldots=5y+2$ $\Rightarrow xy\pm 5y=\ldots$	For an attempt to change the subject. For this mark they must proceed as far as attempting to get the two relevant terms on the same side of the equation. Condone slips in sign, copying errors, etc	M1
	$y(x-5)=3x+2$ $\Rightarrow y=\ldots$	...and then takes out a factor of y and divides by their $(x-5)$. Again, condone slips in sign etc.	dM1
	$y=f^{-1}(x)=\dfrac{3x+2}{x-5}\quad (x\neq 5)$	Correct inverse function or exact equivalent. (Note that the domain is not required). Allow $y=\ldots$ or $f^{-1}(x)=\ldots$	A1
			(3)
(d)	$\dfrac{5x+2}{x-3}=\dfrac{3x+2}{x-5}$ or $\dfrac{5x+2}{x-3}=x$ or $\dfrac{3x+2}{x-5}=x\Rightarrow$ 3TQ in x Attempts to use one of these equations and proceeds to find 3TQ in x. Follow through on their $f^{-1}(x)$		M1
	$x^2-8x-2=0$	Correct quadratic. Accept an equivalent equation in which the terms have been collected such as $x^2-8x=2$ or $2x^2-16x-4=0$	A1
	$x=\dfrac{8\pm\sqrt{64+8}}{2}$	Solves by formula or completing the square. It is dependent upon the first M1. Only accept factorisation if it factorises. Decimal answers correct to 3sf would imply this mark.	dM1
	$x=4\pm 3\sqrt{2}$	Cao (both needed). If both are given and one is subsequently rejected then withhold this mark.	A1
			(4)
			Total 10

Extra note:

In (b) and / or (c) you may see division. Eg in (b) f may be adapted to $5\pm\dfrac{A}{x-3}$ first before $5+\dfrac{17}{2x^2-4}$

In part (c) look for

M1: $x=\dfrac{5y+2}{y-3}\Rightarrow x=5+\dfrac{17}{y-3}\Rightarrow x-5=\dfrac{17}{y-3}$ condoning slips in sign

dM1: $\Rightarrow x-5=\dfrac{17}{y-3}\Rightarrow y=\dfrac{17}{x-5}+3$ condoning slips in sign. The form of the expression must be correct

Question Number	Scheme	Notes	Marks
4(a)	$\dfrac{dy}{dx}=\cos 2x-2x\sin 2x$	Uses the product rule to obtain an expression of the form $\cos 2x\pm kx\sin 2x$ If the rule is stated it must be correct.	M1
		Correct derivative	A1
	$\cos 2x-2x\sin 2x=0\Rightarrow 2x\tan 2x=1$	States or sets $\dfrac{dy}{dx}=0$ (which may be implied by a correct equation) and divides by $\cos 2x$ to form an equation in tan2x Alternatively states or sets $\dfrac{dy}{dx}=0$ and divides by $\sin 2x$ to form an equation in cot2x	M1
	$=\dfrac{1}{2}\arctan\left(\dfrac{1}{2x}\right)*$	Correct completion to printed answer with no errors. Do NOT condone $\arctan\left(\dfrac{1}{2x}\right)\equiv\tan^{-1}\left(\dfrac{1}{2x}\right)$ The bracket is not necessary and all of the lines (or their equivalent) as seen in the scheme must be seen. It cannot be scored following an equation in cot2x and must follow the line $2x\tan 2x=1$ or $\tan 2x=\dfrac{1}{2x}$	A1*
			(4)
(b)	$x_1=\dfrac{1}{2}\arctan\left(\dfrac{1}{2(0.5)}\right)=0.39..$	Attempts $\dfrac{1}{2}\arctan\left(\dfrac{1}{2(0.5)}\right)=....$ and achieves an awrt 0.39	M1
	awrt 0.3927	Awrt 0.3927 or $\dfrac{\pi}{8}$	A1
	$x_2=0.4525$	Awrt 0.4525 Ignore additional values, say x_3 etc	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
5	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{2h^{\frac{3}{2}}}{5t^2}$		
(a)	$\displaystyle\int \frac{1}{2h^{\frac{3}{2}}}\,\mathrm{d}h = \int \frac{1}{5t^2}\,\mathrm{d}t$	Correct separation of variables (allow equivalent forms) There is no need to see the integral signs but the dh and dt must be present and in the correct positions.	B1
	$-h^{-\frac{1}{2}} = -\frac{1}{5}t^{-1} + (c)$	Reaches $\alpha h^{-\frac{1}{2}} = \beta t^{-1}(+c)$	M1
		Correct integration with no requirement for $+c$	A1
	$-(1)^{-\frac{1}{2}} = -\frac{1}{5}(1)^{-1} + c \Rightarrow c = ...$	Uses given conditions at an appropriate point in their integrated form to find a constant of integration. Alt uses $\left[-h^{-\frac{1}{2}}\right]_1^h = \left[-\frac{1}{5}t^{-1} + c\right]_1^t$	M1
	$-h^{-\frac{1}{2}} = -\frac{1}{5}t^{-1} - \frac{4}{5}$	Correct equation, accept all forms. Eg. $5h^{-\frac{1}{2}} = t^{-1} + 4$	A1
	$-h^{-\frac{1}{2}} = -\frac{1}{5}t^{-1} - \frac{4}{5} \Rightarrow \frac{1}{\sqrt{h}} = \frac{1}{5t} + \frac{4}{5} = \frac{1+4t}{5t} \Rightarrow \sqrt{h} = \frac{5t}{1+4t} \Rightarrow h = ...$ Attempts to use correct algebra starting from $\alpha h^{-\frac{1}{2}} = \beta t^{-1} + c$ to make h the subject. Look for an attempt that has • A single fraction being formed before "inverting" oe. Do not allow attempts where the candidate inverts each term • A correct attempt to square. Eg. Each side is squared rather than each term		M1
	$h = \dfrac{25t^2}{(1+4t)^2} \quad (a = 25,\ b = 4)$	cao	A1
			(7)
(b)	Max $h = \dfrac{25}{16}$ or 1.5625	Max $h = \dfrac{\text{Their } a}{\text{Their } b^2}$ from an answer to (a) in the form $h = \dfrac{at^2}{(c+bt)^2} \quad a,b>0$ Implied by 1.56 (ie answer to 3sf) Do NOT award if (a) is in the form $h = \dfrac{at^2}{(c-bt)^2} \quad a,b>0$	M1
		$\dfrac{25}{16}$ oe following $h = \dfrac{25t^2}{(1+4t)^2}$ oe	A1

		Condone for eg $h \leqslant \frac{25}{16}$ or $0 < h < \frac{25}{16}$	
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
6	$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} \equiv 2\text{cosec}^2 x$		
(a)	$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} = \frac{\sec x - \sec^2 x - \sec x - \sec^2 x}{1-\sec^2 x}$ Attempts to express lhs as a single fraction		M1
	$= \frac{-2\sec^2 x}{1-\sec^2 x} = \frac{-2\sec^2 x}{-\tan^2 x}$	Uses $1-\sec^2 x = \pm\tan^2 x$ on the denominator	M1
	$\frac{2}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x} = 2\text{cosec}^2 x$*	Reaches rhs with no errors and at least one intermediate line of working	A1*
			(3)
(b)	$\frac{\sec 2\theta}{1+\sec 2\theta} - \frac{\sec 2\theta}{1-\sec 2\theta} = 3 - 2\cot^2 2\theta$		
	$2\text{cosec}^2 2\theta = 3 - 2\cot^2 2\theta$	Attempts to use part (a) with $x = 2\theta$ Condone slips	M1
	$2\text{cosec}^2 2\theta = 3 - 2\left(\text{cosec}^2 2\theta - 1\right)$ or $2\left(1+\cot^2 2\theta\right) = 3 - 2\cot^2 2\theta$	Uses $\pm 1 \pm \cot^2 2\theta = \pm\text{cosec}^2 2\theta$ Correct bracketing should be seen or implied.	M1
	$\text{cosec}^2 2\theta = \frac{5}{4}$ or $\cot^2 2\theta = \frac{1}{4}$	Correct value for $\text{cosec}^2 2\theta$, $\cot^2 2\theta$ but may be $\sin^2 2\theta = \frac{4}{5}$, $\cos^2 2\theta = \frac{1}{5}$ or $\tan^2 2\theta = 4$	A1
	$\text{cosec} 2\theta = (\pm)\sqrt{\frac{5}{4}} \Rightarrow \sin 2\theta = (\pm)\sqrt{\frac{4}{5}} \Rightarrow 2\theta = ...$ or $\cot 2\theta = (\pm)\sqrt{\frac{1}{4}} \Rightarrow \tan 2\theta = (\pm)2 \Rightarrow 2\theta = ...$ Correct processing to reach values for $2\theta = \ldots$ (may not be called 2θ) May not be awarded from impossible trig. values, eg $\sin^2 2\theta = 4$		M1
	$(2\theta) = 1.107148..., 2.03444..., 4.24874..., 5.1760365...$	FYI	
	$\theta = 0.554, 1.02, 2.12, 2.59$	Awrt 2 of these	A1
		Awrt all 4 angles	A1
	Ignore extra answers outside range and deduct the final mark for extra answers in range		
			(6)
			Total 9

Extra Notes: There are many different ways to proceed from $2\text{cosec}^2 2\theta = 3 - 2\cot^2 2\theta$

2nd M is for an attempt to get in a single trig identity.

Eg $2\text{cosec}^2 2\theta = 3 - 2\cot^2 2\theta \Rightarrow 2 = 3\sin^2 2\theta - 2\cos^2 2\theta \Rightarrow \cos^2 2\theta = \frac{1}{5}\ or\ \sin^2 2\theta = \frac{4}{5}$

6 alt 1	$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} \equiv 2\text{cosec}^2 x$		
(a)	$\frac{1/\cos x}{1+1/\cos x} - \frac{1/\cos x}{1-1/\cos x} = \frac{1}{\cos x+1} - \frac{1}{\cos x-1} = \frac{\cos x - 1 - \cos x - 1}{\cos^2 x - 1}$ Uses $\sec x = 1/\cos x$ and attempts to express lhs as a single fraction		M1
	$\frac{-2}{\cos^2 x - 1} = \frac{-2}{-\sin^2 x}$	Uses $\cos^2 x - 1 = \pm\sin^2 x$ on the denominator	M1
	$= 2\text{cosec}^2 x$*	Reaches rhs with no errors	A1*
			(3)

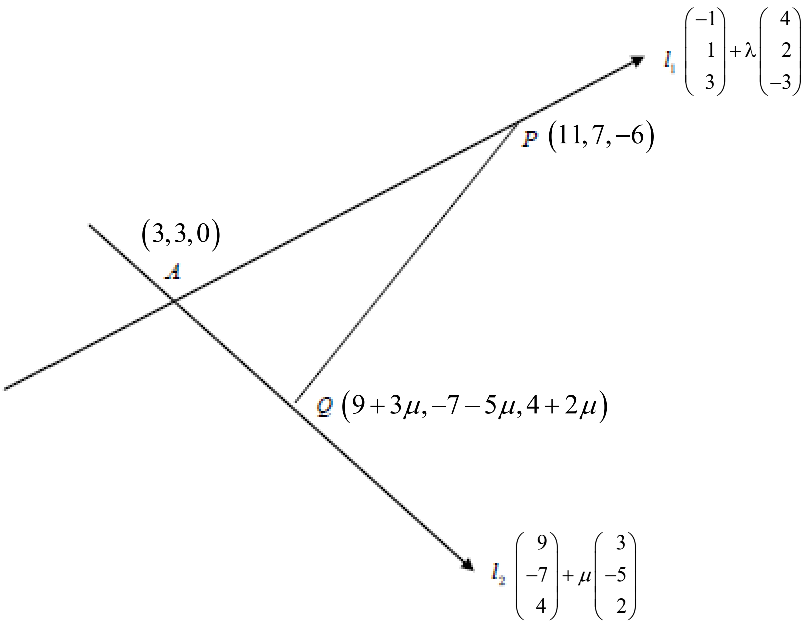
Working from both sides

6 Alt II	$\frac{\sec x}{1+\sec x} - \frac{\sec x}{1-\sec x} \equiv 2\text{cosec}^2 x$		
(a)	$\sec x(1-\sec x) - \sec x(1+\sec x) = 2\text{cosec}^2 x(1-\sec^2 x)$ Attempts to multiply each term by $1-\sec^2 x$		M1
	$-2\sec^2 x = 2\times\frac{1}{\sin^2 x}(-\tan^2 x)$	Uses $1-\sec^2 x = \pm\tan^2 x$	M1
	$-2\sec^2 x = 2\times\frac{1}{\cancel{\sin^2 x}}\left(-\frac{\cancel{\sin^2 x}}{\cos^2 x}\right)$ AND states hence true	Reaches rhs with no errors and at least one intermediate line of working AND states hence true	A1*
			(3)

Question Number	Scheme	Notes	Marks
7(a)	$\dfrac{2x^2-3}{(3-2x)(1-x)^2} \equiv \dfrac{A}{3-2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$		
	$2x^2-3 = A(1-x)^2 + B(3-2x)(1-x) + C(3-2x)$ Multiplies both sides by $(3-2x)(1-x)^2$ in an attempt to form a correct identity. Condone minor slips (say in signs) but there must be an attempt to pair A, B and C with the correct factor(s).		M1
	$x=1 \Rightarrow -1 = C$	M1: Correct method to obtain at least one of A, B or C Allow substitution of a correct value of x or equating terms. Don't be too concerned by the process here. It is not fully dependent on the previous M and may be scored from incorrect identities. A1: One correct constant	M1A1
	$A=6, B=-2, C=-1$	Correct values or correct expression	A1
			(4)
(b)	$\displaystyle\int \frac{2x^2-3}{(3-2x)(1-x)^2}\,dx = -3\ln(3-2x) + 2\ln(1-x) - (1-x)^{-1}\ (+c)$		
	$\displaystyle\int \frac{-1}{(1-x)^2}\,dx \to -1(1-x)^{-1}$ oe	Follow through their C Watch for $\dfrac{-1}{1-x} \to \dfrac{1}{x-1}$ ✓	B1ft
	$p\ln(3-2x)$ or $q\ln(1-x)$	Also accept $p\ln\lvert 3-2x\rvert$, $q\ln\lvert 1-x\rvert$ $p\ln\lvert 2x-3\rvert$ or $q\ln\lvert x-1\rvert$. Also note that any multiple of these are correct. So $p\ln(3-2x) \leftrightarrow p\ln(3k-2kx)$ where k is a constant	M1
	$-3\ln(3-2x) + 2\ln(1-x)$	$-3\ln(3-2x)$ **or** $+2\ln(1-x)$ or with modulus signs. Allow unsimplified and follow through their A **or** their B. Note: $-3\ln(2x-3)$ **or** $+2\ln(x-1)$ are correct as $\dfrac{6}{3-2x} \equiv \dfrac{-6}{2x-3} \to -3\ln(2x-3)$ ✓ Allow with multiples. Eg $2\ln(1-x) \leftrightarrow 2\ln(1k-kx)$ ✓	A1ft
		Fully correct terms: Remember to isw $-3\ln(3-2x)$ **and** $+2\ln(1-x)$ Allow with moduli Allow for example $-3\ln(3-2x) \leftrightarrow -3\ln(2x-3)$	A1

		Allow for example $-3\ln(3-2x) \leftrightarrow -3\ln(3k-2kx)$ where k is a constant. ("+c" not needed)	
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
8(a)	$-1+4\lambda=9+3\mu$ (1) $1+2\lambda=-7-5\mu$ (2) $3-3\lambda=4+2\mu$ (3)	Writes down any **two** of these equations	M1
	Eg. $1\&2:\Rightarrow\lambda=1\ (\mu=-2)$	Full method using any two equations to find λ **or** μ. Don't be too concerned with the mechanics. Allow answers from a calculator	M1
	$\lambda=1$ and $\mu=-2$	Correct values for λ **and** μ	A1
	Eg. Check 3: $3-3(1)=4+2(-2)=0$ so true	Checks values in 3rd equation **and** concludes. Accept ✓ or similar. Alternatively substitutes the correct λ and μ into the equations to give (3, 3, 0) each times and concludes.	B1
	$\begin{pmatrix}-1\\1\\3\end{pmatrix}+"1"\begin{pmatrix}4\\2\\-3\end{pmatrix}$ or $\begin{pmatrix}9\\-7\\4\end{pmatrix}+"-2"\begin{pmatrix}3\\-5\\2\end{pmatrix}$	Uses their λ or μ to find the point of intersection. If no method is seen accept one correct coordinate ft on their λ or μ	M1
	$3\mathbf{i}+3\mathbf{j}$	Correct vector, either form. (Condone coordinate form eg. (3, 3, 0)) Do not condone incorrect notation $(3\mathbf{i},3\mathbf{j},0\mathbf{k})$ This is A0	A1
			(6)
(b)	$\lambda=3\Rightarrow p=7$	Correct value for p	B1
			(1)
(c)	$\overrightarrow{PQ}=\begin{pmatrix}9\\-7\\4\end{pmatrix}+\mu\begin{pmatrix}3\\-5\\2\end{pmatrix}-\begin{pmatrix}11\\"7"\\-6\end{pmatrix}$	Attempts $\overrightarrow{PQ}$ either way around	M1
	$\begin{pmatrix}3\mu-2\\-5\mu-14\\2\mu+10\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\2\end{pmatrix}=0\Rightarrow\mu=...\left(-\frac{42}{19}\right)$	Attempts the scalar product between $\overrightarrow{PQ}$ and the direction of l_2, sets = 0 and solves for μ This is dependent upon the previous M	dM1
	Q is at $\begin{pmatrix}9\\-7\\4\end{pmatrix}\pm"-\frac{42}{19}"\begin{pmatrix}3\\-5\\2\end{pmatrix}$	Attempts to find coordinates of Q using $\begin{pmatrix}9\\-7\\4\end{pmatrix}\pm"\mu"\begin{pmatrix}3\\-5\\2\end{pmatrix}$ with their value of μ. If there is no working allow if one coordinate is correct ft for their μ	ddM1
	$\left(\frac{45}{19},\frac{77}{19},-\frac{8}{19}\right)$	Correct coordinates. Condone answer in vector form $\frac{45}{19}\mathbf{i}+\frac{77}{19}\mathbf{j}-\frac{8}{19}\mathbf{k}$.	A1
			(4)
			Total 11



Note other methods are possible in (c): Look at the solution carefully.

Alt 1:
For the dM1candidates could attempt $\overrightarrow{AQ}.\overrightarrow{PQ}=0$ where A is the point of intersection
Alt 2:
Candidates could minimise the distance PQ
For the dM1 $\quad d^2=(3\mu-2)^2+(-5\mu-14)^2+(2\mu+10)^2$ which is minimised when $\overrightarrow{PQ}$ is perpendicular to l_2

$$6(3\mu-2)-10(-5\mu-14)+4(2\mu+10)=0 \Rightarrow \mu=-\frac{42}{19}$$

Alt 3:
Methods that involve angle PAQ are unlikely to score all marks as they will not produce exact coordinates.

M1: Full method to find distance AQ (which may be done in two steps) using scalar product

Attempts $|AP|\cos\theta=\sqrt{8^2+4^2+6^2}\times\dfrac{3\times4-5\times2+2\times-3}{\sqrt{38}\times\sqrt{29}}=-\dfrac{8}{\sqrt{38}}$

For correct (a) and (b) you may see $\sqrt{116}\cos 96.9°$ or $\sqrt{116}\cos 83.1°$ OR 1.35 units

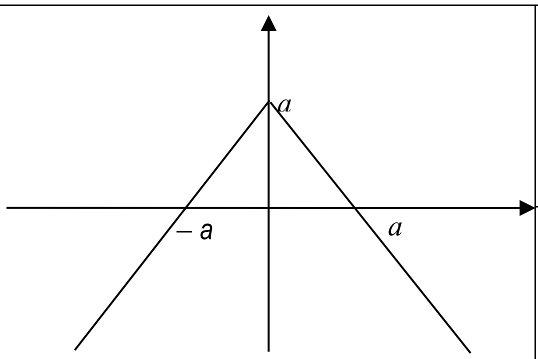
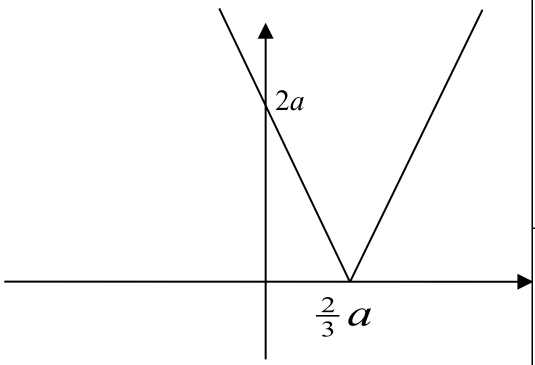
dM1: Full attempt to find the coordinate of Q using $\overrightarrow{OA}\pm\overrightarrow{AQ}=\begin{pmatrix}3\\3\\0\end{pmatrix}\pm\dfrac{1}{\sqrt{9+25+4}}\times\dfrac{8}{\sqrt{38}}\begin{pmatrix}3\\-5\\2\end{pmatrix}$

Accept decimal attempts here: So $\overrightarrow{OA}\pm\overrightarrow{AQ}=\begin{pmatrix}3\\3\\0\end{pmatrix}\pm1.35..\times\dfrac{1}{\sqrt{38}}\begin{pmatrix}3\\-5\\2\end{pmatrix}$

ddM1: Attempts to find the exact coordinates of Q using $\overrightarrow{OA}\pm\overrightarrow{AQ}$ $\begin{pmatrix}3\\3\\0\end{pmatrix}\pm\dfrac{1}{\sqrt{9+25+4}}\times\dfrac{8}{\sqrt{38}}\begin{pmatrix}3\\-5\\2\end{pmatrix}$

A1: $\left(\dfrac{45}{19},\dfrac{77}{19},-\dfrac{8}{19}\right)$ or equivalent vector form

Question Number	Scheme	Notes	Marks

9(a)(i)		B1: "$\wedge$" shape anywhere. The branches may not be symmetrical and the graph may not intersect the x- axis	B1B1
		B1: Fully correct graph in the correct position with correct intercepts. It must cross the x-axis and not just stop there. Don't be too concerned with slight lack of symmetry.	
(ii)		B1: $\vee$ Shape anywhere. The branches may not be symmetrical and the graph may not intersect the y- axis	B1B1
		B1: Fully correct graph in the correct position with correct intercepts. It must cross the y-axis and not just stop there! Don't be too concerned with slight lack of symmetry.	
			(4)
(b)	(As $x > 0$) $a - x = 3x - 2a \Rightarrow x = ...$ **or** $a - x = -3x + 2a \Rightarrow x = ...$ Attempts to solve either equation which must be correct. It cannot be produced from incorrect modulus work. For example $a - \lvert x\rvert = \lvert 3x - 2a\rvert \Rightarrow a = \lvert 3x - 2a\rvert + \lvert x\rvert \Rightarrow a = 3x - 2a + x$ is fine. But $a - \lvert x\rvert = \lvert 3x - 2a\rvert \Rightarrow a - \lvert x\rvert = 3\lvert x\rvert - 2\lvert a\rvert \Rightarrow 4\lvert x\rvert = 3a \Rightarrow x = \frac{3}{4}a$ is not. See bottom of page for SC		M1
	$x = \frac{3}{4}a$ **or** $x = \frac{1}{2}a$	One correct value	A1
	$a - x = 3x - 2a \Rightarrow x = ...$ **and** $a - x = -3x + 2a \Rightarrow x = ...$ Attempt to solve both equations which must be correct		M1
	$x = \frac{3}{4}a$ **and** $x = \frac{1}{2}a$	Both values correct and no other values	A1
			(4)
			Total 8

Note 1: Squaring approaches in (b) will lead to the correct answers but only because the solutions are both positive. Score as follows: (As $x > 0$)

$$\left(a - \lvert x\rvert\right)^2 = \left(3x - 2a\right)^2 \Rightarrow a^2 - 2ax + x^2 = 9x^2 - 12ax + 4a^2 \Rightarrow 8x^2 - 10ax + 3a^2 = 0 \Rightarrow x = \frac{3}{4}a,\ x = \frac{1}{2}a$$

Score M1 (For a correct equation with no incorrect work (as x > 0) A1: One correct answer M1: Attempt at factorisation A1: Second correct answer

Note 2: Watch for candidates who solve using incorrect modulus work.

$$a - \lvert x\rvert = \lvert 3x - 2a\rvert \Rightarrow a - \lvert x\rvert = 3\lvert x\rvert - 2\lvert a\rvert \Rightarrow 4\lvert x\rvert = 3a \Rightarrow x = \frac{3}{4}a$$

Score SC B1 0 0 0 for one correct value

Question Number	Scheme	Notes	Marks

10(a)	$\frac{\mathrm{d}u}{\mathrm{d}x}=2$	Or $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{2}$ or $\mathrm{d}u=2\mathrm{d}x$ etc	B1
	$\int \frac{(3x+2)^2}{2x-1}\mathrm{d}x=\int \frac{\left(3\frac{u+1}{2}+2\right)^2}{u}\frac{1}{2}\mathrm{d}u$	An attempt at a **complete** substitution with all terms (inc the dx) being replaced by 'u' Condone slips	M1
	$=\frac{1}{8}\int\left(9u+42+\frac{49}{u}\right)\mathrm{d}u$	Reaches $\int\left(\alpha u+\beta+\frac{\gamma}{u}\right)\mathrm{d}u$ oe	dM1
	$=\frac{1}{8}\left[\frac{9u^2}{2}+42u+49\ln u\right]$	Correct integration. Allow $\ln cu \leftrightarrow \ln u$ (may see $\ln 8u$)	A1
	$=\frac{1}{8}\left[\left(\frac{9(9)^2}{2}+42(9)+49\ln 9\right)-\left(\frac{9(3)^2}{2}+42(3)+49\ln 3\right)\right]$ Correct use of both limits Eg u= 3 and u=9 within their integrand in 'u' or x = 2 and x = 5 within their integral where 'u' has been changed back to 2x-1		M1
	$72+\frac{49}{8}\ln 3$*	Obtains printed answer with no errors. All steps above, or equivalent must be seen. If candidate writes $=\frac{1}{8}\left[\frac{9u^2}{2}+42u+49\ln u\right]_3^9$ followed by given answer or awrt 78.7 just withhold the final A1*	A1*
			(6)
(b)	$V=k\pi\times\text{Answer to}(a)$	If the formula is quoted it must be correct. Eg. $V=2\pi\int y^2\mathrm{d}x$ is incorrect and M0. Implied by $k\pi\times\left(72+\frac{49}{8}\ln 3\right)$	M1
	$=\pi\left(18+\frac{49}{32}\ln 3\right)$	Correct exact answer (allow equivalent exact forms) E.g. $=\frac{\pi}{4}\left(72+\frac{49}{8}\ln 3\right)$	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks

11	$2x^2 + y^3 = kxy$		
(a)	$\dfrac{\mathrm{d}(y^3)}{\mathrm{d}x} = 3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$	$\dfrac{\mathrm{d}(y^3)}{\mathrm{d}x} = \alpha y^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (See alternative form **)	M1
	$\dfrac{\mathrm{d}(kxy)}{\mathrm{d}x} = ky + kx\dfrac{\mathrm{d}y}{\mathrm{d}x}$	$\dfrac{\mathrm{d}(kxy)}{\mathrm{d}x} = \alpha y + \beta x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (See alternative form **)	M1
	$4x + 3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x} = kx\dfrac{\mathrm{d}y}{\mathrm{d}x} + ky$ Or $4x\,\mathrm{d}x + 3y^2\,\mathrm{d}y = kx\,\mathrm{d}y + ky\,\mathrm{d}x$ (**)	All correct	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}(3y^2 - kx) = ky - 4x \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{ky - 4x}{3y^2 - kx}$	Correct expression oe. Eg. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x - ky}{kx - 3y^2}$	A1
			(4)
(b)	$3y^2 - kx = 0$	Sets the denominator of their answer to (a) = 0	M1
	$2\left(\dfrac{3y^2}{k}\right)^2 + y^3 = ky\left(\dfrac{3y^2}{k}\right)$ or $2x^2 + \left(\dfrac{kx}{3}\right)^{\frac{3}{2}} = kx\left(\dfrac{kx}{3}\right)^{\frac{1}{2}}$	Attempts to substitute an equation in $2x^2 + y^3 = kxy$ to obtain an equation in one variable. The equation substituted must be a result of the denominator, or the numerator, set = 0 Condone slips on a coefficient but expect correct powers to be used.	M1
	$y = \dfrac{k^2}{9}$ or $x = \dfrac{k^3}{27}$	Correct value for x or y. Allow unsimplified, eg. $x = \dfrac{3k^3}{81}$	A1
	$y = \dfrac{k^2}{9} \Rightarrow x = \ldots$ or $x = \dfrac{k^3}{27} \Rightarrow y = \ldots$	Attempts the other coordinate. Dependent upon having scored at least one of the previous two M marks.	M1
	$x = \dfrac{k^3}{27}$ and $y = \dfrac{k^2}{9}$	Correct coordinates, which may be unsimplified	A1
			(5)
			Total 9

A common occurrence is where candidates set their numerator = zero. FYI

M0 "$ky = 4x$"

M1 Substitutes "$y = \dfrac{4x}{k}$" oe in $2x^2 + y^3 = kxy$ to obtain an equation in one variable. eg $2x^2 + \dfrac{64x^3}{k^3} = 4x^2$

A0 $x = \dfrac{k^3}{32}$

dM1 Attempts the other coordinate.

A0 $y = \dfrac{k^2}{8}$

Question Number	Scheme	Notes	Marks

12	$N=\dfrac{250e^{0.2t}}{1+0.25e^{0.2t}}$		
(a)	$\left(\dfrac{250e^{0}}{1+0.25e^{0}}=\right)200$	200	B1
			(1)
(b)	$\dfrac{250e^{0.2t}}{1+0.25e^{0.2t}}=800\Rightarrow 50e^{0.2t}=800$	Puts $N=800$ and solves as far as $pe^{\pm 0.2t}=q$ with $p,q>0$	M1
	$e^{0.2t}=16\Rightarrow 0.2t=\ln 16$	Correctly takes ln's to reach $\pm 0.2t=\ln(\alpha)$ oe Eg allow $\ln p\pm 0.2t=\ln q\Rightarrow \pm 0.2t=$	dM1
	$t=5\ln 16=14\,(\text{nearest integer})$	Awrt 14 but allow 5ln16 isw Provided a correct equation has been written down allow awrt 14 (13.86) so long as no incorrect work is seen.	A1
			(3)
(c)	$\left(\dfrac{dN}{dt}\right)=\dfrac{(1+0.25e^{0.2t})50e^{0.2t}-250e^{0.2t}\times 0.05e^{0.2t}}{(1+0.25e^{0.2t})^{2}}$	Uses quotient rule to obtain an expression of the form: $\dfrac{dN}{dt}=\dfrac{\alpha e^{0.2t}(1+0.25e^{0.2t})-\beta e^{0.2t}\times e^{0.2t}}{(1+0.25e^{0.2t})^{2}}$ Do not withhold this mark if you see incorrect processing on the $e^{0.2t}\times e^{0.2t}$ which may appear as $e^{0.04t}$ or $e^{0.2t^{2}}$ If rule is quoted it must be correct. Allow attempts via the product rule with $u=250e^{0.2t}$, $v=(1+0.25e^{0.2t})^{-1}$ to obtain expression of the form $\alpha e^{0.2t}(1+0.25e^{0.2t})^{-1}\pm\beta(e^{0.2t})^{2}(1+0.25e^{0.2t})^{-2}$	M1
	$\left(\dfrac{dN}{dt}\right)=\dfrac{50e^{0.2t}}{(1+0.25e^{0.2t})^{2}}$ * No need for LHS	Cso. Withhold this mark if you see $e^{0.2t}\times e^{0.2t}=e^{0.04t^{2}}$ or similar	A1*
			(2)
(d)	Sets $10=\dfrac{50e^{0.2t}}{(1+0.25e^{0.2t})^{2}}\Rightarrow$ 3TQ in $e^{0.2t}$ Look for $\alpha e^{0.4t}+\beta e^{0.2t}+\chi=0$ $\alpha(e^{0.2t})^{2}+\beta e^{0.2t}+\chi=0$ **for M1**	Do not withhold any of these marks if the candidate incorrectly processes the $e^{0.2t}\times e^{0.2t}$ **but goes on to treat the expression correctly.**	M1
	$e^{0.4t}-72e^{0.2t}+16=0$ **oe** $(e^{0.2t})^{2}-72e^{0.2t}=-16$ **oe for A1**	A valid alternative is to square root both sides on line one to get a quadratic in $e^{0.1t}$	A1
	For solving the 3TQ in $e^{0.2t}$ **AND** proceeding to value for T using lns FYI exact answer $e^{0.2t}=36\pm 16\sqrt{5}$	FYI $e^{0.2t}-4\sqrt{5}e^{0.1t}+4=0$ $\Rightarrow e^{0.1t}=4+2\sqrt{5}$	**dM1**
	$T=21.4$ only	($T=-7.5$ must be deleted if found)	**A1**
			(4)
			Total 10

Question Number	Scheme	Notes	Marks
13(a)	$0.3^2 \times 4^{0.3} = 0.136414491...$	Awrt 0.1364 Allow if only in their attempt at (b)	B1
			(1)
(b)	States or uses $h = 0.1$	Implied by $0.05\times\{....\}$	B1
	For the correct structure of the trapezium rule or the sum of six trapezia $\text{Area} \approx \frac{1}{2}\times h\left\{(0)+0.8271+2\left(0.0115+0.0528+"0.1364"+0.2786+0.5\right)\right\}$ If candidate writes $\text{Area} \approx \frac{1}{2}\times 0.1\{(0)+0.8271\}+2\left(0.0115+0.0528+"0.1364"+0.2786+0.5\right)=1.75$ Then score M0 A0 but if followed by awrt 0.139 allow.		M1
	Awrt 0.139		A1
			(3)
(c)	Be aware of candidates who use $4^x \equiv \mathrm{e}^{x\ln 4}$ throughout the question		
	$\int x^2 4^x \mathrm{d}x = x^2 \frac{4^x}{\ln 4} - \frac{2}{\ln 4}\int x4^x \mathrm{d}x$	M1: Integrates by parts the right way around to obtain an expression of the form $ax^2 4^x - \int bx4^x \mathrm{d}x$, $b>0$ A1: $x^2 \frac{4^x}{\ln 4} - \frac{2}{\ln 4}\int x4^x \mathrm{d}x$	M1A1
	$\int x4^x \mathrm{d}x = x\frac{4^x}{\ln 4} - \frac{1}{\ln 4}\int 4^x \mathrm{d}x$	M1: Integrates by parts again, an expression of the form $kx4^x$, the right way around to obtain an expression of the form $ax4^x - \int b4^x \mathrm{d}x$. A1: $x\frac{4^x}{\ln 4} - \frac{1}{\ln 4}\int 4^x \mathrm{d}x$	dM1A1
	$\int x^2 4^x \mathrm{d}x = \frac{x^2 4^x}{\ln 4} - \frac{2x4^x}{(\ln 4)^2} + \frac{2\times 4^x}{(\ln 4)^3}(+c)$	All correct or equivalent expression. Allow notation $\ln^3 4$ for $(\ln 4)^3$	A1
			(5)
(d)	$\left[\frac{x^2 4^x}{\ln 4} - \frac{2x4^x}{(\ln 4)^2} + \frac{2\times 4^x}{(\ln 4)^3}\right]_0^{0.6} = \frac{0.6^2 4^{0.6}}{\ln 4} - \frac{2\times 0.6\times 4^{0.6}}{(\ln 4)^2} + \frac{2\times 4^{0.6}}{(\ln 4)^3} - \frac{2}{(\ln 4)^3}$ For an attempt to use correct limits within an answer to (c) of the form $px^2 4^x + qx4^x + r4^x, \quad p,q,r \neq 0$		M1
	$(0.887 - 0.751) \quad = 0.136$	Awrt 0.136 Note that 0.136 without any working is M0 A0 as 0.136 is the calculator answer to the definite integral	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
14(a)	$\frac{dy}{dx}=\frac{3t^2-1}{2t}$	M1: Attempts to use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1A1
		A1: Correct derivative	
	At $t=2, x=3\ y=6$	Correct coordinates	B1
	$y-6=-\frac{2(2)}{3\times2^2-1}(x-3)$	Fully correct method for the normal using the negative reciprocal of their $\frac{dy}{dx}$ at $t=2$ and their stated coordinates. Allow the gradient to be written down from their $\frac{dy}{dx}$	M1
	$4x+11y-78=0$*	cso	A1*
			(5)
(b) Way 1 **Via trig or Rcos**	$4(12.5+a\cos t)+11(15+a\sin t)-78=0$	Substitutes the parametric form of C_2 into the normal	M1
	$4a\cos t+11a\sin t=-137$	Rearranges to $p\cos t+q\sin t=k$ But may be implied by further work.	M1
	Max lhs $=\sqrt{(4a)^2+(11a)^2}\Rightarrow a=\ldots$	Attempt Pythagoras to find a value for a Dependent upon both previous M's	ddM1
	$a=\sqrt{137}$ or $-\sqrt{137}$	A correct value for a	A1
	$-\sqrt{137}<a<\sqrt{137}$	Correct inequality	A1
			(5)
(b) Way 2 **Via Circle geometry**	$y-15=\frac{3\times2^2-1}{2(2)}(x-12.5)$	Attempts equation of perpendicular to normal passing through (12.5, 15)	M1
	$4x+11y-78=0,\ 22x-8y-155=0$ $\Rightarrow x=8.5,\quad y=4$	Solves simultaneously	M1
	Max a $=\sqrt{(12.5-"8.5")^2+(15-"4")^2}$	Attempt distance between (12.5, 15) and their coordinates. Dependent upon both previous M's	ddM1
	$a=\sqrt{137}$ or $-\sqrt{137}$	Correct value for a	A1
	$-\sqrt{137}<a<\sqrt{137}$	Correct inequality	A1
			(5)
(b) Way 3 **Via simultaneous equations and discriminant**	$(x-12.5)^2+(y-15)^2=a^2$	Attempt Cartesian equation. This may appear as $y=$ using the identity $\sin t=\sqrt{1-\cos^2 t}$	M1
	$x=\frac{78-11y}{4}$ or $y=\frac{78-4x}{11}$ $\Rightarrow\left(\frac{78-11y}{4}-12.5\right)^2+(y-15)^2=a^2$ or $\Rightarrow(x-12.5)^2+\left(\frac{78-4x}{11}-15\right)^2=a^2$ Substitutes to set up a quadratic equation (not an inequation) in x or y		M1
	$137y^2-1096y+4384-16a^2=0$ or $137x^2-2329x+26475.25-121a^2=0$ Attempts to find the critical value for a using $b^2-4ac\ldots0\Rightarrow$ oe $1096^2-4\times137(4384-16a^2)<0$ or $2329^2-4\times137(26475.25-121a^2)<0$ Dependent upon both previous M's		ddM1
	$a=\sqrt{137}$ or $-\sqrt{137}$	Correct value for a	A1
	$-\sqrt{137}<a<\sqrt{137}$	Correct inequality	A1
			(5)
			Total 10

Note: There may be many other ways of attempting this question. Consider each one carefully please.
(a) Via Cartesian coordinates:

M1: Attempts y in terms of x. FYI $y=(x+1)^{\frac{3}{2}}-(x+1)^{\frac{1}{2}}$ and attempts chain rule condoning slips

Alt $y=x(x+1)^{\frac{1}{2}}$ and attempts product rule condoning slips

A1: $\frac{dy}{dx}=\frac{3}{2}(x+1)^{\frac{1}{2}}-\frac{1}{2}(x+1)^{-\frac{1}{2}}$ or $\frac{dy}{dx}=(x+1)^{\frac{1}{2}}+\frac{1}{2}x(x+1)^{-\frac{1}{2}}$

In (b)

Alternatively, for all three M's, finds the distance from $(\alpha,\beta)=(12.5,15)$ to $ax+by+c=0$ ($4x+11y-78=0$) using $\frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}}$

A1: $a=\sqrt{137}$

A1: $-\sqrt{137}<a<\sqrt{137}$

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