Pearson Edexcel

Mark Scheme (Results)

January 2019

Pearson Edexcel International Advanced Level In Core Mathematics C34 (WMA02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1(a)	$R=\sqrt{53}$	cao	B1
	$\tan\alpha=\frac{2}{7}\Rightarrow\alpha=...$ $\tan\alpha=\pm\frac{2}{7}$ or $\tan\alpha=\pm\frac{7}{2}$ or $\sin\alpha=\pm\frac{2}{'\sqrt{53}'}$ or $\sin\alpha=\pm\frac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\frac{7}{'\sqrt{53}'}$ or $\cos\alpha=\pm\frac{2}{'\sqrt{53}'}$ $\Rightarrow\alpha=...$ Uses one of these equations to find a value for α		M1
	$\alpha=15.95^{\circ}$	Awrt 15.95° (Allow awrt 0.28 (rad))	A1
			(3)
(b)	$\sqrt{53}\sin(2\theta-15.95^{\circ})=4\Rightarrow\sin(2\theta-15.95^{\circ})=\frac{4}{\sqrt{53}}(0.549)$ Attempts to use part (a) $"\sqrt{53}"\sin(2\theta-"15.95^{\circ}")=4$ and proceeds to $\sin(2\theta\pm"15.95^{\circ}")=K,\quad \lvert K\rvert<1$ Allow the letter α for "15.95"		M1
	$2\theta-15.95^{\circ}=33.3287...\Rightarrow\theta=24.6^{\circ}$	Awrt 24.6° (Allow awrt 0.43 (rad))	A1
	$2\theta-15.95^{\circ}=180^{\circ}-33.287...\Rightarrow\theta=...$ Correct attempt at a second solution in the range. E.g. $2\theta_2\mp'15.95^{\circ}'=180^{\circ}-'33.3287...^{\circ}'\Rightarrow\theta_2=\frac{180^{\circ}-'33.3287...^{\circ}'\pm'15.95^{\circ}'}{2}$ (May be implied by their θ_2) It is **dependent** upon having scored the previous M. **Do not allow mixing of radians and degrees so if working in radians must be using π not 180**		dM1
	$\theta=81.3^{\circ}$	Awrt 81.3° **only**	A1
	Ignore extra answers outside range but deduct the final A for extra answers in range.		
			(4)
(c)	$28\sin\theta\cos\theta=a\sin2\theta\Rightarrow a=14$	$a=14$	B1
	$8\sin^2\theta=b\left(\pm1\pm2\sin^2\theta\right)+c$ or $8\sin^2\theta=8\left(\frac{1}{2}\left(\pm1\pm\cos2\theta\right)\right)$ or $8\sin^2\theta=4\sin^2\theta+4\sin^2\theta=4\sin^2\theta+4\left(1-\cos^2\theta\right)=\pm4\cos2\theta\pm4$ Attempts to use a $\cos2\theta$ identity e.g. $\cos2\theta=\pm1\pm2\sin^2\theta$ or $\sin^2\theta=\frac{1}{2}\left(\pm1\pm\cos2\theta\right)$ at some point in their working and applies it to the given expression.		M1
	$b=-4,\ c=4$ or $...-4\cos2\theta+4$	Correct values or correct expression	A1
			(3)
(d)	$\left(28\sin\theta\cos\theta+8\sin^2\theta\right)_{\max}=2'\sqrt{53}'+'4'$	Maximum $=2\times$ their $\sqrt{53}+$ their c May be implied e.g. by their decimal answer.	M1
	$2\sqrt{53}+4$	Cao (must be exact not decimals)	A1
	Attempts to use calculus for the maximum should reach $2R+c$ as above for M1 .		
			(2)
			Total 12

Question Number	Scheme	Notes	Marks
2	$$\frac{3x^2+4x-7}{(x+1)(x-3)} \equiv A+\frac{B}{x+1}+\frac{C}{x-3}$$		
(a)	$A=3$	Must be clearly identified as the value of A. (May be implied by their partial fractions)	B1
	$3x^2+4x-7=A(x+1)(x-3)+B(x-3)+C(x+1)$ And then expands and compares coefficients or substitutes values of x leading to a value for B or C **Or** $3x^2+4x-7\div(x+1)(x-3)=3+\frac{\text{"}10x+2\text{"}}{(x+1)(x-3)}$ $\Rightarrow 10x+2=B(x-3)+C(x+1)$ And then expands and compares coefficients or substitutes values of x leading to a value for B or C A correct method may be implied by correct values provided no incorrect work is seen		M1
	$B=2$ or $C=8$	One of B or C correct	A1
	$B=2$ and $C=8$	Both B and C correct	A1
			(4)

(b) Way 1	**If correct values for *A*, *B* and *C* are obtained by an incorrect method in part (a), allow a full recovery in (b)**		
	$\frac{1}{x+1}=(1+x)^{-1}=(1-x+x^2...)$	Attempts to expand $(1+x)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	$\frac{1}{x-3}=-(3-x)^{-1}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$	$\frac{1}{x-3}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$. Takes out a correct factor including the minus sign **and** a correct bracket.	B1
	$\left(1-\frac{1}{3}x\right)^{-1}=1+\frac{1}{3}x+\frac{1}{9}x^2...$	Attempts to expand $\left(1\pm\frac{1}{3}x\right)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.	M1
	Note $-(3-x)^{-1}$ can be expanded as $-\left(3^{-1}+(-1)3^{-2}(-x)+\frac{(-1)(-2)}{2!}3^{-3}(-x)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term **or** $\frac{1}{x-3}$ can be expanded as $(x-3)^{-1}=3^{-1}\left(\frac{x}{3}-1\right)^{-1}\left(=3^{-1}\left(-1+\frac{x}{3}\right)^{-1}\right)$ $=3^{-1}\left(-1-(-1)^{-2}\left(\frac{x}{3}\right)+\frac{-1(-2)}{2}(-1)^{-3}\left(\frac{x}{3}\right)^2+...\right)$ Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term		
	$\frac{3x^2+4x-7}{(x+1)(x-3)}\approx(3+)2(1-x+x^2)-\frac{8}{3}\left(1+\frac{1}{3}x+\frac{1}{9}x^2\right)$ Combines using their expansions and at least their *B* and *C* (so allow if they forget/don't add their *A*)		M1
	$=\frac{7}{3}-\frac{26}{9}x+\frac{46}{27}x^2$	Any 2 correct terms	A1
		All terms correct	A1
	Allow $2\frac{1}{3}$ for $\frac{7}{3}$, $-2\frac{8}{9}$ for $-\frac{26}{9}$, $1\frac{19}{27}$ for $\frac{46}{27}$		
			(6)
			Total 10

<table>
<tr><td rowspan="9">2(b)</td><td colspan="2">(b) Way 2 not requiring part (a) using
$\frac{3x^2+4x-7}{(x+1)(x-3)}=(3x^2+4x-7)(x+1)^{-1}(x-3)^{-1}$</td><td></td></tr>
<tr><td>$(1+x)^{-1}=(1-x+x^2...)$</td><td>Attempts to expand $(1+x)^{-1}$.
Look for 1 + a correct simplified or unsimplified second or third term.</td><td>M1</td></tr>
<tr><td>$\frac{1}{x-3}=-(3-x)^{-1}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$</td><td>$\frac{1}{x-3}=-\frac{1}{3}\left(1-\frac{1}{3}x\right)^{-1}$ or $-3^{-1}\left(1-\frac{1}{3}x\right)^{-1}$
Takes out a correct factor including the minus sign.</td><td>B1</td></tr>
<tr><td>$\left(1-\frac{1}{3}x\right)^{-1}=1+\frac{1}{3}x+\frac{1}{9}x^2...$</td><td>Attempts to expand $\left(1\pm\frac{1}{3}x\right)^{-1}$. Look for 1 + a correct simplified or unsimplified second or third term.</td><td>M1</td></tr>
<tr><td colspan="2">Note
$-(3-x)^{-1}$ can be expanded as $-\left(3^{-1}+(-1)3^{-2}(-x)+\frac{(-1)(-2)}{2!}3^{-3}(-x)^2+...\right)$
Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term
or
$\frac{1}{x-3}$ can be expanded as $(x-3)^{-1}=3^{-1}\left(\frac{x}{3}-1\right)^{-1}\left(=3^{-1}\left(-1+\frac{x}{3}\right)^{-1}\right)$
$=3^{-1}\left(-1-(-1)^{-2}\left(\frac{x}{3}\right)+\frac{-1(-2)}{2}(-1)^{-3}\left(\frac{x}{3}\right)^2+...\right)$
Score B1 for -3^{-1} as the first term and M1 for correct attempt at the 2nd or 3rd term</td><td></td></tr>
<tr><td colspan="2">$\frac{3x^2+4x-7}{(x+1)(x-3)}\approx(3x^2+4x-7)\left(-\frac{1}{3}\right)\left(1+\frac{1}{3}x+\frac{1}{9}x^2\right)(1-x+x^2)=...$
Attempts to multiply out all 3 brackets</td><td>M1</td></tr>
<tr><td rowspan="2">$=\frac{7}{3}-\frac{26}{9}x+\frac{46}{27}x^2$</td><td>Any 2 correct terms</td><td>A1</td></tr>
<tr><td>All terms correct</td><td>A1</td></tr>
<tr><td></td><td></td><td>(6)</td></tr>
</table>

Question Number	Scheme	Notes	Marks
3(a)	$\mathrm{f}\left(-\frac{3k}{4}\right)=\ldots$ or $\mathrm{f}(-4k)=\ldots$	Attempts $\mathrm{f}\left(-\frac{3k}{4}\right)$ **or** $\mathrm{f}(-4k)$	M1
	Note: Candidates who use completion of the square to obtain e.g. $a\left(x+\frac{3k}{4}\right)^2+b$ must then identify the "b" as an "end point" if they do not explicitly find $\mathrm{f}\left(-\frac{3k}{4}\right)$		
	$y_{\min}=-\frac{k^2}{8}$ or $y>-\frac{k^2}{8}$ or $y\geq-\frac{k^2}{8}$ or $y_{\max}=21k^2$ or $y<21k^2$ or $y\leq 21k^2$	One correct "end" of the range. May be implied by their final answer. Allow strict and non-strict inequality symbols or other indications that values are max or min.	A1
	$\mathrm{f}\left(-\frac{3k}{4}\right)=\ldots$ and $\mathrm{f}(-4k)=\ldots$	Attempts $\mathrm{f}\left(-\frac{3k}{4}\right)$ **and** $\mathrm{f}(-4k)$	M1
	Note: Candidates who use completion of the square to obtain e.g. $a\left(x+\frac{3k}{4}\right)^2+b$ must then identify the "b" as an "end point" if they do not explicitly find $\mathrm{f}\left(-\frac{3k}{4}\right)$		
	$-\frac{k^2}{8}\leq \mathrm{f}(x)\leq 21k^2$ $\left[-\frac{k^2}{8}, 21k^2\right]$ $\mathrm{f}(x)\geq-\frac{k^2}{8}$ **and** $\mathrm{f}(x)\leq 21k^2$ $\mathrm{f}(x)\geq-\frac{k^2}{8}\cap \mathrm{f}(x)\leq 21k^2$	Correct range. Allow alternative notation as shown and allow y or "range" for f(x) but do not allow x for f(x).	A1
			(4)
(b)	$\mathrm{gf}(-2)=2k-3\left(2(-2)^2+3k(-2)+k^2\right)$ or $\mathrm{gf}(x)=2k-3\left(2x^2+3kx+k^2\right)$	Correct expression for gf(−2) or gf(x). Award this mark as soon as a correct expression is seen.	B1
	$2k-3\left(2(-2)^2+3k(-2)+k^2\right)=-12$	Puts their gf(−2) = ±12 to obtain an equation in k only. Must be using $x=-2$.	M1
	$3k^2-20k+12=0$ $\Rightarrow(3k-2)(k-6)=0\Rightarrow k=\frac{2}{3},\ 6$	Solves a 3TQ – see general guidance. **Dependent on the previous M.**	dM1
		Correct values. Allow equivalent fractions for $\frac{2}{3}$ or 0.6 with a clear dot over the 6.	A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
4	$81y^3 + 64x^2 y + 256x = 0$		
(a)	$\frac{\mathrm{d}(81y^3)}{\mathrm{d}x} = 243y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	$\frac{\mathrm{d}(81y^3)}{\mathrm{d}x} = ky^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}(64x^2 y)}{\mathrm{d}x} = 128xy + 64x^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	$\frac{\mathrm{d}(64x^2 y)}{\mathrm{d}x} = \alpha xy + \beta x^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$243y^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 128xy + 64x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 256(= 0)$	Correct differentiation. The "= 0" is not required but there should be no extra terms.	A1
	For the first 3 marks you can ignore any spurious "$\frac{\mathrm{d}y}{\mathrm{d}x} =$" at the start.		
	$243y^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 64x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = -128xy - 256 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}(243y^2 + 64x^2) = -128xy - 256$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ Makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject allowing sign errors only with "= 0" seen or implied. This depends on there being **exactly two** $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms. One coming from the differentiation of $81y^3$ and one coming from the differentiation of $64x^2 y$		M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-128xy - 256}{243y^2 + 64x^2}$	Correct expression (oe)	A1
	Note that the final M1A1 in (a) <u>can</u> be recovered in part (b)		
			(5)

(b)	**Note that full marks are available in (b) following an incorrect <u>denominator</u> in (a)**		
	$-128xy-256=0$	Sets their numerator = 0. Note that this may appear from putting $\frac{dy}{dx}=0$ into their differentiation in part (a) before making $\frac{dy}{dx}$ the subject.	M1
	$81y^3+64y\left(-\frac{2}{y}\right)^2+256\left(-\frac{2}{y}\right)=0$ or $81\left(-\frac{2}{x}\right)^3+64x^2\left(-\frac{2}{x}\right)+256x=0$	Substitutes to obtain an equation in one variable. **Dependent on the first M.**	**d**M1
	$y^4=\frac{256}{81}\Rightarrow y=...$ or $x^4=\frac{81}{16}\Rightarrow x=...$	Solves an equation of the form $y^4=p$ or $x^4=q$ $(p, q>0)$ **Depends on the previous M.**	**d**M1
	$y=\pm\frac{4}{3}$ or $x=\pm\frac{3}{2}$	2 Correct values for x or 2 correct values for y. Allow unsimplified for this mark.	A1
	$y=(\pm)"\frac{4}{3}"\Rightarrow x=...$ or $x=(\pm)"\frac{3}{2}"\Rightarrow y=...$	Attempts at least one value of the other variable having previously found and solved an equation in one variable.	M1
	Examples: $\left(\pm\frac{3}{2},\mp\frac{4}{3}\right)$ or $x=\pm\frac{3}{2},\ y=\mp\frac{4}{3}$ or $x=\frac{3}{2},\ y=-\frac{4}{3}$, and $x=-\frac{3}{2},\ y=\frac{4}{3}$ or $\left(\frac{3}{2},-\frac{4}{3}\right),\left(-\frac{3}{2},\frac{4}{3}\right)$	Correct values which must now be simplified and paired correctly. **Do not isw and mark their final answer.**	A1
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
5	$\tan x = m$ and $4\tan y = 8m + 5$		
(a)	Examples: $\sec^2 x = 1 + m^2$ or $\sec^2 y = 1 + \left(\frac{8m+5}{4}\right)^2$ or $16\sec^2 y = 16 + 16(8m+5)^2$	Attempts to express $\sec^2 x$ or $\sec^2 y$ in terms of m using a **correct** identity.	M1
	$16\left(\sec^2 x + \sec^2 y\right) = 16\left(1 + m^2 + 1 + \left(\frac{8m+5}{4}\right)^2\right) = 537$ Uses their expressions in m and 537 to obtain a quadratic equation in terms of m which may be unsimplified.		M1
	$m^2 + m - 6 = 0 \Rightarrow m = 2, -3$	Solves their 3TQ as far as $m = ...$	M1
		Correct values	A1
			(4)
(b)	$\tan x = 2 \Rightarrow \sin x = \frac{2}{\sqrt{1^2 + 2^2}} = ...$ Correct method for the value of $\sin x$. Must be from an appropriate identity or exact work but m does not need to be exact. So e.g. $\sin\left(\tan^{-1} 2\right) = 0.8944..$ scores M0 Can be for using either of their values of m.		M1
	$= \frac{2}{\sqrt{5}}$	cao (oe) **and no other values**	A1
			(2)
(c)	$\tan y = \frac{21}{4} \Rightarrow \cot y = \frac{4}{21}$	Correct method to obtain a value for $\cot y$. So uses $4\tan y = 8m + 5$ and their m to find a value for $\tan y$ and finds reciprocal. Can be for using either of their values of m.	M1
		cao (oe) **and no other values**	A1
			(2)
			Total 8

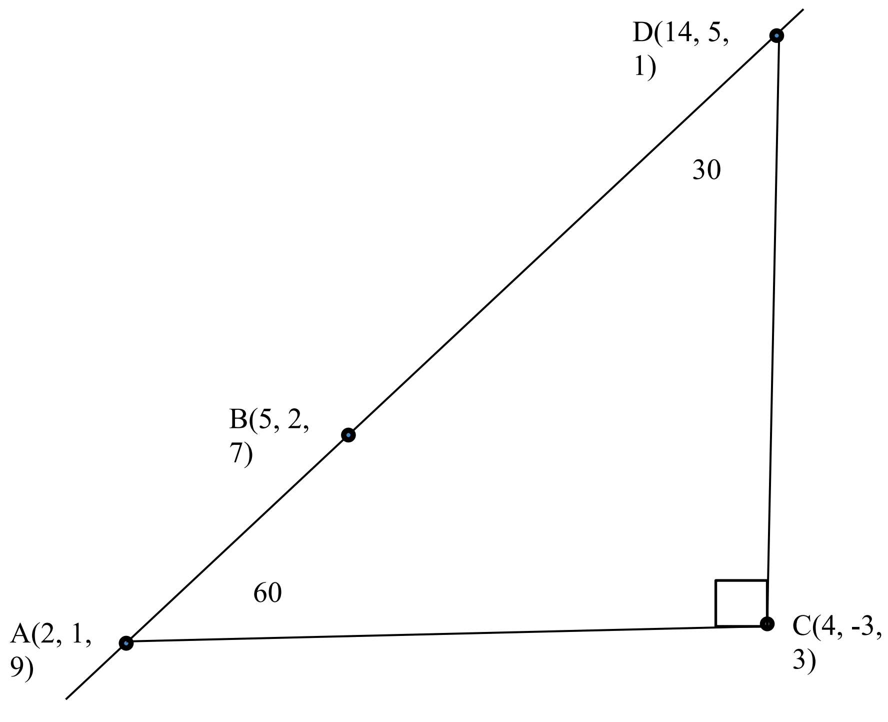
Question Number	Scheme	Notes	Marks
6(a)	$\pm\overrightarrow{AB} = \pm\left(\begin{pmatrix}5\\2\\7\end{pmatrix} - \begin{pmatrix}2\\1\\9\end{pmatrix}\right)$	Correct attempt at direction. May be implied by at least 2 correct components if no method seen.	M1
	$\mathbf{r} = \begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$ or $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 9\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	Accept equivalents but it must be an equation and it must be "r =" or $\begin{pmatrix}x\\y\\z\end{pmatrix} = \ldots$	A1
	Equivalent correct answers include: $\mathbf{r} = \begin{pmatrix}2\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}-3\\-1\\2\end{pmatrix}$ $\mathbf{r} = \begin{pmatrix}5\\2\\7\end{pmatrix} + \lambda\begin{pmatrix}-3\\-1\\2\end{pmatrix}$, $\mathbf{r} = \begin{pmatrix}5\\2\\7\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$		
	Do not allow e.g. $\mathbf{r} = \begin{pmatrix}2\mathbf{i}\\\mathbf{j}\\9\mathbf{k}\end{pmatrix} + \lambda\begin{pmatrix}3\mathbf{i}\\\mathbf{j}\\-2\boldsymbol{k}\end{pmatrix}$ unless a correct form is seen earlier then isw		
			(2)

(b) Way 1	$\overrightarrow{AC}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$	Attempts $\pm\overrightarrow{AC}$. May be implied by at least 2 correct components if no method seen.	M1
	$\pm\overrightarrow{AB}.\pm\overrightarrow{AC}=\lvert AB\rvert\lvert AC\rvert\cos\theta\Rightarrow 6-4+12=\sqrt{14}\sqrt{56}\cos\theta$ $\Rightarrow\cos\theta\Rightarrow\dfrac{14}{\sqrt{14}\sqrt{56}}\Rightarrow\theta=...$ Attempt the scalar product of $\pm\overrightarrow{AB}$ or their direction vector from part (a) and their $\pm\overrightarrow{AC}$ and proceeds to $\theta=\ldots$		dM1
	$\theta=60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)
	(b) Way 2 (cosine rule on triangle *ABC*)		
	$AB=\sqrt{14},\ AC=2\sqrt{14},\ BC=\sqrt{42}$	Attempts the lengths of all 3 sides	M1
	$42=14+56-2\sqrt{14}\sqrt{56}\cos\theta$ $\Rightarrow\cos\theta=\dfrac{28}{2\sqrt{14}\sqrt{56}}\Rightarrow\theta=...$	Attempt cosine rule and proceeds to $\theta=\ldots$	dM1
	$\theta=60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)
	(b) Way 3 using vector product		
	$\overrightarrow{AC}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$	Attempts $\pm\overrightarrow{AC}$	M1
	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-4&-6\\3&1&-2\end{vmatrix}=\begin{pmatrix}14\\-14\\14\end{pmatrix}\Rightarrow\sin\theta=\dfrac{\sqrt{14^2+14^2+14^2}}{\sqrt{2^2+4^2+6^2}\sqrt{3^2+1^2+2^2}}$ Attempt the vector product of $\pm\overrightarrow{AB}$ or their direction vector from part (a) and their $\pm\overrightarrow{AC}$ and proceeds to $\theta=\ldots$		M1
	$\theta=60^{\circ}$	Cao (Must be **degrees** not radians)	A1
			(3)

(c) Way 1	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on AB) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$\begin{pmatrix}2\\-4\\-6\end{pmatrix}\bullet\left\{\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}\right\}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}\bullet\begin{pmatrix}3\lambda-2\\\lambda+4\\-2\lambda+6\end{pmatrix}$ $6\lambda-4-4\lambda-16-36+12\lambda=0\Rightarrow\lambda=...$ Attempts $\overrightarrow{AC}\bullet\overrightarrow{CD}=0$ and solves for λ. It must be a correct AC or their attempt at AC and a correct attempt at CD or what they think is CD as long as it is clearly identified as CD.		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses **their** value of λ to find D. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1
			(4)
	(c) Way 2:		
	$AC=2\sqrt{14}\Rightarrow AD=\dfrac{2\sqrt{14}}{\cos 60°}\left(=4\sqrt{14}\right)$	Correct attempt at the length of AD	M1
	$AB=\sqrt{14}\Rightarrow AD=4AB$ or $(3\lambda)^2+\lambda^2+(2\lambda)^2=\left(4\sqrt{14}\right)^2\Rightarrow\lambda=...$	Uses ratio of AB to AD to find a value for "λ" or uses the length of AD and applies Pythagoras to "λ"×their direction of l to find a value for "λ"	M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of "λ" to find D. **Dependent on both previous M's**	**dd**M1
	D(14, 5, 1) or 14**i** + 5**j** + **k** etc.	Correct coordinates or vector and **no other points or vectors**	A1
	(c) Way 3		
	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on AB) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$(3\lambda-2)^2+(\lambda+4)^2+(6-2\lambda)^2=(AC\tan 60)^2$ $\lambda^2-2\lambda-8=0\Rightarrow\lambda=...$ Attempts $(CD)^2=(AC\tan 60)^2$ and solves for λ		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of λ to find D. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1

	(c) Way 4		
	$\begin{pmatrix}2\\1\\9\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}-\begin{pmatrix}4\\-3\\3\end{pmatrix}$	Attempts $\overrightarrow{CD}$ by finding: (a general point on AB) – $\overrightarrow{OC}$ or (their part (a)) – $\overrightarrow{OC}$	M1
	$(3\lambda-2)^2+(\lambda+4)^2+(6-2\lambda)^2+AC^2=(3\lambda)^2+\lambda^2+(2\lambda)^2$ $28\lambda-112=0\Rightarrow\lambda=...$ Attempts $AC^2+CD^2=AD^2$ and solves for λ		M1
	$\lambda="4"\Rightarrow OD=\begin{pmatrix}2\\1\\9\end{pmatrix}+"4"\begin{pmatrix}3\\1\\-2\end{pmatrix}$	Uses their value of λ to find *D*. **Dependent on both previous M's**	**dd**M1
	(14, 5, 1) or 14**i** + 5**j** + **k** or $\begin{pmatrix}14\\5\\1\end{pmatrix}$	Correct coordinates or vector and no other points or vectors.	A1

(d)	Area ADC = $\frac{1}{2}AC\times CD=\frac{1}{2}\sqrt{56}\sqrt{168}$	Correct triangle area method	M1
	$=28\sqrt{3}$	cao	A1
			(2)
	Alternatives for (d)		
	$\frac{1}{2}AC\times AD\sin 60^\circ=\frac{1}{2}\sqrt{56}\sqrt{224}\frac{\sqrt{3}}{2}$, $\frac{1}{2}AD\times DC\sin 30^\circ=\frac{1}{2}\sqrt{168}\sqrt{224}\frac{1}{2}$ $\frac{1}{2}AC\times AC\tan 60^\circ=\frac{1}{2}\sqrt{56}\sqrt{56}\sqrt{3}$		
			Total 11



Question Number	Scheme	Notes	Marks
7(a)	Strip width = 0.5	Correct value stated or used within the formula.	B1
	$\frac{11\sqrt{5}}{5}+\frac{13}{3}+2\left(\frac{23\sqrt{6}}{12}+\frac{12\sqrt{7}}{7}+\frac{25\sqrt{2}}{8}\right)$ or $\left(4.91...+4.33...+2\left(4.69...+4.53...+4.41...\right)\right)$ Correct structure for their *y* values (if their values need checking, look for 2sf) Must have *y* values starting at *x* = 4 and ending at *x* = 6		M1
	Area $\approx\frac{1}{2}\times\frac{1}{2}\left(\frac{11\sqrt{5}}{5}+\frac{13}{3}+2\left(\frac{23\sqrt{6}}{12}+\frac{12\sqrt{7}}{7}+\frac{25\sqrt{2}}{8}\right)\right)$ or Area $\approx\frac{1}{2}\times\frac{1}{2}\left(4.91...+4.33...+2\left(4.69...+4.53...+4.41...\right)\right)$ Correct numerical expression for the area (allow decimal values to 2sf), may be implied by their area value		A1
	9.14	9.14 **only**	A1
			(4)
(b)	$u=2x-3\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=2$	Correct derivative. Accept any correct equivalents e.g. d*u* = 2d*x*	B1
	$\int\frac{x+7}{\sqrt{2x-3}}\mathrm{d}x=\int\frac{\frac{u+3}{2}+7}{\sqrt{u}}\,\frac{1}{2}\mathrm{d}u$	M1: Fully substitutes. Just replacing "d*x*" with "d*u*" with no evidence of where the "d*u*" has come from is M0 but allow slips e.g. omission of " + 7" A1: Fully correct expression.	M1A1
	$\frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}+34u^{\frac{1}{2}}\right)(+c)$	Fully correct integration in any form (+ *c* not required)	A1
	Note: Integration by parts gives $\frac{1}{4}\int(u+17)u^{-\frac{1}{2}}\ \mathrm{d}u=\frac{1}{4}\left(2u^{\frac{1}{2}}\right)(u+17)-\frac{1}{4}\int2u^{-\frac{1}{2}}\ \mathrm{d}u=\frac{1}{2}u^{\frac{3}{2}}+\frac{17}{2}u^{\frac{1}{2}}-\frac{1}{3}u^{\frac{3}{2}}(+c)$		
	$x=4,u=5\quad x=6,u=9$	Correct *u* limits seen anywhere.	B1
	If they return to *x* then this B1 is for replacing *u* with $2x-3$		
	$\frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}}+34u^{\frac{1}{2}}\right]_5^9=\frac{1}{4}\left\{\left(\frac{2}{3}(9)^{\frac{3}{2}}+34(9)^{\frac{1}{2}}\right)-\left(\frac{2}{3}(5)^{\frac{3}{2}}+34(5)^{\frac{1}{2}}\right)\right\}$ Substitutes their (changed) *u* limits into a changed function and subtracts either way round or substitutes *x* limits if they undo the substitution and subtracts either way round		M1
	$=30-\frac{28}{3}\sqrt{5}$	cao	A1
			(7)
			Total 11

	Note that 7(b) is hence or otherwise so other substitutions will be seen. The mark scheme will follow the same structure as in the example below:		
7(b)	$u^2 = 2x-3 \Rightarrow 2u\dfrac{du}{dx} = 2$	Correct derivative. Accept correct equivalents e.g. $2u = 2\dfrac{dx}{du}$, $dx = u\mathrm{d}u$	B1
	$\displaystyle\int \frac{x+7}{\sqrt{2x-3}}\,dx = \int \frac{\frac{u^2+3}{2}+7}{u}\,u\,du$	M1: Fully substitutes. Allow slips e.g. omission of "+ 7" A1: Fully correct expression.	M1A1
	$\dfrac{u^3}{6}+\dfrac{17}{2}u(+c)$	Fully correct integration in any form (+ c not required)	A1
	$x=4, u=\sqrt{5}\quad x=6, u=3$	Correct u limits seen anywhere.	B1
	If they return to x then this B1 is for replacing u with $\sqrt{2x-3}$		
	$\left[\dfrac{1}{6}u^3+\dfrac{17}{2}u\right]_{\sqrt{5}}^{3} = \left\{\left(\dfrac{27}{6}+\dfrac{17}{2}(3)\right)-\left(\dfrac{1}{6}\left(\sqrt{5}\right)^3+\dfrac{17}{2}\left(\sqrt{5}\right)\right)\right\}$ Substitutes their (changed) u limits into a changed function and subtracts either way round or substitutes x limits if they undo the substitution and subtracts either way round		M1
	$=30-\dfrac{28}{3}\sqrt{5}$	cao	A1
			(7)

	Note that 7(b) can also be done by parts:		
7(b)	$\displaystyle\int \frac{x+7}{\sqrt{2x-3}}\,dx = \int (x+7)(2x-3)^{-\frac{1}{2}}\,dx$	Uses $\dfrac{x+7}{\sqrt{2x-3}}$ as $(x+7)(2x-3)^{-\frac{1}{2}}$ and makes some progress with attempting to integrate even if it is incorrect.	B1
	$\displaystyle\int (x+7)(2x-3)^{-\frac{1}{2}}\,dx = (x+7)(2x-3)^{\frac{1}{2}} - \int (2x-3)^{\frac{1}{2}}\,dx$ M1: Integration by parts in the correct direction A1: Correct expression		M1A1
	$\displaystyle\int (2x-3)^{\frac{1}{2}}\,dx = \frac{1}{3}(2x-3)^{\frac{3}{2}}$	$\displaystyle\int (2x-3)^{\frac{1}{2}}\,dx = k(2x-3)^{\frac{3}{2}}$	M1
		$\displaystyle\int (2x-3)^{\frac{1}{2}}\,dx = \frac{1}{3}(2x-3)^{\frac{3}{2}}$	A1
	$\left[(x+7)(2x-3)^{\frac{1}{2}}-\dfrac{1}{3}(2x-3)^{\frac{3}{2}}\right]_4^6 = \left\{\left(11(9)^{\frac{1}{2}}-\dfrac{1}{3}(9)^{\frac{3}{2}}\right)-\left(11(5)^{\frac{1}{2}}-\dfrac{1}{3}(5)^{\frac{3}{2}}\right)\right\}$ Substitutes the limits 4 and 6 into a changed function and subtracts either way round		M1
	$=30-\dfrac{28}{3}\sqrt{5}$	cao	A1
			(7)

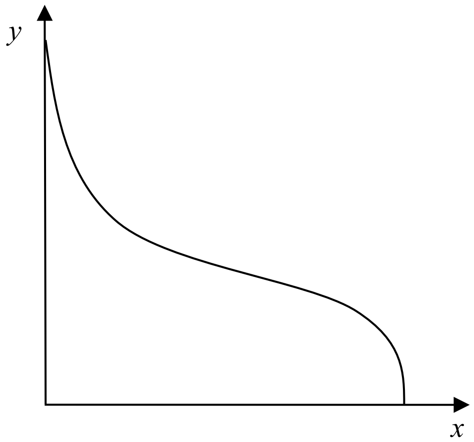
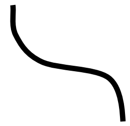
Question Number	Scheme	Notes	Marks
8(a)	$\frac{dx}{dt} = 2t - 1$	Correct derivative	B1
	Quotient rule $\frac{dy}{dt} = \frac{(1-t)\times 4 - 4t \times (-1)}{(1-t)^2}$ Obtains $\frac{dy}{dt} = \frac{\alpha(1-t) \pm \beta t}{(1-t)^2}, \quad \alpha > 0, \ \beta > 0$ **or product rule** $\frac{dy}{dt} = 4t(1-t)^{-2} + 4(1-t)^{-1}$ Obtains $\frac{dy}{dt} = p(1-t)^{-1} \pm qt(1-t)^{-2}, \quad p > 0, \ q > 0$ **If an incorrect formula is quoted this scores M0**		M1
	NB: May see $\frac{4t}{1-t} = -4 + \frac{4}{1-t} \Rightarrow \frac{dy}{dt} = -4(1-t)^{-2} \times -1$ Allow M1 for $\frac{4t}{1-t} = A + \frac{B}{1-t} \Rightarrow \frac{dy}{dt} = k(1-t)^{-2}$		
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{(1-t)\times 4 - 4t \times (-1)}{(1-t)^2} \times \frac{1}{2t-1}$ **Correct** application of the chain rule using their derivatives. This is an independent method mark. Their $\mathbf{\frac{dy}{dt}}$ divided by their $\mathbf{\frac{dx}{dt}}$ **or** their $\mathbf{\frac{dy}{dt}}$ multiplied by their $\mathbf{\frac{dt}{dx}}$		M1
	$\frac{dy}{dx} = \frac{4}{(2t-1)(1-t)^2}$ Allow e.g. $\frac{4}{(2t-1)(1-2t+t^2)}, \frac{4}{2t^3 - 5t^2 + 4t - 1}$ but not $\frac{1}{(2t-1)} \times \frac{4}{(1-t)^2}$ **But isw once a correct answer is seen**		A1
			(4)
(b)	$t = -1 \to (2, -2)$ or $x = 2, y = -2$	Correct coordinates for P	B1
	$t = -1 \Rightarrow \frac{dy}{dx} = \frac{4}{(2(-1)-1)(1-(-1))^2} \left(= -\frac{1}{3} \right)$	Attempts gradient. May be implied by their value for the gradient.	M1
	$y + 2 = -\frac{1}{3}(x - 2)$	Correct straight line method for the tangent **not the normal**. If using $y = mx + c$ must reach as far as finding a value for c.	M1
	$x + 3y + 4 = 0$	Any integer multiple.	A1
			(4)

(c) **Way 1**	$t^2 - t + 3\left(\dfrac{4t}{1-t}\right) + 4 = 0$	Substitutes to obtain an equation in *t* only.	M1
	$t^3 - 2t^2 - 7t - 4 = 0$	Correct cubic	A1
	$(t+1)(t^2-3t-4)=0$ or $(t+1)^2(t-4)=0$ Attempt to factorise using $(t \pm 1)$ or $(t \pm 1)^2$ as a factor. Look for $(t\pm1)(at^2+...)$ or $(t\pm1)^2(at+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $at^2 + \ldots$ or $at + \ldots$ **<u>This mark is dependent on having obtained a cubic equation that has a constant term</u>** **This mark is not for just solving their cubic e.g. using a calculator. However, if they have a correct cubic equation and the root *t* = 4 is seen, this method can be implied.**		M1
	$t = 4$	Correct value of *t*	A1
	$\left(12, -\dfrac{16}{3}\right)$	Correct coordinates	A1
			(5)
			Total 13
(c) **Way 2**	$y = \dfrac{4t}{1-t} \Rightarrow t = \dfrac{y}{4+y} \Rightarrow x = \left(\dfrac{y}{4+y}\right)^2 - \dfrac{y}{4+y}$ $\Rightarrow \left(\dfrac{y}{4+y}\right)^2 - \dfrac{y}{4+y} + 3y + 4 = 0$	Finds *x* in terms of *y* by eliminating *t* and substitutes to obtain an equation in *y* only. When eliminating *t* using *y*, the algebra must be correct so allow sign errors only for making *t* the subject from *y*.	M1
	$3y^3 + 28y^2 + 76y + 64 = 0$	Correct cubic	A1
	$(y+2)(3y^2+22y+32)=0$ or $(y+2)^2(3y+16)=0$ Attempt to factorise using $(y \pm 2)$ or $(y \pm 2)^2$ as a factor. Look for $(y\pm"-2")(ay^2+...)$ or $(y\pm"-2")^2(ay+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $ay^2 + \ldots$ or $ay + \ldots$ **<u>This mark is dependent on having obtained a cubic equation that has a constant term</u>** **This mark is not for just solving their cubic e.g. using a calculator. However, if they have a correct cubic equation and the root *y* = -16/3 is seen, this method can be implied.**		M1
	$y = -\dfrac{16}{3}$	Correct value of *y*	A1
	$\left(12, -\dfrac{16}{3}\right)$	Correct coordinates	A1
			(5)

Question Number	Scheme	Notes	Marks
9(a)	$\int x\sin 2x\,\mathrm{d}x = -x.\frac{1}{2}\cos 2x + \frac{1}{2}\int \cos 2x\,(\,\mathrm{d}x)$	$\int x\sin 2x\,\mathrm{d}x = \pm px.\cos 2x \pm q\int \cos 2x\,(\,\mathrm{d}x)$	M1
		Correct expression (dx not required)	A1
	$\int x\sin 2x\,\mathrm{d}x = \frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x\,(+c)$	Correct integration in any form – does not need to be simplified but is cso so e.g. any double sign errors should be penalised here. Condone poor notation e.g. cos2x.x rather than $x$cos2x. **The constant of integration is not required.**	A1 **cso**
			(3)
(b)	$(x+\sin 2x)^2 = x^2 + 2x\sin 2x + \sin^2 2x$	Correct (possibly unsimplified) expansion. Condone poor notation so allow e.g. 2sin2x.x for 2$x$sin2x and sin2x^2 for sin^{2}2x	B1
	$\int \sin^2 2x\,\mathrm{d}x = \frac{1}{2}\int (1-\cos 4x)\,\mathrm{d}x$	Uses $\cos 4x = \pm 1 \pm 2\sin^2 2x$	M1
	$\int \sin^2 2x\,\mathrm{d}x = \frac{1}{2}x - \frac{1}{8}\sin 4x\,(+c)$	Correct integration	A1
	$\int (x+\sin 2x)^2\,\mathrm{d}x = \frac{x^3}{3} + \frac{1}{2}\sin 2x - x\cos 2x + \frac{1}{2}x - \frac{1}{8}\sin 4x\,(+c)$ Allow in any correct possibly unsimplified form. Follow through their answer to part (a) so allow for: $\int (x+\sin 2x)^2\,\mathrm{d}x = \frac{x^3}{3} + 2\times \text{their part (a)} + \frac{1}{2}x - \frac{1}{8}\sin 4x\,(+c)$ **The constant of integration not required.**		A1ft
	In part (b) allow mixed variables for the first 3 marks but for the final mark the expression must be in terms of *x* only.		
			(4)
(c)	$(\text{Volume} =)\,\pi\int (x+\sin 2x)^2\,\mathrm{d}x$	States or implies that the volume required is $\pi\int (x+\sin 2x)^2$ **Note that the π <u>is</u> required but may appear later in their working.**	M1
	$= (\pi)\left(\frac{\pi^3}{24} + 0 + \frac{\pi}{2} + \frac{\pi}{4} - 0 - (0)\right)$ Applies at least the limit $\frac{\pi}{2}$ to an expression of the form: $\alpha x^3 + \beta x + (\text{at least one trig function})$. The substitution of $x = 0$ does not need to be seen. Must be exact work and not just decimals.		M1
	$= \frac{\pi^4 + 18\pi^2}{24}$ or $\frac{\pi^2(\pi^2+18)}{24}$	Cso. Allow any equivalent exact single fraction but come from correct integration. Note that incorrect coefficients of sin… will fortuitously give the correct answer.	A1 **cso**
	Note: Condone mixing *x* with *θ*		
			(3)
			Total 10

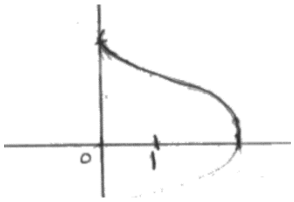
Question Number	Scheme	Notes	Marks
10(a)	E.g. $\frac{r}{h}=\frac{3}{5},\frac{3}{r}=\frac{5}{h},5r=3h,r=\frac{3}{5}h,h=\frac{5}{3}r$	Any correct equation connecting r and h	B1
	$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ $\left(V=\frac{9}{75}\pi h^3\right)$	Obtains $V=kh^3$ or equivalent using their equation connecting h and r. $V=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ is sufficient.	M1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{27}{75}\pi h^2$	Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$. Allow for $\frac{\mathrm{d}V}{\mathrm{d}h}=\alpha h^2$. **Dependent on the first M.**	**d**M1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}h}\Rightarrow\frac{27}{75}\pi h^2=-0.02\frac{\mathrm{d}t}{\mathrm{d}h}$	Uses e.g. $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}h}$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm0.02$ May be implied by their work.	M1
	$h^2\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{18\pi}$ *	Correct equation or states $k=18$	A1 **cso**
			(5)
(a) Way 2	**Avoids the need to find $\frac{\mathrm{d}V}{\mathrm{d}h}$**		
	$\frac{r}{h}=\frac{3}{5},\frac{3}{r}=\frac{5}{h},5r=3h,r=\frac{3}{5}h,h=\frac{5}{3}r$	Any correct equation connecting r and h	B1
	$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{3}{5}h\right)^2h$ $\left(V=\frac{9}{75}\pi h^3\right)$	Obtains $V=kh^3$ (or equivalent) using their equation connecting h and r.	M1
	Examples: $V=\frac{9}{75}\pi h^3\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=3\times\frac{9}{75}\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ or $h^3=\frac{75V}{9\pi}\Rightarrow3h^2\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75V}{9\pi}\frac{\mathrm{d}V}{\mathrm{d}t}$ The M1 is for differentiating both sides with respect to t to obtain $\alpha\frac{\mathrm{d}V}{\mathrm{d}t}=\beta h^2\frac{\mathrm{d}h}{\mathrm{d}t}$		dM1
	$-0.02=3\times\frac{9}{75}\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ or $3h^2\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75V}{9\pi}\times-0.02$	Replaces $\frac{\mathrm{d}V}{\mathrm{d}t}$ with ±0.02	M1
	$h^2\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{18\pi}$ *	Correct equation or states $k=18$	A1
			(5)

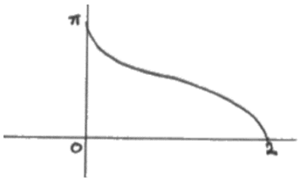
(b) Way 1	$\frac{h^3}{3} = -\frac{1}{18\pi}t(+c)$	$ph^3 = qt(+c)$. **Note that "+ *c*" is not required for this mark.**	M1
	$t = 0, h = 5 \Rightarrow c = \frac{125}{3}$	Uses h = 5 and t = 0 to find c. **There must be a constant of integration for this mark.**	M1
	Note that both M marks are available if the letter k is used i.e. if they haven't obtained a value for k in part (a)		
	$h = \sqrt[3]{125 - \frac{t}{6\pi}}$	Correct equation (oe)	A1
			(3)
(b) Way 2	$\frac{\mathrm{d}V}{\mathrm{d}t} = -0.02 \Rightarrow V = -0.02t + c$ $15\pi = c \Rightarrow V = 15\pi - 0.02t$	Uses $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm 0.02$ to obtain $V = \pm 0.02t + c$ and then uses r = 3 and h = 5 when t = 0 to find c. This may be implied by sight of $V = 15\pi - 0.02t$ (**but must be *V* = …**)	M1
	$\frac{9}{75}\pi h^3 = 15\pi - 0.02t$ $h^3 = 125 - \frac{t}{6\pi} \Rightarrow h = ...$	Replaces V with V in terms of h and rearranges to find h	M1
	$h = \sqrt[3]{125 - \frac{t}{6\pi}}$	Correct equation (oe)	A1
			(3)
(c) Way 1	$h = 0 \Rightarrow 125 - \frac{t}{6\pi} = 0 \Rightarrow t = ...$	Puts h = 0 and solves for t	M1
	$t = 750\pi$ seconds		
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
(c) Way 2	$\left[\frac{h^3}{3}\right]_5^0 = \left[-\frac{1}{18\pi}t\right]_0^T \Rightarrow 0 - \frac{125}{3} = -\frac{1}{18\pi}T \Rightarrow T = ...$ Uses the limits 0 and 5 with their ph^3 and 0 and T or t with their qt and solves for T (or t). The limits can be either way round and the substitution of 0 does not need to be seen. A minimum could be $\frac{125}{3} = \frac{1}{18\pi}t \Rightarrow t = ...$(as in the main scheme)		M1
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
(c) Way 3	$\frac{1}{3}\pi(3)^2 \times 5 \div 0.02 = ...$ or e.g. solves $15\pi - 0.02t = 0$	Calculates the volume of the cone and divides by 0.02	M1
	39 (minutes)	Cao. Must be positive. Allow awrt 39 (minutes) and isw.	A1
			(2)
			Total 10

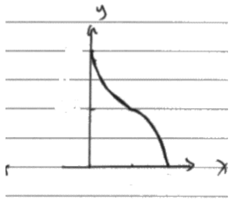
Question Number	Scheme	Notes	Marks
11(a)		 Correct shape anywhere. Ignore any extra "cycles" or other parts of graphs. The curve should become steeper at each end.	M1
		Correct shape in the correct position with no "extra cycles" or other parts of graphs. Ignore any labels on axes, correct or otherwise.	A1
	See next page for example marking		
			(2)
(b)	$\mathrm{f}(0.9)=0.4108...,\ \mathrm{f}(1.1)=-0.4941...$	Substitutes both x = 0.9 and x = 1.1 and obtains at least one answer correct to 1sf or truncated so allow 0.4 and – 0.4 or – 0.5.	M1
	Change of sign therefore $0.9 < \alpha < 1.1$ Both values correct (to one sig fig or truncated), change of sign + conclusion Allow equivalent statements e.g. positive, negative therefore root etc. but this mark may be withheld if there are any contradictory statements e.g. therefore root lies between f(0.9) and f(1.1)		A1
			(2)
(c)	$\arctan\left(\arccos(1.1-1)\right)$	Attempt the given formula with x = 1.1 Score for $\arctan\left(\arccos(1.1-1)\right)$ This may be implied by awrt 0.97 (using radians) or awrt 89 (using degrees) for x_1	M1
	$(x_1 =)0.974,\ (x_2 =)1.011$	$(x_1 =)$awrt 0.974, $(x_2 =)$awrt 1.011. Ignore any subsequent iterations and ignore labelling if answers are clearly the second and third terms.	A1
			(2)
			Total 6

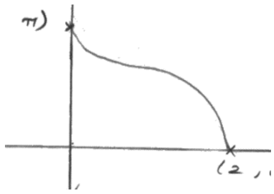
Example marking of Q11 part (a)

These sketches score both marks:

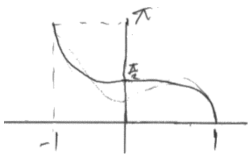


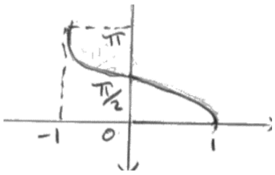


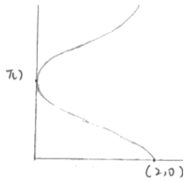


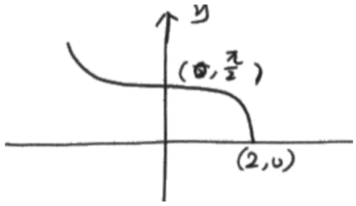


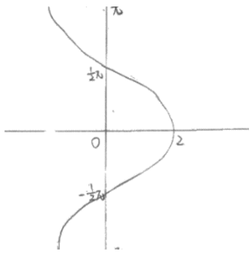
These sketches score M1A0:

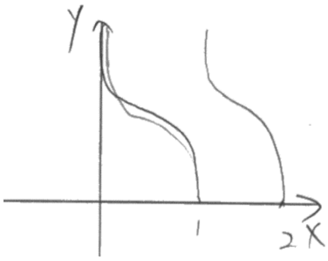




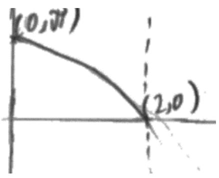


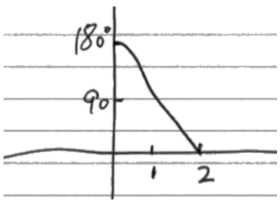


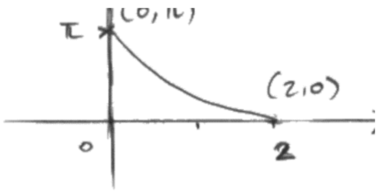


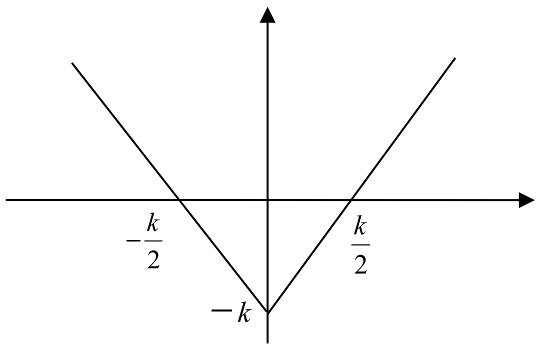


These sketches score M0A0:







Question Number	Scheme	Notes	Marks
12(a)			
	V Shape with the vertex anywhere on the y-axis with the branches approximately symmetrical about the y-axis. Ignore any dashed or dotted lines.		B1
	There must be a sketch for this mark. Intercepts (must be **<u>crossing</u>**) at $\left(-\frac{k}{2},0\right)$, $\left(\frac{k}{2},\ 0\right)$ and $\left(0,-k\right)$ and no others. Allow if the coordinates are the wrong way round provided the positioning is correct. The zeros are not needed as long as the expressions are correct (as above). Allow if the correct **coordinates** are seen away from the sketch but they must be the right way round in this case and must correspond with the sketch. If there is any ambiguity, the sketch has precedence.		B1
			(2)
(b)	$2x-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow x=...$ or $-2x-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow x=...$ Attempt to solve either equation to make x or k the subject		M1
	$x=\frac{5k}{6}$ or $x=-\frac{k}{2}$	One correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	$x=\frac{5k}{6}$ and $x=-\frac{k}{2}$	Both x values correct for. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	Note that the $x=-\frac{k}{2}$ must clearly be from work in (b) and not from work in (a) when attempting the sketch unless it is clearly stated as an answer to (b).		
			(3)

	(b) Alternative by squaring:		
	$$2\|x\|-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow 2\|x\|=\frac{1}{2}x+\frac{5k}{4}\Rightarrow 4x^2=\frac{1}{4}x^2+\frac{5k}{4}x+\frac{25}{16}k^2$$ $$\Rightarrow 60x^2-20kx-25k^2=0\Rightarrow x=...$$ Adds k to both sides, squares and solves to obtain a 3TQ and solves for x		M1
	$x=\frac{5k}{6}$ or $x=-\frac{k}{2}$	One correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
	$x=\frac{5k}{6}$ and $x=-\frac{k}{2}$	Both x values correct for. Allow equivalent fractions e.g. $\frac{10k}{12}$, $-\frac{2k}{4}$ etc.	A1
			(3)
			Total 5

	(b) Special case		
	$$2x-k=\frac{1}{2}x+\frac{k}{4}\Rightarrow 4x^2-4kx+k^2=\frac{1}{4}x^2+\frac{k}{4}x+\frac{1}{16}k^2$$ $$\Rightarrow 60x^2-68kx+15k^2=0\Rightarrow x=...$$ Squares both sides to obtain 3 terms each time and solves the resulting 3TQ solves for x		M1
	$x=\frac{5k}{6}$	Correct value for x. Allow equivalent fractions e.g. $\frac{10k}{12}$	A1
	If this is all they do, 2 marks will be the maximum		

Question Number	Scheme	Notes	Marks
13	$N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}$		
(a)	$\dfrac{240}{1+k\mathrm{e}^{(0)}}=50\Rightarrow k=...$	Substitutes $t=0$ and $N=50$ and solves for k	M1
	$k=3.8\left(=\dfrac{19}{5}\right)$	cao	A1
			(2)
(b)	$100=\dfrac{240}{1+3.8\mathrm{e}^{-\frac{t}{16}}}\Rightarrow 380\mathrm{e}^{-\frac{t}{16}}=140$	Puts $N=100$ and solves as far as $p\mathrm{e}^{-\frac{t}{16}}=q$ using correct processing (allow sign/copying/arithmetic slips)	M1
	$\mathrm{e}^{-\frac{t}{16}}=\dfrac{7}{19}\Rightarrow -\dfrac{t}{16}=\ln\left(\dfrac{7}{19}\right)$	Takes ln's **correctly** to reach $\pm\dfrac{t}{16}=\ln(\alpha),\alpha>0$ **Dependent on the previous M**	**d**M1
	$t=16\ln\left(\dfrac{19}{7}\right)$ or $-16\ln\left(\dfrac{7}{19}\right)$ or $8\ln\left(\dfrac{361}{49}\right)$ or $4\ln\left(\dfrac{130321}{2401}\right)$ etc	Cao (accept equivalents) or awrt 16	A1
			(3)
	(b) For mis-read $N=\dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 2/3)**		
	$100=\dfrac{240}{1+3.8\mathrm{e}^{\frac{t}{16}}}\Rightarrow 380\mathrm{e}^{\frac{t}{16}}=140$	Puts $N=100$ and solves as far as $p\mathrm{e}^{\frac{t}{16}}=q$ using correct processing (allow sign/copying/arithmetic slips)	M1
	$\mathrm{e}^{\frac{t}{16}}=\dfrac{7}{19}\Rightarrow \dfrac{t}{16}=\ln\left(\dfrac{7}{19}\right)$	Takes ln's **correctly** to reach $\pm\dfrac{t}{16}=\ln(\alpha),\alpha>0$ **Dependent on the previous M**	**d**M1
	$t=16\ln\left(\dfrac{7}{19}\right)$ etc.		A0

Part (c) General Guidance for Marking:

M1 is for their attempt at differentiating
A1 is for correct differentiation (in terms of k or follow through their k)
M1 is for $\mathrm{e}^{\cdots}$ or $k\mathrm{e}^{\cdots}$ or $1+k\mathrm{e}^{\cdots}$ in terms of N
M1 is for obtaining $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N
A1 fully correct

Note that a value of *k* is not necessary to do part (c)

(c) **Way 1**	$N=240\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{-1}$	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{-\frac{t}{16}}\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$	M1
	$\Rightarrow\dfrac{\mathrm{d}N}{\mathrm{d}t}=-240\left(1+3.8\mathrm{e}^{-\frac{t}{16}}\right)^{-2}\times-\dfrac{3.8}{16}\mathrm{e}^{-\frac{t}{16}}$	Correct derivative. Follow through their *k* or the letter *k*	A1ft
	May see **quotient rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{(0)-240\times-\dfrac{k}{16}\mathrm{e}^{-\frac{t}{16}}}{\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{2}}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{A\mathrm{e}^{-\frac{t}{16}}}{\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{2}}$ May see **product rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=0+\dfrac{240k\mathrm{e}^{-\frac{t}{16}}}{16}\left(1+k\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{-\frac{t}{16}}\left(1+B\mathrm{e}^{-\frac{t}{16}}\right)^{-2}$ **If an incorrect rule is quoted this scores M0**		
	$N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow 1+k\mathrm{e}^{-\frac{t}{16}}=\dfrac{240}{N}$	Attempt to find $\mathrm{e}^{-\frac{t}{16}}$ or $k\mathrm{e}^{-\frac{t}{16}}$ or $1+k\mathrm{e}^{-\frac{t}{16}}$ in terms of *N*	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{-\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{57\left(\dfrac{240-N}{3.8N}\right)}{\left(\dfrac{240}{N}\right)^{2}}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of *N* only (may include *k*'s)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{1}{16}N-\dfrac{1}{3840}N^{2}$	Cao (Allow $p = 16$, $q = 3840$)	A1
			(5)

	(c) Way 1 mis-read $N=\dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$N=240\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^{-1}$ $\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=-240\left(1+3.8\mathrm{e}^{\frac{t}{16}}\right)^{-2}\times\dfrac{3.8}{16}\mathrm{e}^{\frac{t}{16}}$	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{\frac{t}{16}}\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ Correct derivative. Follow through their k or the letter k	M1 A1ft
	May see **quotient rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{(0)-240\times\frac{k}{16}\mathrm{e}^{\frac{t}{16}}}{\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^2}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{A\mathrm{e}^{\frac{t}{16}}}{\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^2}$ May see **product rule**: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=0-\dfrac{240k\mathrm{e}^{\frac{t}{16}}}{16}\left(1+k\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ But this must satisfy the conditions above i.e. they need to obtain $\dfrac{\mathrm{d}N}{\mathrm{d}t}=A\mathrm{e}^{\frac{t}{16}}\left(1+B\mathrm{e}^{\frac{t}{16}}\right)^{-2}$ **If an incorrect formula is quoted this scores M0**		
	$N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow 1+k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}$	Attempt to find $\mathrm{e}^{\frac{t}{16}}$ or $k\mathrm{e}^{\frac{t}{16}}$ or $1+k\mathrm{e}^{\frac{t}{16}}$ in terms of N	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{-57\left(\frac{240-N}{3.8N}\right)}{\left(\frac{240}{N}\right)^2}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

	(c) Way 2		
	$\left(N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow k\mathrm{e}^{-\frac{t}{16}}=\dfrac{240}{N}-1\right)$		
	$\Rightarrow -\dfrac{k}{16}\mathrm{e}^{-\frac{t}{16}}\dfrac{\mathrm{d}t}{\mathrm{d}N}=-\dfrac{240}{N^2}$ Or	Differentiates to obtain $A\mathrm{e}^{-\frac{t}{16}}\dfrac{\mathrm{d}t}{\mathrm{d}N}=\dfrac{B}{N^2}$ or $A\mathrm{e}^{-\frac{t}{16}}=\dfrac{B}{N^2}\dfrac{\mathrm{d}N}{\mathrm{d}t}$	M1
	$\Rightarrow -\dfrac{k}{16}\mathrm{e}^{-\frac{t}{16}}=-\dfrac{240}{N^2}\dfrac{\mathrm{d}N}{\mathrm{d}t}$	Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow k\mathrm{e}^{-\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $\mathrm{e}^{-\frac{t}{16}}$ or $k\mathrm{e}^{-\frac{t}{16}}$ or $1+k\mathrm{e}^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{-\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{\frac{1}{16}\left(\frac{240}{N}-1\right)}{\frac{240}{N^2}}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{1}{16}N-\dfrac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
			(5)
	(c) Way 2 mis-read $N=\dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$\left(N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}-1\right)$		
	$\Rightarrow \dfrac{k}{16}\mathrm{e}^{\frac{t}{16}}\dfrac{\mathrm{d}t}{\mathrm{d}N}=-\dfrac{240}{N^2}$ or	Differentiates to obtain $A\mathrm{e}^{\frac{t}{16}}\dfrac{\mathrm{d}t}{\mathrm{d}N}=\dfrac{B}{N^2}$ or $A\mathrm{e}^{\frac{t}{16}}=\dfrac{B}{N^2}\dfrac{\mathrm{d}N}{\mathrm{d}t}$	M1
	$\Rightarrow \dfrac{k}{16}\mathrm{e}^{\frac{t}{16}}=-\dfrac{240}{N^2}\dfrac{\mathrm{d}N}{\mathrm{d}t}$	Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $\mathrm{e}^{\frac{t}{16}}$ or $k\mathrm{e}^{\frac{t}{16}}$ or $1+k\mathrm{e}^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{\frac{1}{16}\left(1-\frac{240}{N}\right)}{\frac{240}{N^2}}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

	(c) Way 3		
	$\left(N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow k\mathrm{e}^{-\frac{t}{16}}=\dfrac{240}{N}-1\right)$		
	$\Rightarrow -\dfrac{t}{16}=\ln\dfrac{1}{k}\left(\dfrac{240}{N}-1\right)$ $\Rightarrow t=-16\ln\dfrac{1}{k}-16\ln\left(\dfrac{240}{N}-1\right)$	Makes t the subject, takes ln's and differentiates using the chain rule.	M1
	$\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N}=-16\left(\dfrac{N}{240-N}\right)\left(-\dfrac{240}{N^2}\right)$	Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+k\mathrm{e}^{-\frac{t}{16}}}\Rightarrow k\mathrm{e}^{-\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $\mathrm{e}^{-\frac{t}{16}}$ or $k\mathrm{e}^{-\frac{t}{16}}$ or $1+k\mathrm{e}^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{-\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$=\dfrac{3840}{N(240-N)}$ $\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{N(240-N)}{3840}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1
	$\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{1}{16}N-\dfrac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
	(c) Way 3 mis-read $N=\dfrac{240}{1+k\mathrm{e}^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}-1$		
	$\dfrac{t}{16}=\ln\dfrac{1}{k}\left(\dfrac{240}{N}-1\right)$ $\Rightarrow t=16\ln\dfrac{1}{k}+16\ln\left(\dfrac{240}{N}-1\right)$	Makes t the subject, takes ln's and differentiates using the chain rule.	M1
	$\Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N}=16\left(\dfrac{N}{240-N}\right)\left(-\dfrac{240}{N^2}\right)$	Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+k\mathrm{e}^{\frac{t}{16}}}\Rightarrow k\mathrm{e}^{\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $\mathrm{e}^{\frac{t}{16}}$ or $k\mathrm{e}^{\frac{t}{16}}$ or $1+k\mathrm{e}^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+k\mathrm{e}^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$=\dfrac{3840}{N(N-240)}$	Obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N only (may include k's)	M1

	$\Rightarrow \dfrac{dN}{dt}=\dfrac{N(N-240)}{3840}$		
	$\dfrac{dN}{dt}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

	(c) Way 4		
	$\left(1+ke^{-\frac{t}{16}}\right)N=240$ $\Rightarrow N\times-\dfrac{k}{16}e^{-\frac{t}{16}}+\left(1+ke^{-\frac{t}{16}}\right)\dfrac{dN}{dt}=0$ or $\left(1+ke^{-\frac{t}{16}}\right)+N\times-\dfrac{k}{16}e^{-\frac{t}{16}}\dfrac{dt}{dN}=0$	Multiplies by $\left(1+ke^{-\frac{t}{16}}\right)$ and differentiates with respect to t or N using the product rule	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+ke^{-\frac{t}{16}}}\Rightarrow ke^{-\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $e^{-\frac{t}{16}}$ or $ke^{-\frac{t}{16}}$ or $1+ke^{-\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{-\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{N(240-N)}{3840}$	Obtains $\dfrac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\dfrac{dN}{dt}=\dfrac{1}{16}N-\dfrac{1}{3840}N^2$	Cao (Allow p = 16, q = 3840)	A1
	(c) Way 4 mis-read $N=\dfrac{240}{1+ke^{+\frac{t}{16}}}$ **(Max 4/5)**		
	$\left(1+ke^{\frac{t}{16}}\right)N=240$ $\Rightarrow N\times\dfrac{k}{16}e^{\frac{t}{16}}+\left(1+ke^{\frac{t}{16}}\right)\dfrac{dN}{dt}=0$ or $\left(1+ke^{\frac{t}{16}}\right)+N\times\dfrac{k}{16}e^{\frac{t}{16}}\dfrac{dt}{dN}=0$	Multiplies by $\left(1+ke^{\frac{t}{16}}\right)$ and differentiates with respect to t or N using the product rule	M1
		Correct differentiation. Follow through their k or the letter k	A1ft
	$N=\dfrac{240}{1+ke^{\frac{t}{16}}}\Rightarrow ke^{\frac{t}{16}}=\dfrac{240}{N}-1$	Attempt to find $e^{\frac{t}{16}}$ or $ke^{\frac{t}{16}}$ or $1+ke^{\frac{t}{16}}$ in terms of N.	M1
	Note that this mark may be scored by e.g. replacing $1+ke^{\frac{t}{16}}$ by $\dfrac{240}{N}$ in their solution		
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{N(N-240)}{3840}$	Obtains $\dfrac{dN}{dt}$ in terms of N only (may include k's)	M1
	$\dfrac{dN}{dt}=-\dfrac{1}{16}N+\dfrac{1}{3840}N^2$		A0

There may be other methods not covered in the MS but the marking should follow the same pattern.

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