Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Tuesday 20 June 2017 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

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Question 1 continued

(Total 6 marks)

Q1

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2. Use integration by parts to find the exact value of $\int_1^{e} \frac{\ln x}{x^2}\,dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

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Question 2 continued

Q2

(Total 6 marks)

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3. The function g is defined by

$$g(x) = \frac{6x}{2x+3} \qquad x > 0$$

(a) Find the range of g.

(2)

(b) Find $g^{-1}(x)$ and state its domain.

(3)

(c) Find the function $gg(x)$, writing your answer as a single fraction in its simplest form.

(3)

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Question 3 continued

Q3

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4.

$$f(x) = \frac{27}{(3 - 5x)^2} \qquad |x| < \frac{3}{5}$$

(a) Find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(5)

Use your answer to part (a) to find the series expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{27}{(3 + 5x)^2} \qquad |x| < \frac{3}{5}$

(1)

(c) $h(x) = \frac{27}{(3 - x)^2} \qquad |x| < 3$

(2)

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

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5.

$$\frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \equiv A + \frac{B}{(2 - x)} + \frac{C}{(1 + 2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

(b) Using part (a), find $f'(x)$.

(3)

(c) Prove that $f(x)$ is a decreasing function.

(1)

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Question 5 continued

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Question 5 continued

Q5

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6. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 10 \\ 5 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Justify, giving reasons in each case, whether the lines l_1 and l_2 are parallel, intersecting or skew.

(6)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 6 marks)

Q6

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7. (a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°, n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve, for $-90° < \theta < 90°$,

$$\frac{2-2\cos 2\theta}{1+\cos 2\theta} - 2 = 7\sec\theta$$

Give your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

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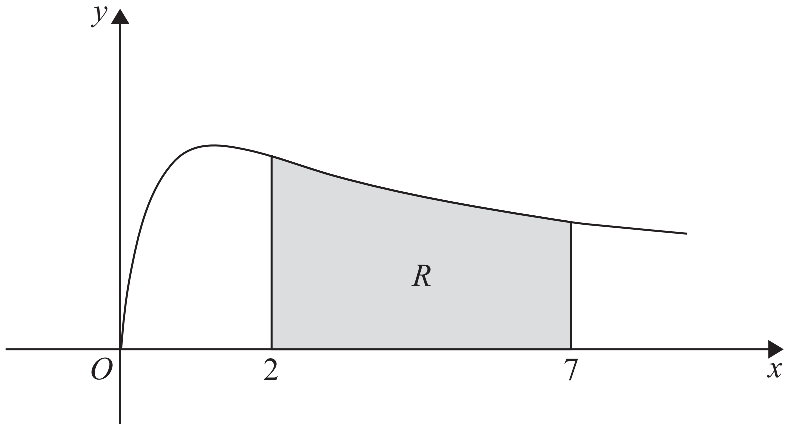

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{\dfrac{x}{x^2+1}}, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 7$

The table below shows corresponding values of x and y for $y = \sqrt{\dfrac{x}{x^2+1}}$

x	2	3	4	5	6	7
y	0.6325	0.5477	0.4851	0.4385	0.4027	0.3742

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

The region R is rotated 360° about the x-axis to form a solid of revolution.

(b) Use calculus to find the exact volume of the solid of revolution formed. Write your answer in its simplest form.

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Question 8 continued

(Total 7 marks)

Q8

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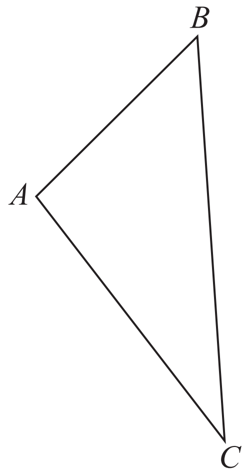

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{AC} = 5\mathbf{i} - 6\mathbf{j} + \mathbf{k}$,

(a) find the size of angle *CAB*, giving your answer in degrees to 2 decimal places,
(3)

(b) find the area of triangle *ABC*, giving your answer to 2 decimal places.
(2)

Using your answer to part (b), or otherwise,

(c) find the shortest distance from *A* to *BC*, giving your answer to 2 decimal places.
(3)

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Question 9 continued

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Q9

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10. (a) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to one decimal place.
(3)

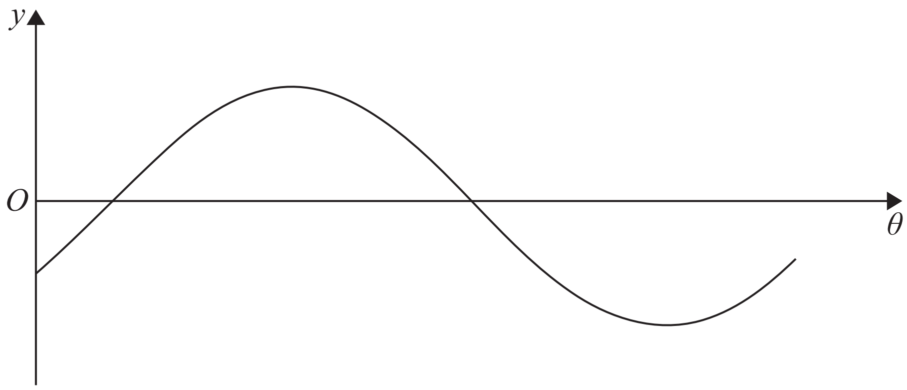

Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta$, $0 \leqslant \theta < 360°$

(b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \qquad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.
(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |2\sin(15t)° - \cos(15t)°|, \qquad 0 \leqslant t < 24$$

where $f(t)$ is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

(c) (i) the maximum value of $f(t)$,

(ii) the largest value of t, for $0 \leqslant t < 24$, at which this maximum value occurs. Give your answer to one decimal place.
(3)

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 9 marks)

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11. $$y = (2x^2 - 3)\tan\left(\frac{1}{2}x\right), \qquad 0 < x < \pi$$

(a) Find the exact value of x when $y = 0$

(1)

Given that $\dfrac{dy}{dx} = 0$ when $x = \alpha$,

(b) show that

$$2\alpha^2 - 3 + 4\alpha \sin \alpha = 0$$

(6)

The iterative formula

$$x_{n+1} = \frac{3}{(2x_n + 4\sin x_n)}$$

can be used to find an approximation for α.

(c) Taking $x_1 = 0.7$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 0.7283$ to 4 decimal places.

(2)

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Q11

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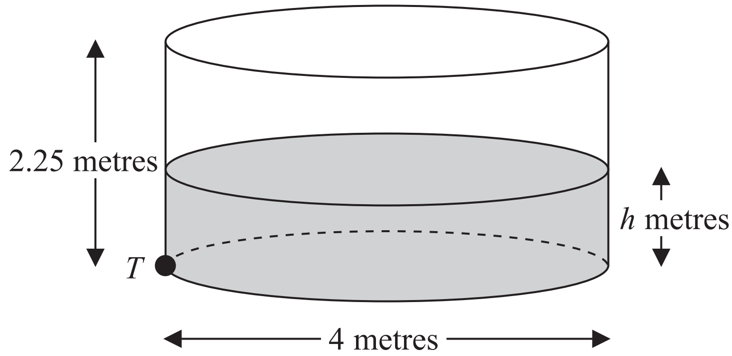

Figure 4

Figure 4 shows a right cylindrical water tank. The diameter of the circular cross section of the tank is 4 m and the height is 2.25 m. Water is flowing into the tank at a constant rate of 0.4π m^3 min^{-1}. There is a tap at a point T at the base of the tank. When the tap is open, water leaves the tank at a rate of $0.2\pi\sqrt{h}$ m^3 min^{-1}, where h is the height of the water in metres.

(a) Show that at time t minutes after the tap has been opened, the height h m of the water in the tank satisfies the differential equation

$$20\frac{\mathrm{d}h}{\mathrm{d}t} = 2 - \sqrt{h}$$

(5)

At the instant when the tap is opened, $t = 0$ and $h = 0.16$

(b) Use the differential equation to show that the time taken to fill the tank to a height of 2.25 m is given by

$$\int_{0.16}^{2.25} \frac{20}{2 - \sqrt{h}}\,\mathrm{d}h$$

(2)

Using the substitution $h = (2 - x)^2$, or otherwise,

(c) find the time taken to fill the tank to a height of 2.25 m.

Give your answer in minutes to the nearest minute.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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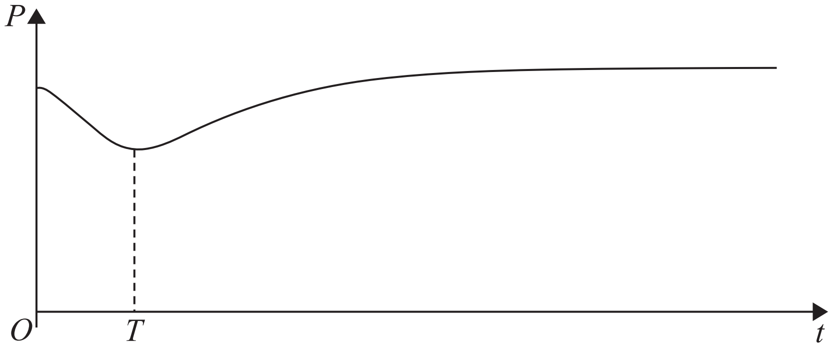

Figure 5

A colony of ants is being studied. The number of ants in the colony is modelled by the equation

$$P = 200 - \frac{160e^{0.6t}}{15 + e^{0.8t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of ants, measured in thousands, t years after the study started.
A sketch of the graph of P against t is shown in Figure 5

(a) Calculate the number of ants in the colony at the start of the study.

(2)

(b) Find $\frac{dP}{dt}$

(3)

The population of ants initially decreases, reaching a minimum value after T years, as shown in Figure 5

(c) Using your answer to part (b), calculate the value of T to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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14.

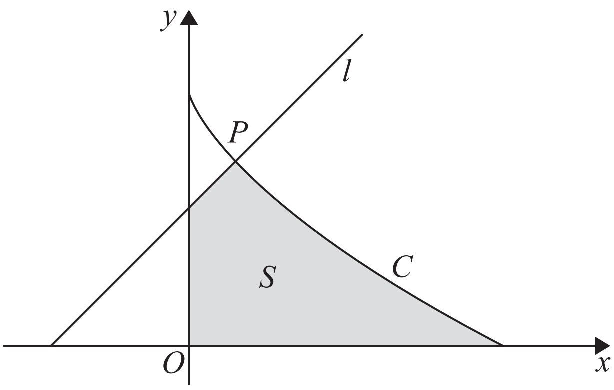

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta, \qquad y = 6\sin^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Show that an equation of l is $y = x + 3.5$

(5)

The finite region S, shown shaded in Figure 6, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, by integration, find the exact area of S.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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