Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
In Core Mathematics C12 (WMA01) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- o.e. – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\lfloor$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread, however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles.*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the <u>correct</u> formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \; q \neq 0$, leading to $x = \ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1.</td><td>$6x^3+5x^2-6x=0$</td><td></td><td></td></tr>
<tr><td>(a)</td><td>$x(6x^2+5x-6)=0$</td><td>For dividing or factorising out the 'x'.
This may be awarded for an answer of $x=0$
or for sight of $6x^2+5x-6$ or $(3x-2)(2x+3)$
or attempting to apply the formula or complete the square on $6x^2+5x-6\{=0\}$</td><td>M1</td></tr>
<tr><td></td><td>$\left\{6x^2+5x-6=0 \text{ or } x^2+\frac{5}{6}x-1=0 \Rightarrow\right\}$
e.g. $(3x-2)(2x+3)=0 \Rightarrow x=...$</td><td>dependent on the previous M mark
A valid correct method of solving their $3\text{TQ}=0$ to give $x=...$</td><td>dM1</td></tr>
<tr><td></td><td>$x=0,\frac{2}{3},-\frac{3}{2}$</td><td>$x=0,\frac{2}{3},-\frac{3}{2}$
Note: Give A0 for any extra values</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>(3)</td></tr>
<tr><td>(b)</td><td>$6\sin^3\theta+5\sin^2\theta-6\sin\theta=0;\ 0\le\theta<\pi$</td><td></td><td></td></tr>
<tr><td></td><td>$\sin\theta=0$ or $\sin\theta=\frac{2}{3} \Rightarrow \theta=...$</td><td>Finds at least one value of θ for $\sin\theta=(\text{their } k \text{ from } (a)),\ 0<k<1$ (where $0<\theta<\pi$) or for finds at least one of $\theta=0$, awrt 0.73, awrt 2.41
Note: Allow equivalent answers in degrees. E.g. θ = awrt 41.8, awrt 138</td><td>M1</td></tr>
<tr><td></td><td rowspan="2">$\theta=0,\ 0.730,\ 2.41$</td><td>For at least two of $\theta=0$, awrt 0.73 or awrt 2.41
Note: Allow equivalent answers in degrees. E.g. θ = awrt 41.8, awrt 138</td><td>A1</td></tr>
<tr><td></td><td>$\theta=0$, awrt 0.730, awrt 2.41 and no extra values within the range $0\le\theta\le\pi$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Note: Ignore π or awrt 3.14 for the final A mark</td><td>(3)</td></tr>
<tr><td></td><td></td><td></td><td>6</td></tr>
</table>

<table>
<tr><th></th><th colspan="2">Question 1 Notes</th></tr>
<tr><td>1. (a)</td><td>Note</td><td>A valid correct attempt of solving their $6x^2+5x-6=0$ or their $x^2+\frac{5}{6}x-1=0$ includes any of
• $(3x-2)(2x+3)=0 \Rightarrow x=...$
• $\left(x+\frac{5}{12}\right)^2-\frac{25}{144}-1=0 \Rightarrow x=...$
• $x=\frac{-5\pm\sqrt{5^2-4(6)(-6)}}{2(6)} \Rightarrow x=...$
• using their calculator to write down at least one correct root for their $3\text{TQ}=0$</td></tr>
<tr><td></td><td>Note</td><td>Completing the square: Give 2nd M1 for either $6\left(x\pm\frac{5}{12}\right)^2\pm q\pm 6=0\Rightarrow x=...$
or for $\left(x\pm\frac{5}{12}\right)^2\pm q\pm 1=0\Rightarrow x=...;\ q\ne 0$</td></tr>
<tr><td></td><td>Note</td><td>Give M1 dM0 A0 for writing down $x=0,\frac{2}{3},-\frac{3}{2}$ from no working</td></tr>
<tr><td></td><td>Note</td><td>Give M0 dM0 A0 for writing down only $x=\frac{2}{3},-\frac{3}{2}$ from no working</td></tr>
</table>

		Question 1 Notes Continued
1. (a)	**Note**	Give M1 dM1 A0 for $\{6x^3+5x^2-6x=0 \Rightarrow\}\ 6x^2+5x-6=0 \Rightarrow x=\frac{2}{3}, -\frac{3}{2}$
	Note	Give M1 dM1 A1 for $\{6x^3+5x^2-6x=0 \Rightarrow\}\ 6x^2+5x-6=0 \Rightarrow x=0, \frac{2}{3}, -\frac{3}{2}$
	Note	Give M1 dM1 A1 for $\{6x^3+5x^2-6x=0 \Rightarrow\}\ x(6x^2+5x-6)=0 \Rightarrow x=0, \frac{2}{3}, -\frac{3}{2}$
(b)	**Note**	Give M1 A1 A1 for $\theta = 0, 0.730, 2.41, 3.14$
	Note	Give M1 A1 A1 for $\theta = 0, 0.730, 2.41, \pi$
	Note	Give M1 A1 A0 for $\theta = 0, 0.73, 2.41, \pi$
	Note	Condone $x = ...$ instead of $\theta = ...$ if it is clear that they are working with angle $x \equiv \theta$ and not $x = \sin\theta$
	Note	Allow 0.00 written in place of 0

Question Number	Scheme	Notes	Marks
2.	$\int\left(15x^4+\dfrac{4}{3x^3}-4\right)\mathrm{d}x\ ;\quad x>0$		
	$=15\left(\dfrac{x^5}{5}\right)+\dfrac{4}{3}\left(\dfrac{x^{-2}}{-2}\right)-4x+c$	At least one of either $15x^4\to\pm Ax^5$, $\dfrac{4}{3x^3}\to\pm Bx^{-2}$ or $\pm\dfrac{B}{x^2}$, or $-4\to -4x$; $A,\ B\neq 0$	M1
		At least two correct integrated terms which can be simplified or un-simplified	A1
		At least three correct integrated terms which can be simplified or un-simplified	A1
	$=3x^5-\dfrac{2}{3}x^{-2}-4x+c$ or $3x^5-\dfrac{2}{3x^2}-4x+c$	Correct simplified integration contained on the same line of working	A1
	Note: $+c$ is counted as an integrated term		**(4)**
			4
	Question 2 Notes		

Note	You can ignore subsequent working after a correct final answer.	
Note	Poor notation (i.e. incorrect use of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $\int$) can be condoned for any or all of the marks.	
Note	$+c$ is counted as 'integrated term' for all the A marks.	

Question Number	Scheme	Notes	Marks
3.	$u_1 = 5,\ u_{n+1} = ku_n + 2\ \{\Rightarrow u_2 = ku_1 + 2,\ u_3 = ku_2 + 2\}$		
(a)	$u_2 = 5k + 2$	$u_2 = 5k + 2$ or $u_2 = 2 + 5k$	B1
	$u_3 = k(5k + 2) + 2$	Substitutes their u_2 which is in terms of k into $u_3 = ku_2 + 2$	M1
	$u_3 = 5k^2 + 2k + 2$	$u_3 = 5k^2 + 2k + 2$	A1
			(3)
(b) **Way 1**	$\{u_3 = 2 \Rightarrow\}\ 5k^2 + 2k + 2 = 2 \Rightarrow k = ...\quad \{k = -0.4\}$	Sets their $u_3 = 2$, where u_3 is a 3TQ in k, and uses a valid method of solving a quadratic equation in k to give $k = ...$ **Note:** Allow M1 if a relevant value of k is subsequently rejected.	M1
	$u_2 = 5("-0.4") + 2 = 0\quad \Rightarrow \sum_{n=1}^{3} u_n = 5 + "0" + 2$	**dependent on the previous M mark** Uses their value for k to calculate u_2 **and** adds their value for u_2 to 5 and 2	dM1
	$= 7$ **cso**	7	A1 **cso**
	Note: Do not give dM1 for using $u_2 = 2$ (which is found by using $k = 0$)		**(3)**
(b) **Way 2**	$\{u_3 = 2 \Rightarrow\}\ 5k^2 + 2k + 2 = 2 \Rightarrow k = ...\quad \{k = -0.4\}$	Sets their $u_3 = 2$, where u_3 is a 3TQ in k, and uses a valid method of solving a quadratic equation in k to give $k = ...$ **Note:** Allow M1 if a relevant value of k is subsequently rejected.	M1
	$u_2 = ("-0.4")(5) + 2 = 0,\ \{u_3 = 2\},$ $u_4 = ("-0.4")(2) + 2 = 1.2$ $\sum_{n=1}^{3} u_n = \sum_{n=1}^{3}\left(\frac{u_{n+1} - 2}{k}\right) = \frac{1}{"-0.4"}("0" + 2 + "1.2" - 6)$	**dependent on the previous M mark** Uses their value for k to calculate u_2 and u_4 and applies $\frac{1}{\text{their } k}(\text{their } u_2 + 2 + \text{their } u_4 - 6)$	dM1
	$= 7$ **cso**	7	A1 **cso**
	Note: Do not give dM1 for using $u_2 = 2$ (which is found by using $k = 0$)		**(3)**
			6

		Question 3 Notes
3. (a)	**Note**	Give M0 A0 for $u_3 = k(5k + 2)$
(b)	**Note**	dM1 can also be given for a **correct substitution** of $k = -0.4$ into $5k^2 + 7k + 9$ o.e.
		Give dM1 for $5 + 5(-0.4) + 2 + 5(-0.4)^2 + 2(-0.4) + 2$
		Give dM1 for $5(-0.4)^2 + 7(-0.4) + 9$
		Give dM0 for $5(-0.4) + 7(-0.4) + 9\ \{= 4.2\}$. {This is a common error.}
	Note	**Way 1:** Give M1 dM1 A0 for • $5k^2 + 2k + 2 = 2 \Rightarrow k(5k + 2) = 0 \Rightarrow k = \frac{2}{5}$; $u_2 = 5(0.4) + 2 = 4 \Rightarrow \sum_{n=1}^{3} u_n = 5 + "4" + 2 = 11$
	Note	**Way 1:** Give M1 dM0 A0 for • $5k^2 + 2k + 2 = 2 \Rightarrow k(5k + 2) = 0 \Rightarrow k = \frac{2}{5}$; $u_2 = 5(0.4) + 2 = 4,\ u_3 = 5(0.4)^2 + 2(0.4) + 2 = 3.6$ $\Rightarrow \sum_{n=1}^{3} u_n = 5 + 4 + 3.6 = 12.6$
	Note	There must be some evidence of using their k to find their value of u_2

		Question 3 Notes Continued
3. (b)	**Note**	Give dM0 for an incorrect follow through value of u_2 from their *k* with no supporting working.
	Note	Send to review applying $u_3 = 3$ **consistently** to give $\sum_{n=1}^{3} u_n$ = any of $9-\sqrt{6}$, $9+\sqrt{6}$ or awrt 6.55 or awrt 11.4 Otherwise give M0 dM0 A0 for applying $u_3 = 3$

Question Number	Scheme	Notes	Marks
4.	(i) $\dfrac{8^y}{4^{2x}} = \dfrac{\sqrt{2}}{32}$; (ii) $x\sqrt{3} = 4\sqrt{2} + x$		
(i) **Way 1**	$\dfrac{2^{3y}}{2^{4x}} = \dfrac{2^{\frac{1}{2}}}{2^5} \Rightarrow 2^{3y-4x} = 2^{\frac{1}{2}-5}$		M1 A1
	$3y - 4x = -\dfrac{9}{2} \Rightarrow y = \dfrac{4}{3}x - \dfrac{3}{2}$ or $y = \dfrac{1}{6}(8x-9)$ **cso**		dM1 A1 **cso**
(i) **Way 2**	$\log\left(\dfrac{8^y}{4^{2x}}\right) = \log\left(\dfrac{\sqrt{2}}{32}\right) \Rightarrow y\log 8 - 2x\log 4 = \log\left(\dfrac{\sqrt{2}}{32}\right)$		M1
	$y\log 8 - 2x\log 4 = \log(\sqrt{2}) - \log(32)$		A1
	$y = \dfrac{2x\log 4 + \log(\sqrt{2}) - \log(32)}{\log 8} \Rightarrow y = \dfrac{2x(2\log 2) + \frac{1}{2}\log 2 - 5\log 2}{3\log 2}$		dM1
	$\Rightarrow y = \dfrac{4}{3}x - \dfrac{3}{2}$ or $y = \dfrac{1}{6}(8x-9)$ **cso**		A1 **cso**
			(4)

	Scheme	Notes	Marks
(ii)	$x\sqrt{3} - x = 4\sqrt{2} \Rightarrow x(\sqrt{3}-1) = 4\sqrt{2}$	For sight of **an equation** containing $(\pm\sqrt{3} \pm 1)x$	M1
	$x = \dfrac{4\sqrt{2}}{\sqrt{3}-1}$	$x = \dfrac{4\sqrt{2}}{\sqrt{3}-1}$ or $x = \dfrac{-4\sqrt{2}}{1-\sqrt{3}}$ o.e.	A1
	$x = \dfrac{4\sqrt{2}}{(\sqrt{3}-1)}.\dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	**dependent on the previous M mark** Attempt to rationalise the denominator	dM1
	$x = \dfrac{4\sqrt{6} + 4\sqrt{2}}{2} \Rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ **cso**	Uses a non-calculator process to obtain $x = 2\sqrt{6} + 2\sqrt{2}$ or equivalent	A1 **cso**
			(4)

Question 4 Notes

4. (i) **Way 1**	**M1**	Uses index laws to correctly combine two **relevant** terms as listed below: • $\dfrac{8^y}{4^{2x}} \to 2^{3y-4x}$ or $\dfrac{\sqrt{2}}{32} \to 2^{\frac{1}{2}-5}$ • $(8^y)(32) \to 2^{3y+5}$ or $(4^{2x})(\sqrt{2}) \to 2^{4x+\frac{1}{2}}$ • $\dfrac{(8^y)(32)}{4^{2x}} \to 2^{3y+5+\dots}$ or $2^{3y-4x+\dots}$ or $2^{5-4x+\dots}$ or $2^{3y+5-4x}$
	A1	Correct equation in powers of 2 of the form $2^{\dots} = 2^{\dots}$
	dM1	**dependent on the previous M mark** Writes their equation in the form $2^{\dots} = 2^{\dots}$, equates their powers of 2 and rearranges to make y the subject.
	A1	Obtains $y = \dfrac{4}{3}x - \dfrac{3}{2}$ or $y = \dfrac{4}{3}x - 1.5$ or $y = \dfrac{1}{6}(8x-9)$ or $y = \dfrac{8x-9}{6}$ by correct solution only
4. (i) **Way 2**	**M1**	**Starts from a correct equation and** writes down a correct equation in logarithms with some evidence of applying either the addition or subtraction law of logarithms **and** the power law of logarithms.
	A1	Progresses as far as a correct $y\log 8 - 2x\log 4 = \log(\sqrt{2}) - \log(32)$, o.e.
	dM1	Rearranges to make y the subject **and** converts all logs in terms of $\log 2$
	A1	**Uses a non-calculator process** to obtain $y = \dfrac{4}{3}x - \dfrac{3}{2}$ or $y = \dfrac{4}{3}x - 1.5$ or exact equivalent by correct solution only.

		Question 4 Notes Continued
4. (i)	**Note**	The following solution in powers of 4 can be marked using the same principles as Way 1. • $\frac{8^y}{4^{2x}} = \frac{\sqrt{2}}{32} \Rightarrow \frac{4^{\frac{3}{2}y}}{4^{2x}} = \frac{4^{\frac{1}{4}}}{4^{\frac{5}{2}}} \Rightarrow 4^{\frac{3}{2}y - 2x} = 4^{\frac{1}{4} - \frac{5}{2}} \Rightarrow \frac{3}{2}y - 2x = -\frac{9}{4} \Rightarrow y = \frac{4}{3}x - \frac{3}{2}$ or $y = \frac{1}{6}(8x-9)$
	Note	Give M0 A0 dM0 A0 for $y = \log_8\left(\frac{4^{2x}\sqrt{2}}{32}\right)$ or $y = \frac{\log\left(\frac{\sqrt{2}}{32}4^{2x}\right)}{\log 8}$
4. (ii)	**Note**	Exact equivalent forms of $x = 2\sqrt{6} + 2\sqrt{2}$ include $x = 2\sqrt{2} + 2\sqrt{6}$, $x = \sqrt{24} + \sqrt{8}$, $x = 2\sqrt{6} + \sqrt{8}$, $x = \sqrt{24} + 2\sqrt{2}$, etc. for the final A mark.
	Note	Give • M0 A0 dM0 A0 for $x\sqrt{3} - x = 4\sqrt{2} \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • M1 A0 dM0 A0 for $x(\sqrt{3} - 1) = 4\sqrt{2} \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • (M1 A1) dM0 A0 for $x = \frac{4\sqrt{2}}{\sqrt{3}-1} \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • (M1 A1) dM1 A1 for $x = \frac{4\sqrt{2}}{\sqrt{3}-1} \rightarrow x = \frac{4\sqrt{6} + 4\sqrt{2}}{2} \Rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • (M1 A1 dM1) A1 for $x = \frac{4\sqrt{2}}{(\sqrt{3}-1)}.\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ with no intermediate working.

Question Number	Scheme	Notes	Marks
4.	(ii) $x\sqrt{3} = 4\sqrt{2} + x$		
(ii) **Way 2**	$(x\sqrt{3})^2 = (4\sqrt{2} + x)^2$ $3x^2 = 32 + 4\sqrt{2}x + 4\sqrt{2}x + x^2$	Squares both sides, followed by an attempt to form a 3-term quadratic.	M1
	e.g. $2x^2 = 8\sqrt{2}x + 32$ or $x^2 = 4\sqrt{2}x + 16$ or $2x^2 - 8\sqrt{2}x - 32 = 0$ or $x^2 - 4\sqrt{2}x - 16 = 0$	A correct 3-term quadratic. **Note**: $2x^2 - 8\sqrt{2}x = 32$ or $x^2 - 4\sqrt{2}x - 16$ $\{= 0\}$ are acceptable for this mark.	A1
	e.g. $x = \frac{4\sqrt{2} \pm \sqrt{32 - 4(1)(-16)}}{2}$ **or** $(x - (\sqrt{8} + \sqrt{24}))(x - (\sqrt{8} + \sqrt{24})) = 0 \Rightarrow x = ...$ **or** $(x - 2\sqrt{2})^2 - 8 - 16 = 0 \Rightarrow x = ...$	**dependent on the previous M mark** Correct method (applying the quadratic formula, factorising or completing the square) for solving a $3\text{TQ} = 0$ to find $x = ...$	dM1
	$x = 2\sqrt{2} + 2\sqrt{6}$ or $x = \sqrt{24} + 2\sqrt{2}$ o.e. **cso**	$x = 2\sqrt{6} + 2\sqrt{2}$ or equivalent	A1 **cso**
			(4)

		Question 4 Notes
4. (ii) **Way 2**	**Note**	The 3-term quadratic must involve surds for the 1st M mark.
	Note	The 3-term quadratic must involve surds for the 1st A mark.
	Note	Give 2nd A0 for giving more than one answer for x as their final answer.
	Note	Give • M0 A0 dM0 A0 for $x\sqrt{3} - x = 4\sqrt{2} \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • (M1 A1) dM0 A0 for $2x^2 = 8\sqrt{2}x + 32 \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$ • (M1 A1) dM0 A0 for $x^2 - 4\sqrt{2}x - 16 = 0 \rightarrow x = 2\sqrt{6} + 2\sqrt{2}$

		with no intermediate working.

Question Number	Scheme	Notes	Marks
5.	$\text{Area}(R) = 9 \Rightarrow \int_4^a \frac{4}{\sqrt{x}}\,dx = 9$		
	Note: You can mark part (a) and part (b) together.		
(a)(i) **Way 1**	$\left\{\int_4^a \frac{4}{\sqrt{3x}}\,dx = \frac{1}{\sqrt{3}}\int_4^a \frac{4}{\sqrt{x}}\,dx = \right\}\ \frac{1}{\sqrt{3}}(9) = 3\sqrt{3}$	For $\frac{1}{\sqrt{3}}(9)$ or awrt 5.2	M1
		$3\sqrt{3}$. Condone $\sqrt{27}$	A1
(a)(ii) **Way 1**	$\left\{\int_1^a \frac{4}{\sqrt{x}}\,dx = \int_1^4 \frac{4}{\sqrt{x}}\,dx + \int_4^a \frac{4}{\sqrt{x}}\,dx\right\}$		
	$= \left[\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 + 9$	Integrates so that $\frac{4}{\sqrt{x}} \to kx^{\frac{1}{2}};\ k \neq 0$, is seen anywhere in Q5. Also allow M1 for integrating so that $\frac{4}{\sqrt{3x}} \to kx^{\frac{1}{2}};\ k \neq 0$ is seen anywhere in Q5.	M1
		dependent on the previous M mark $\left[kx^{\frac{1}{2}}\right]_1^4$ and adding 9; $k \neq 0$, **Note:** Limits need to be correct, but do not need to be evaluated for this mark	dM1
	$= \left[8x^{\frac{1}{2}}\right]_1^4 + 9 = 8\sqrt{4} - 8\sqrt{1} + 9 = 16 - 8 + 9$		
	$= 17$	17	A1
			(5)
(b)	$\left[\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^a = 9$	Integrates to give $\left[kx^{\frac{1}{2}}\right]_4^a, k \neq 0$, and sets this result equal to 9. **Note:** Limits need to be correct, but do not need to be applied for this mark	M1
	$8\sqrt{a} - 8\sqrt{4} = 9$	Applies limits to obtain a correct equation in $\sqrt{a}$	A1
	$\sqrt{a} = \frac{25}{8}$	**dependent on the previous M mark** Proceeds from $p\sqrt{a} \pm b = 9$ to $\sqrt{a} = \lambda;\ p, b, \lambda \neq 0$	dM1
	$a = \frac{625}{64}$	$a = \frac{625}{64}$ or $9\frac{49}{64}$ or 9.765625	A1
	Note: The mark scheme for part (b) can be applied anywhere in a student's solution to Q5.		**(4)**
			9
	Question 5 Notes		
5.	**Note**	Some students may use their answer to (b) to answer (a)(i) and/or (a)(ii). See next page.	

Question Number	Scheme	Notes	Marks
5. (a)(i) **Way 2**	$\left\{\int_4^a \frac{4}{\sqrt{3x}}\,dx = \frac{1}{\sqrt{3}}\int_4^a \frac{4}{\sqrt{x}}\,dx = \frac{1}{\sqrt{3}}\left[8x^{\frac{1}{2}}\right]_4^{\frac{625}{64}}\right\}$ $= \frac{8}{\sqrt{3}}\left(\sqrt{\frac{625}{64}} - \sqrt{4}\right)$	**dependent on gaining both M marks in (b) and their $a > 4$ or their $\sqrt{a} > 2$** For $\frac{8}{\sqrt{3}}\left(\sqrt{(\text{their } a)} - \sqrt{4}\right)$	dM1
	$= \frac{8}{\sqrt{3}}\left(\frac{25}{8} - 2\right) = \frac{8}{\sqrt{3}}\left(\frac{9}{8}\right) = 3\sqrt{3}$	$3\sqrt{3}$. Condone $\sqrt{27}$	A1
			(2)
(a)(i) **Way 3**	$\left\{\int_4^a \frac{4}{\sqrt{3x}}\,dx = \int_4^a 4(3x)^{-\frac{1}{2}}\,dx = \left[\frac{8}{3}(3x)^{\frac{1}{2}}\right]_4^{\frac{625}{64}}\right\}$	**dependent on gaining both M marks in (b) and their $a > 4$ or their $\sqrt{a} > 2$** For $\frac{8}{3}\left(\sqrt{(3)(\text{their } a)} - \sqrt{(3)(4)}\right)$ or $\frac{8}{\sqrt{3}}\left(\sqrt{(\text{their } a)} - \sqrt{4}\right)$	dM1
	$= \frac{8}{3}\left(\sqrt{(3)\left(\frac{625}{64}\right)} - \sqrt{(3)(4)}\right)$ or $\frac{8}{\sqrt{3}}\left(\sqrt{\frac{625}{64}} - \sqrt{4}\right)$		
	$= \frac{8}{\sqrt{3}}\left(\frac{25}{8}\sqrt{3} - 2\sqrt{3}\right) = \frac{8}{3}\left(\frac{9}{8}\sqrt{3}\right) = 3\sqrt{3}$	$3\sqrt{3}$. Condone $\sqrt{27}$	A1
			(2)
(a)(ii) **Way 2**	$\left\{\int_1^a \frac{4}{\sqrt{x}}\,dx = \int_1^{\frac{625}{64}} \frac{4}{\sqrt{x}}\,dx\right\}$		
	$= \left[\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{\frac{625}{64}}$	Integrates so that $\frac{4}{\sqrt{x}} \to kx^{\frac{1}{2}};\ k \neq 0$, is seen anywhere in Q5. Also allow M1 for integrating so that $\frac{4}{\sqrt{3x}} \to kx^{\frac{1}{2}};\ k \neq 0$ is seen anywhere in Q5.	M1
		dependent on the previous M mark, dependent on gaining both M marks in (b) and their $a > 4$ or their $\sqrt{a} > 2$ For $\left[kx^{\frac{1}{2}}\right]_1^{\text{their stated } a}$ $;k \neq 0$ **Note:** Limits do not need to be applied for this mark .	dM1
	$= \left[8x^{\frac{1}{2}}\right]_1^{\frac{625}{64}} = 8\sqrt{\frac{625}{64}} - 8\sqrt{1} = 25 - 8$		
	$= 17$	17	A1
			(3)

	Question 5 Notes Continued	
5. (b)	**Note**	Give M0 A0 dM0 A0 for setting their part (a)(i) answer (which is in terms of a) equal to 9. • E.g. Give M0 A0 dM0 A0 for $\frac{8}{\sqrt{3}}\left(\sqrt{a} - \sqrt{4}\right) = 9$ seen in part (b).

Question Number	Scheme	Notes	Marks
6.	(a) $y = x(x+3)(x-2)$; (b) $\frac{\mathrm{d}y}{\mathrm{d}x} \geq 2$		
(a) **Way 1**	$y = x(x^2 - 2x + 3x - 6)$ $\Rightarrow y = x^3 - 2x^2 + 3x^2 - 6x$	$\{y =\}\ x^3 + Ax^2 + Bx;\ A, B \neq 0,$ where A, B can be simplified or un-simplified	M1 B1 on ePEN
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x + 6x - 6$	Obtains a cubic expression and differentiates to give either $x^3 \to \lambda x^2$, $Ax^2 \to \mu x$ or $Bx \to B$; $A, B, \lambda, \mu \neq 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 6$	Correct differentiation in simplest form	A1
			(3)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 6 \geq 2$ $3x^2 + 2x - 6 = 2 \Rightarrow 3x^2 + 2x - 8 = 0$ $(x+2)(3x-4) = 0 \Rightarrow x = ...$	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$, forms a $3\text{TQ} = 0$ and uses a correct valid method of solving their $3\text{TQ} = 0$ to give $x = ...$	M1
	{Critical values are} $x = -2, \frac{4}{3}$	Critical values of $x = -2, \frac{4}{3}$ or $x = -2$, awrt 1.33, These may be implied by their inequalities	A1
		Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$, forms a $3\text{TQ} = 0$ and uses their two distinct critical values to write down an ***outside region***	M1
	$x \leq -2$ or $x \geq \frac{4}{3}$	$x \leq -2$ or $x \geq \frac{4}{3}$ o.e., e.g. $(-\infty, -2] \cup [\frac{4}{3}, \infty)$. Allow ",", "or" or a space between the answers but give final M1 A0 for $x \leq -2$ **and** $x \geq \frac{4}{3}$ or for $-2 \geq x \geq \frac{4}{3}$ as their final answer. This answer can be a ft for their two distinct critical values.	A1ft
	Note: $x \leq \frac{4}{3}$ or $x \geq -2$ is final M0 A0		**(4)**
			7

		Question 6 Notes
6. (b)	**Note**	Give M0 A0 M0 A0 where the critical values are found from solving $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 6 = 0$
	Note	A valid correct attempt of solving their $3x^2 + 2x - 8 = 0$ or their $x^2 + \frac{2}{3}x - \frac{8}{3} = 0$ includes any of • $(x+2)(3x-4) = 0 \Rightarrow x = ...$ • $\left(x + \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{8}{3} = 0 \Rightarrow x = ...$ • $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-8)}}{2(3)} \Rightarrow x = ...$ • using their calculator to write down at least one correct root for their $3\text{TQ} = 0$
	Note	**Completing the square:** Give 1st M1 for either $3\left(x \pm \frac{1}{3}\right)^2 \pm q \pm 8 = 0 \Rightarrow x = ...$ or for $\left(x \pm \frac{1}{3}\right)^2 \pm q \pm \frac{8}{3} = 0 \Rightarrow x = ...$; $q \neq 0$
	Note:	E.g. $\{x : x \in \mathbb{R}, x \leq -2\} \cup \{x : x \in \mathbb{R}, x \geq \frac{4}{3}\}$, o.e., is acceptable for the 2nd A mark.

Question Number	Scheme	Notes	Marks
6.	(a) $y = x(x+3)(x-2)$; (b) $\frac{\mathrm{d}y}{\mathrm{d}x} \geq 2$		
	Way 2, Way 3 and Way 4: Product Rule		
(a) **Way 2**	$y = (x^2+3x)(x-2) \Rightarrow \begin{array}{ll} u = x^2+3x & v = x-2 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x+3 & \frac{\mathrm{d}v}{\mathrm{d}x} = 1 \end{array}$	Differentiates so that $x^2+3x \to Cx+3$; $C \neq 0$	M1 B1 on ePEN
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2+3x + (x-2)(2x+3)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2+3x + (x-2)(Cx+3)$; $C \neq 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+2x-6$	Correct simplified differentiation	A1
			(3)
(a) **Way 3**	$y = (x^2-2x)(x+3) \Rightarrow \begin{array}{ll} u = x^2-2x & v = x+3 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x-2 & \frac{\mathrm{d}v}{\mathrm{d}x} = 1 \end{array}$	Differentiates so that $x^2-2x \to Cx-2$; $C \neq 0$	M1 B1 on ePEN
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2-2x + (x+3)(2x-2)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2-2x + (x+3)(Cx-2)$; $C \neq 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+2x-6$	Correct simplified differentiation	A1
			(3)
(a) **Way 4**	$y = x(x^2+x-6)$ $\Rightarrow \begin{array}{ll} u = x & v = x^2+x-6 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 1 & \frac{\mathrm{d}v}{\mathrm{d}x} = 2x+1 \end{array}$	Differentiates so that $x^2-2x+3x-6 \to Cx+1$; $C \neq 0$	M1 B1 on ePEN
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2+x-6 + x(2x+1)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2+Ax-6 + x(2x+A)$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = (x+3)(x-2) + x(2x+A)$; $A \neq 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+2x-6$	Correct simplified differentiation	A1
			(3)

	Question 6 Notes Continued	
6. (b)	**Note**	The critical values found from solving $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+2x-6 = 0$ are $x = \frac{-2 \pm \sqrt{76}}{6}$ $x = \frac{-1 \pm \sqrt{19}}{3}$ or $x = -1.78629...,\ 1.1196...$

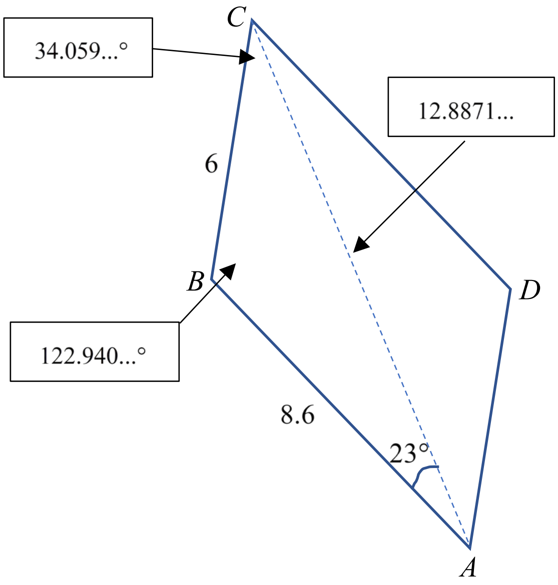
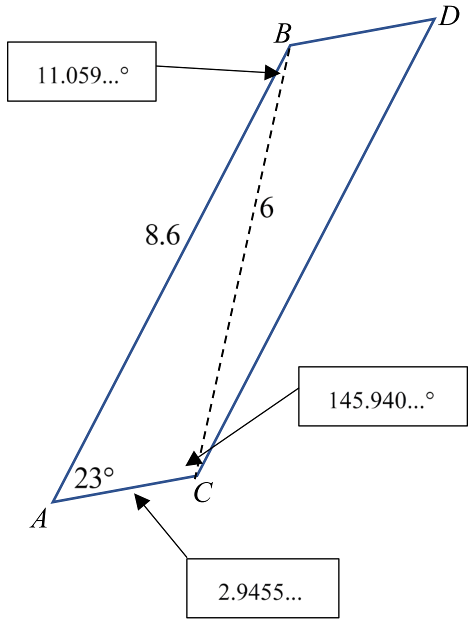
Question Number	Scheme	Notes	Marks
7.	(i) $3\times\left(\frac{1}{2}\right)^{p-1}=1.3$; (ii) $\log_4 2x+2\log_4 x=8$		
(i) **Way 1**	$\left(\frac{1}{2}\right)^{p-1}=\frac{1.3}{3}$ $\left\{\text{or } 2^{p-1}=\frac{3}{1.3}\right\}$		M1
	$\log\left(\frac{1}{2}\right)^{p-1}=\log\left(\frac{1.3}{3}\right) \Rightarrow (p-1)\log\left(\frac{1}{2}\right)=\log\left(\frac{1.3}{3}\right) \Rightarrow p-1=\frac{\log\left(\frac{1.3}{3}\right)}{\log\left(\frac{1}{2}\right)}$		M1
	$p=\frac{\log\left(\frac{1.3}{3}\right)}{\log\left(\frac{1}{2}\right)}+1 \Rightarrow p=\text{awrt } 2.206$ $\{\Rightarrow p=2.206\ (3\text{ dp})\}$		A1
			(3)
(i) **Way 2**	$\log\left(3\times\left(\frac{1}{2}\right)^{p-1}\right)=\log 1.3$		M1
	$\log 3+\log\left(\frac{1}{2}\right)^{p-1}=\log 1.3 \Rightarrow \log 3+(p-1)\log\left(\frac{1}{2}\right)=\log 1.3 \Rightarrow p-1=\frac{\log 1.3-\log 3}{\log\left(\frac{1}{2}\right)}$		M1
	$p=\frac{\log 1.3-\log 3}{\log\left(\frac{1}{2}\right)}+1 \Rightarrow p=\text{awrt } 2.206$ $\{\Rightarrow p=2.206\ (3\text{ dp})\}$		A1
			(3)
(i) **Way 3**	$3\left(\frac{1}{2}\right)^{p}\left(\frac{1}{2}\right)^{-1}=1.3 \Rightarrow 3(2)\left(\frac{1}{2}\right)^{p}=1.3 \Rightarrow \left(\frac{1}{2}\right)^{p}=\frac{1.3}{6}$ $\left\{\text{or } 2^{p}=\frac{6}{1.3}\right\}$		M1
	$\log\left(\frac{1}{2}\right)^{p}=\log\left(\frac{1.3}{6}\right) \Rightarrow p\log\left(\frac{1}{2}\right)=\log\left(\frac{1.3}{6}\right) \Rightarrow p=\frac{\log\left(\frac{1.3}{6}\right)}{\log\left(\frac{1}{2}\right)}$		M1
	$p=\text{awrt } 2.206$ $\{\Rightarrow p=2.206\ (3\text{ dp})\}$		A1
			(3)
(i) **Notes**	**Way 1, Way 2, Way 3 and Way 4 (on next page)**		
	For correctly making $\left(\frac{1}{2}\right)^{p-1}$, 2^{p-1}, $\left(\frac{1}{2}\right)^{p}$ or 2^{p} the subject or for writing a correct equation involving logarithms.		M1
	Complete process of writing a correct equation involving logarithms and using correct log laws (and correct index laws, where appropriate) to make $p-1$ or p the subject.		M1
	$p=\text{awrt } 2.206$		A1
	Note: See next page for how to mark Special Case M1 M0 A0		**(3)**
(ii)	$\log_4 2x+\log_4 x^2=8 \Rightarrow \log_4(2x(x^2))=8$	Correct method for combining the log terms. $\log_4 2x+2\log_4 x\to\log_4(2x(x^2))$ Condone $\log_4 2x+2\log_4 x\to\log(2x(x^2))$	M1
	$2x^3=4^8$ $\{\Rightarrow 2x^3=65536\}$	$\log_4(ax^n)=8\Rightarrow ax^n=4^8$ or 2^{16} or 65536, where $ax^n=2x^3$, $4x^4$ or $2x^2$ only	M1
	$x^3=32768\Rightarrow x=(32768)^{\frac{1}{3}}\Rightarrow x=32$	$x=32$	A1
			(3)
			6

Question Number	Scheme	Notes	Marks
7. (i) **Way 4**	$\left\{3\times\left(\frac{1}{2}\right)^{p-1}=1.3\Rightarrow\right\}\left(\frac{1}{2}\right)^{p-1}=\frac{1.3}{3}$		M1
	$\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{p-1}=\log_{\frac{1}{2}}\left(\frac{1.3}{3}\right)\Rightarrow p-1=\log_{\frac{1}{2}}\left(\frac{1.3}{3}\right)$		M1
	$p=\log_{\frac{1}{2}}\left(\frac{1.3}{3}\right)+1\Rightarrow p=\text{awrt }2.206\ \{\Rightarrow p=2.206\text{ (3 dp)}\}$		A1
			(3)

Question 7 Notes

7. (i)	**Note**	Allow Special Case M1 M0 A0 (unless recovered) for • $3\times\left(\frac{1}{2}\right)^{p-1}=1.3\rightarrow\log 3+p-1\log\left(\frac{1}{2}\right)=\log 1.3$ (i.e. 'invisible' brackets) • $3\times\left(\frac{1}{2}\right)^{p-1}=1.3\rightarrow\left(\frac{1}{2}\right)^{p-1}=\frac{13}{20}$ (i.e. for a division slip)
	Note	Give M1 M1 A1 (recovered bracketing slip) for • $3\times\left(\frac{1}{2}\right)^{p-1}=1.3\Rightarrow\log 3+p-1\log\left(\frac{1}{2}\right)=\log 1.3\Rightarrow p=\frac{\log\left(\frac{1.3}{3}\right)}{\log\left(\frac{1}{2}\right)}+1\Rightarrow p=2.206$
	Note	Give M0 M0 A0 for any of $3\times\left(\frac{1}{2}\right)^{p-1}=1.3\Rightarrow\left(\frac{3}{2}\right)^{p-1}=1.3$ or $\left(\frac{1}{2}\right)^{p-1}=-2.7$
	Note	Give M0 M0 A0 for • $3\times\left(\frac{1}{2}\right)^{p-1}=1.3\Rightarrow\log 3\times\log\left(\frac{1}{2}\right)^{p-1}=\log 1.3\Rightarrow p=\frac{\log\left(\frac{1.3}{3}\right)}{\log\left(\frac{1}{2}\right)}+1\Rightarrow p=2.206$
	Note	Give M1 dM1 A1 {for using a calculator to write down} $p=\text{awrt }2.206$ from no working.
	Note	Give M1 dM1 A1 for correct work leading to $p=\text{awrt }2.206$ E.g. • give M1 dM1 A1 for $\left(\frac{1}{2}\right)^{p-1}=\frac{1.3}{3}\Rightarrow p=\text{awrt }2.206$ • give (M1) M1 A1 for $\log 3+(p-1)\log\left(\frac{1}{2}\right)=\log 1.3\Rightarrow p=\text{awrt }2.206$ with no intermediate working.
	Note	Give M0 M0 A0 for $(\log 3)\log\left(\frac{1}{2}\right)^{p-1}=\log 1.3\Rightarrow p=\text{awrt }2.206$ with no intermediate working.
	Note	The M marks can be gained by working in decimals to at least 2 dp. (or 1 dp for $\log 2=0.3010...$) • e.g. Give M1 M1 A0 for $\left(\frac{1}{2}\right)^{p-1}=0.43\Rightarrow p=1+\frac{(-0.37)}{(-0.3)}\Rightarrow p=2.233$ (3 dp)

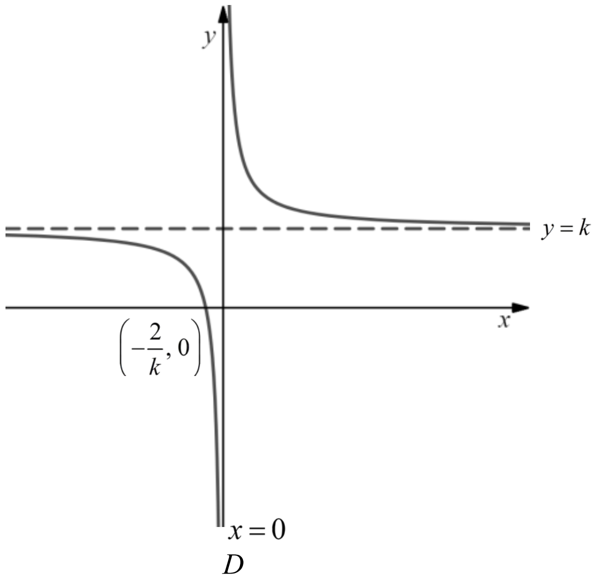
		Question 7 Notes
7. (ii)	**Note**	Give M1 M1 A1 {for using a calculator to write down} $x = 32$ from no working
	Note	Give M1 M1 A1 for correct work leading to $x = 32$. E.g. • give M1 M1 A1 for $\log_4 2x + \log_4 x^2 = 8 \Rightarrow x = 32$ • give M1 M1 A1 for $\log_4 2x + \log_4 x^2 = 8 \Rightarrow \log_4(2x^3) = 8 \Rightarrow x = 32$ with no intermediate working.
	Note	Give M0 M1 A0 for $\log_4 2x + 2\log_4 x = 8 \Rightarrow \log_4 2x^2 = 8 \Rightarrow 2x^2 = 65536 \Rightarrow x = 128\sqrt{2}$
	Note	Give M0 M1 (implied) A0 for $\log_4 2x + 2\log_4 x = 8 \Rightarrow \log_4 2x^2 = 8 \Rightarrow x = 128\sqrt{2}$
	Note	Give M0 M0 A0 for $\log_4 2x + 2\log_4 x = 8 \Rightarrow \log_4 2x^2 = 4 \Rightarrow x = 8\sqrt{2}$
	Note	Give A0 for $x = \pm 32$ unless recovered
	Note	Allow final A1 for (incorrect notation recovered) $x^3 = 32768 \Rightarrow x = \sqrt{32768} \Rightarrow x = 32$
	Note	Give M0 M1 A0 for $\log_4 2x + 2\log_4 x = 8 \Rightarrow (\log_4 2x)(\log_4 x^2) = 8 \Rightarrow \log_4 2x^3 = 8 \Rightarrow x = 32$

Question Number	Scheme	Notes	Marks
8.	 **Relevant $ABCD$ for parts (b) and (c)**		
(a)	$\dfrac{\sin B\hat{C}A}{8.6} = \dfrac{\sin 23}{6}$	Attempts sine rule with one unknown, $B\hat{C}A$, and with the edges and relevant angles in the correct position	M1
	$\{B\hat{C}A =\}$ 34.05911... or 145.94088...	awrt 34 or awrt 146. This may be implied by $A\hat{B}C =$ awrt 123 or awrt 11	A1
	$A\hat{B}C = 180 - 23 - 34.05911... = 122.94088...$ $A\hat{B}C = 180 - 23 - 145.94088... = 11.05911...$	**dependent on the previous M mark** Complete correct method to find ***at least one*** value of angle $A\hat{B}C$ **Note:** This mark can be implied by either $A\hat{B}C =$ awrt 123 or awrt 11	dM1
		Both awrt 122.9 **and** awrt 11.1	A1
			(4)
(b)	E.g. • $AC^2 = 8.6^2 + 6^2 - 2(8.6)(6)\cos\text{"}122.9\text{"}$ • $\dfrac{AC}{\sin\text{"}122.9\text{"}} = \dfrac{6}{\sin 23}$ • $\dfrac{AC}{\sin\text{"}122.9\text{"}} = \dfrac{8.6}{\sin(180 - 23 - \text{"}122.9\text{"})}$	Complete correct method to find angle $A\hat{B}C$ **and** **either** uses the cosine rule to find AC^2 or AC with their obtuse angle $A\hat{B}C$ (and not $A\hat{B}C =$ their $B\hat{C}A = 145.9$) **or** uses the sine rule with one unknown, AC, and with edges and relevant angles in the correct position	M1
	$AC =$ awrt 12.88 {cm} or awrt 12.89 {cm}	$AC =$ awrt 12.88 or awrt 12.89 **Note:** Ignore the units.	A1
			(2)
(c)	Area $ABCD = (8.6)(6)\sin\text{"}122.9\text{"}$ **or** $= 2 \times [(0.5)(8.6)(6)\sin\text{"}122.9\text{"}]$ **or** $= (8.6)(\text{"}12.89\text{"})\sin 23$ **or** $= (6)(\text{"}12.89\text{"})\sin(180 - 23 - \text{"}122.9\text{"})$ **or** $= (8.6)[(6)\sin(23 + \text{"}34.059...\text{"})]$	Complete correct method to find angle $A\hat{B}C$ **and** a correct complete method for finding area $ABCD$, where angle $A\hat{B}C$ is obtuse	M1
	$=$ awrt 43.3 {cm^2} (3 sf)	awrt 43.3 **Note:** Ignore the units.	A1
			(2)
			8

		Question 8 Notes
8. (b)	**Note**	$A\hat{B}C = 122.9408861...$ gives $AC = 12.8871029...$
	Note	$A\hat{B}C = 122.9$ gives $AC = 12.8847042...$
(c)	**Note**	Give M0 A0 for Area $ABCD = (8.6)(6)\sin"11.059" = 9.897998172...$
	Note	Condone M1 for $(8.6)(6)[\sin(\text{awrt } 57.1)]$ **and** A1 for awrt 43.3; ignoring how (awrt 57.1) has been derived in part (a) and/or part (b).
	Note	$(8.6)(6)\sin 122.9 = 43.32438501...$
	Note	$(8.6)(6)\sin 122.9408861... = 43.30437342...$
	Note	$(8.6)(12.89)\sin 23 = 43.31410852...$
	Note	$(8.6)(12.88)\sin 23 = 43.28050564...$
	Note	$(8.6)(12.8871029...)\sin 23 = 43.30437343...$

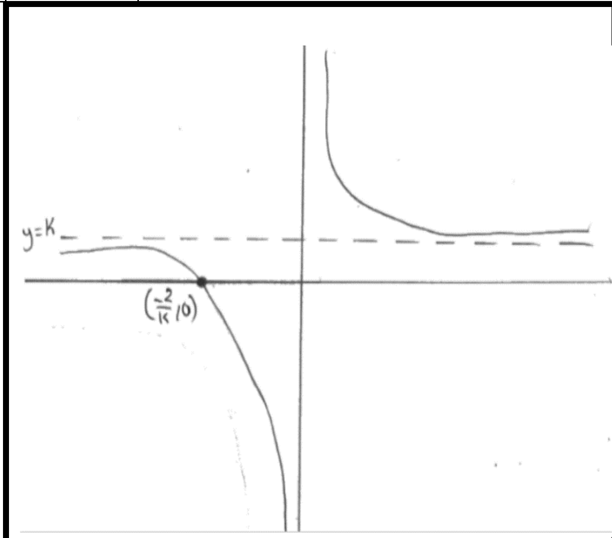
ALT (a), (b) **ALT**	**Alternative Method of initially using Cosine Rule with $6, 8.6$ and $AC = x$**		
	Note: Mark part (a) and part (b) together if this alternative method is used		
	$6^2 = 8.6^2 + x^2 - 2(8.6)(x)\cos 23$ $x^2 - 2(8.6)(x)\cos 23 + 8.6^2 - 6^2 = 0$ $x^2 - (17.2\cos 23)x + 37.96 = 0$ $x = \dfrac{17.2\cos 23 \pm \sqrt{(17.2\cos 23)^2 - 4(1)(37.96)}}{2(1)}$	Applies cosine rule with edges in the correct position, forms a 3TQ and uses a correct method (e.g. quadratic formula, completing the square or calculator) to solve their 3TQ $= 0$ to give at least one of $x = ...$	**1st M1 in (a)** **and** **1st M1 in (b)**
	$x = \dfrac{15.83268348... \pm \sqrt{98.83386616...}}{2}$		
	$x = 2.945580577..., 12.8871029...$	2.95 or awrt 2.9 or awrt 12.9	**1st A1 in (a)**
		Clearly identifies in part (b) that $AC =$ awrt 12.89 or awrt 12.88 **Note:** Units are not required	**1st A1 in (b)**
	E.g. • $\cos A\hat{B}C = \dfrac{8.6^2 + 6^2 - "2.9455...^2"}{2(8.6)(6)} \Rightarrow A\hat{B}C = 11.0591...$ • $\cos A\hat{B}C = \dfrac{8.6^2 + 6^2 - "12.8871...^2"}{2(8.6)(6)} \Rightarrow A\hat{B}C = 122.9408...$ • For $"AC" < 8.6$, $A\hat{B}C = \sin^{-1}\left("2.9455..." \times \dfrac{\sin 23}{6}\right)$ $= 11.0591...$ • For $"AC" > 8.6$, $A\hat{B}C = 180 - \sin^{-1}\left("12.8871..." \times \dfrac{\sin 23}{6}\right)$ $= 122.9408..$	**dependent on the 1st M mark in part (a)** Complete method to find ***at least one*** value of angle $A\hat{B}C$ **Note:** This mark can be implied by either $A\hat{B}C =$ awrt 123 or awrt 11	**dM1 in (a)**
		Both awrt 11.1 **and either** awrt 122.8 or awrt 122.9 or awrt 123.0	**2nd A1 in (a)**
			(4)

8. ALT	**Note**	Only apply the alternative mark scheme if it is clear that the candidate using the Cosine Rule with $6, 8.6$ **and** $AC = x$
	Note	A calculator can be used to write down at least one correct root for their 3TQ $= 0$
(c)	**Note**	Allow A1 for awrt 43.4 or awrt 43.3 in part (c) if $A\hat{B}C =$ awrt $122.8°$ is found using the ALT method in part (b)

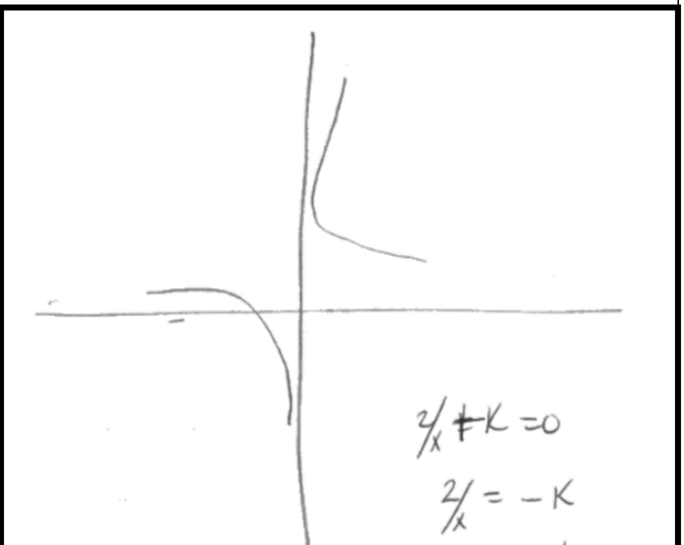
Question Number	Scheme	Notes	Marks
9.	(a) $y=\dfrac{2}{x}+k$; $k>0$ (b) $y=5-3x$, l and C do not meet		
(a)		**Either** a hyperbolic branch drawn in quadrant 1 **only** for $x>0$ **or** a hyperbolic branch drawn in both quadrant 2 and quadratic 3 for $x<0$	M1
		Correct graph – see notes	A1
		Curve cuts or meets the axes once only where $x<0$ and $\left(-\dfrac{2}{k},0\right)$ is stated **or** $-\dfrac{2}{k}$ marked on the negative x-axis. Allow $\left(0,-\dfrac{2}{k}\right)$ rather than $\left(-\dfrac{2}{k},0\right)$ if marked in the correct place on the x-axis.	B1
		Only asymptotes $x=0$ and $y=k$ stated **or** seen stated in the correct positions on their graph.	B1
	Note: If curve cuts/meets the negative x-axis once then allow coordinates stated elsewhere.		**(4)**
(b) **Way 1**	$\dfrac{2}{x}+k=5-3x$ $2+kx=5x-3x^2$ $3x^2-5x+2+kx=0$	Sets $\dfrac{2}{x}+k=5-3x$ and attempts to multiply both sides by x and collects **all terms onto one side**. Allow e.g. $">0"$ or $"<0"$ for $"="$. At least 3 of the terms must be multiplied by x, e.g. allow one slip. The $'=0'$ may be implied.	M1
	$3x^2+(k-5)x+2=0$ **or** $-3x^2+(5-k)x-2=0$ **or** $a=3,\ b=k-5,\ c=2$	Correct 3TQ $\{=0\}$. If terms are not collected this mark may be implied by correct a, b and c stated or applied in b^2-4ac	A1
	$\{b^2-4ac=\}$ $(k-5)^2-4(3)(2)$	Applies b^2-4ac with their a, b and c from their equation where $a=\pm3$, $b=\pm k\pm5$ and $c=\pm2$. This could be part of the quadratic formula (only look for the b^2-4ac part) or as e.g. $b^2-4ac=0$, $b^2<4ac$, $b^2>4ac$, $\sqrt{b^2-4ac}$, etc. **Note:** There must be no $x's$ in their b^2-4ac.	M1
	$\{b^2-4ac<0\Rightarrow(k-5)^2-24<0\}$ $(k-5)^2-24=0$ $k=5\pm\sqrt{24}$ or $k=$ awrt 0.1..., awrt 9.9 $5-2\sqrt{6}<k<5+2\sqrt{6}$ (**Note:** $5+\sqrt{24}>k>5-\sqrt{24}$ is a correct answer)	**dependent on the previous M mark** Uses a correct valid method of solving their quadratic $=0$ to give **two distinct** critical values for k {and applies $b^2-4ac<0$ } to write down an inside region with both critical values for k. **Note:** Allow this mark for $0.1<k<9.9$; $5-2\sqrt{6}\le k\le5+2\sqrt{6}$; $[5-\sqrt{24},5+\sqrt{24}]$; **Note:** Give final dM0 A0 for $5+\sqrt{24}<k<5-\sqrt{24}$, o.e.	dM1
		$5-2\sqrt{6}<k<5+2\sqrt{6}$ **or exact** equivalent. Accept e.g. $5-\sqrt{24}<k<5+\sqrt{24}$; $(5-\sqrt{24},5+\sqrt{24})$; $k\in(5-\sqrt{24},5+\sqrt{24})$	A1
			(5)
			9

		Question 9 Notes
9. (a)	**M1**	For $x > 0$, condone the hyperbolic branch being asymptotic to both the x-axis and y-axis. Condone the hyperbolic branch significantly 'bending back up' when $x \to \infty$ Condone the hyperbolic branch significantly 'bending back down' for $x \to -\infty$ Condone the hyperbolic branch 'bending back' when approaching the y-axis asymptote. Condone the hyperbolic branch touching the y-axis or touching the horizontal asymptote.
	A1	The graph must not touch the y-axis and must not touch the horizontal asymptote (where the horizontal asymptote is clearly above the y-axis). **Note:** The horizontal and/or vertical asymptotes do not need to be marked or labelled for the A mark. The hyperbolic branch must not **significantly** 'bend back up' for $x \to \infty$ The hyperbolic branch must not **significantly** 'bend back down' for $x \to -\infty$ The hyperbolic branch must not **significantly** 'bend back' when approaching the y-axis asymptote.
	Note	Allow 2nd B1 for $y = 0$ marked on the x-axis in addition to $x = 0$ and $y = k$ marked in the correct positions.
	Note	Do not allow 2nd B1 for y-axis stated as their asymptote without reference to $x = 0$

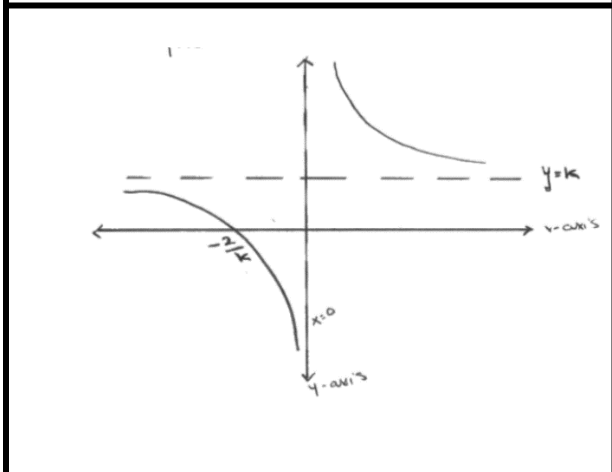
Egs.



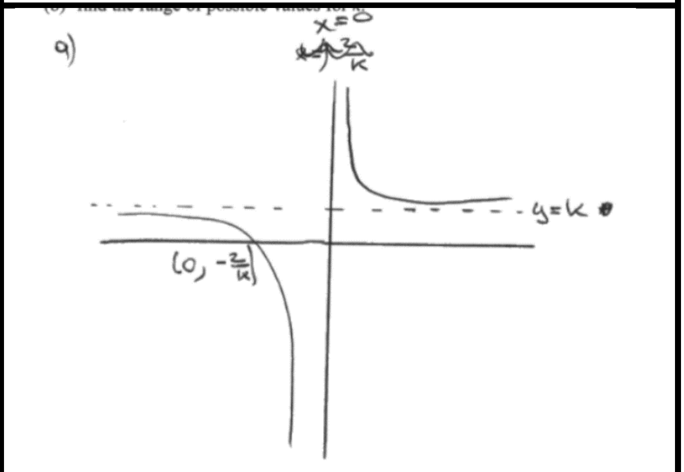
E.g. 1: Scores M1 A1 (just), B1 B0



E.g. 2: Scores M1 A0



E.g. 3: Scores M1, A1 (just), B1, B1



E.g. 4: Scores M1 A1 (just) B1 B1

Question Number	Scheme	Notes	Marks
9.	(a) $y=\frac{2}{x}+k;\ k>0$ (b) $y=5-3x$, l and C do not meet		
(b) **Way 2**	$\left\{\frac{d}{dx}\left(\frac{2}{x}+k\right)=-3 \Rightarrow\right\}\ -\frac{2}{x^2}=-3$	Differentiates $y=\frac{2}{x}+k$ to give $\frac{dy}{dx}=\pm Ax^{-2}$; $A\neq 0$, and sets the result equal to -3	M1
	$\left\{x^2=\frac{2}{3} \Rightarrow\right\}\ x=\pm\sqrt{\frac{2}{3}}$	$x=\pm\sqrt{\frac{2}{3}}$ **or** $x=\pm$ awrt 0.82 **or** $x=\pm\frac{1}{3}\sqrt{6}$	A1
	$\left\{\frac{2}{x}+k=5-3x,\ x=\sqrt{\frac{2}{3}},\ -\sqrt{\frac{2}{3}} \Rightarrow\right\}$ Either $\frac{2}{\sqrt{\frac{2}{3}}}+k=5-3\left(\sqrt{\frac{2}{3}}\right)$ or $\frac{2}{-\sqrt{\frac{2}{3}}}+k=5-3\left(-\sqrt{\frac{2}{3}}\right)$	Substitutes at least one of their x, (which has been found from solving $\pm Ax^{-2}=-3$), into the equation $\frac{2}{x}+k=5-3x$	M1
	$k=5-2\sqrt{6}$, $5+2\sqrt{6}$ or $k=$ awrt 0.1..., awrt 9.9	**dependent on the previous M mark** Uses a complete method to find **both** critical values for k **and** writes down an inside region with both critical values for k.	dM1
	$5-2\sqrt{6}<k<5+2\sqrt{6}$	$5-2\sqrt{6}<k<5+2\sqrt{6}$ **or exact** equivalent.	A1
			(5)

		Question 9 Notes Continued
9. (b)	**Note**	For the final A mark accept exact equivalents such as $\frac{10-\sqrt{96}}{2}<k<\frac{10+\sqrt{96}}{2}$; $k>5-2\sqrt{6}$ **and** $k<5+2\sqrt{6}$.
	Note	Give final dM0 A0 (unless recovered) for $k>5-2\sqrt{6}$ **or** $k<5+2\sqrt{6}$; $k>5-2\sqrt{6}$, $k<5+2\sqrt{6}$
	Note	Give final dM1 A0 (unless recovered) for $5-2\sqrt{6}<x<5+2\sqrt{6}$, o.e.
	Note	$3x^2+kx-5x+2=0$ by itself is 1st A0, but $3x^2+kx-5x+2=0$ followed by $(k-5)^2-4(3)(2)$ is final 1st A1 (implied), 2nd M1

Question Number	Scheme	Notes	Marks
10.	(a) $\left(2-\frac{1}{3}x\right)^9$ (b) $\mathrm{f}(x)=\left(3+\frac{a}{x}\right)\left(2-\frac{1}{3}x\right)^9$; coefficient of x in $\mathrm{f}(x)$ is 0		
(a) **Way 1**	$=2^9+{}^9C_1(2)^8\left(-\frac{1}{3}x\right)+{}^9C_2(2)^7\left(-\frac{1}{3}x\right)^2+{}^9C_3(2)^6\left(-\frac{1}{3}x\right)^3+\dots$	Constant term of 2^9 or 512	B1
		See notes	M1
		See notes	A1
	$\left\{=512+(9)(256)\left(-\frac{1}{3}x\right)+(36)(128)\left(\frac{1}{9}x^2\right)+(84)(64)\left(-\frac{1}{27}x^3\right)+\dots\right\}$		
	$=512-768x+512x^2-\frac{1792}{9}x^3+\dots$	At least one correctly simplified x term or x^2 term or x^3 term	A1
		$512-768x+512x^2-\frac{1792}{9}x^3$	A1
	Note: Any of the final two A marks may not be found on the final line of working. **Note:** Work for the final A mark must be seen on one line and you can apply isw		**(5)**
(a) **Way 2**	$\left\{2^9\left(1-\frac{1}{6}x\right)^9\right\}=2^9\left(1+{}^9C_1\left(-\frac{1}{6}x\right)+{}^9C_2\left(-\frac{1}{6}x\right)^2+{}^9C_3\left(-\frac{1}{6}x\right)^3+\dots\right)$	See notes	B1
		See notes	M1
		See notes	A1
	$\left\{=512\left(1+(9)\left(-\frac{1}{6}x\right)+(36)\left(\frac{1}{36}x^2\right)+(84)\left(-\frac{1}{216}x^3\right)+\dots\right)\right\}$		
	$=512-768x+512x^2-\frac{1792}{9}x^3+\dots$	At least one correctly simplified x term or x^2 term or x^3 term	A1
		$512-768x+512x^2-\frac{1792}{9}x^3$	A1
			(5)
(b)	$\mathrm{f}(x)=\left(3+\frac{a}{x}\right)\left(512-768x+512x^2-\frac{1792}{9}x^3\right)$		
	Either $\mathrm{f}(x)=1536\underline{-2304x}+1536x^2-\frac{1792}{3}x^3+\frac{512a}{x}-768\underline{+512ax}-\frac{1792}{9}ax^2$	**Either** writes down $\pm3('768')x\pm'512'ax$ as part of their expansion of $\mathrm{f}(x)$ **or** identifies their x terms as $\pm3('768')x\pm'512'ax$ **or** identifies their coefficient of x as $\pm3('768')\pm'512'a$	M1
	or x terms: $3(-768)x+512ax$		
	or coefficient of x: $3(-768)+512a$		
	$3(-768)x+512ax=0\Rightarrow a=\dots$ or $3(-768)+512a=0\Rightarrow a=\dots$	**dependent on the previous M mark** Sets their x term equal to 0 or sets their coefficient of x equal to 0 and solves to give $a=\dots$	dM1
	$\left\{a=\frac{3(768)}{512}\Rightarrow\right\}$ $a=\frac{9}{2}$	Correct simplified a. E.g. $a=\frac{9}{2}$ or $4\frac{1}{2}$ or 4.5	A1
			(3)
			8

		Question 10 Notes
10. (a)	**B1**	Constant term of 2^9 or 512. Do not allow B1 for $512x^0$ unless simplified to 2^9 or 512.
Way 1	**1st M1**	$({}^9C_1)(...)(x)$ or $({}^9C_2)(...)(x^2)$ or $({}^9C_3)(...)(x^3)$. Requires **correct** binomial coefficient in any form **with the correct power of *x***, but the other part of the coefficient may be wrong or missing.
	1st A1	At least two correct terms from ${}^9C_1(2)^8\left(-\frac{1}{3}x\right) + {}^9C_2(2)^7\left(-\frac{1}{3}x\right)^2 + {}^9C_2(2)^6\left(-\frac{1}{3}x\right)^3$, or equivalent, which can be un-simplified or simplified.
	Note	${}^9C_1(2)^8 -\frac{1}{3}x + {}^9C_2(2)^7 -\frac{1}{3}x^2 + {}^9C_2(2)^6 -\frac{1}{3}x^3 + ..$ {bad bracketing} scores M0 unless later work implies a correct method.
	Note	The common error $2^9 + {}^9C_1(2)^8\left(-\frac{1}{3}x\right) + {}^9C_2(2)^7\left(-\frac{1}{3}x^2\right) + {}^9C_3(2)^6\left(-\frac{1}{3}x^3\right)$ $512 - 768x + 1536x^2 - 1792x^3$ is B1 M1 A0 A1 A0
	Note	The common error ${}^9C_1(2)^8\left(\frac{1}{3}x\right) + {}^9C_2(2)^7\left(\frac{1}{3}x\right)^2 + {}^9C_3(2)^6\left(\frac{1}{3}x\right)^3$ $512 + 768x + 562x^2 + \frac{1792}{9}x^3$ is B1 M1 A0 A1 A0
	Note	$2^9 + {}^9C_8(2)^8\left(-\frac{1}{3}x\right) + {}^9C_7(2)^7\left(-\frac{1}{3}x\right)^2 + {}^9C_6(2)^6\left(-\frac{1}{3}x\right)^3 + ...$ is also a correct expansion.
(a)	**B1**	$2^9(1 \pm ...)$ or $512(1 \pm ...)$. Award when first seen.
Way 2	**1st M1**	Expands $(1 \pm kx)^9$; $k \neq \pm\frac{1}{3}$ to give either $({}^9C_1)(...)(x)$ or $({}^9C_2)(...)(x^2)$ or $({}^9C_3)(...)(x^3)$. Requires **correct** binomial coefficient in any form **with the correct power of *x***, but the other part of the coefficient may be wrong or missing.
	1st A1	At least two correct terms from ${}^9C_1\left(-\frac{1}{6}x\right) + {}^9C_2\left(-\frac{1}{6}x\right)^2 + {}^9C_3\left(-\frac{1}{6}x\right)^3$ or $-\frac{3}{2}x + x^2 - \frac{7}{18}x^3$, or equivalent, which can be un-simplified or simplified.
	SC	Allow Special Case B1 M1 A1 for Way 2: $K\left(1 + {}^9C_1\left(-\frac{1}{6}x\right) + {}^9C_2\left(-\frac{1}{6}x\right)^2 + {}^9C_3\left(-\frac{1}{6}x\right)^3\right)$ or $K\left(1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3\right)$ where $K \neq 2^9$ or $K \neq 512$
	Note	$2\left(1 + {}^9C_1\left(-\frac{1}{6}x\right) + {}^9C_2\left(-\frac{1}{6}x\right)^2 + {}^9C_3\left(-\frac{1}{6}x\right)^3 + ...\right)$ would get SC B1 M1 A1 A0 A0
(a)	**Note**	E.g. $\binom{9}{3}$ or $\frac{9(8)(7)}{3!}$ or $\frac{9!}{3!6!}$ or 84 or even $\binom{9}{3}$ can be written in place of ${}^9C_3$
	Note	Condone giving the final A mark for a 'simplified' $512 + -768x + 512x^2 + -\frac{1792}{9}x^3$.
	Note	$-\frac{1792}{9}x^3$ may be written as either $-199\frac{1}{9}x^3$ or $-199.\dot{1}\,x^3$ but do not allow $-199.1x^3$ or $-199x^3$
	Note	Condone terms in reverse order $-\frac{1792}{9}x^3 + 512x^2 - 768x + 512$ for B1 M1 A1 A1 A1.

	Question 10 Notes Continued	
10. (a)	**Note**	The terms may be "listed" rather than added for any of the first 4 marks.
	Note	Any higher order terms can be ignored in part (a).
	SC	**Special Case:** If a candidate expands in descending powers of x, i.e. $\left\{\left(2-\frac{1}{3}x\right)^9\right\} = \left(-\frac{1}{3}x\right)^9 + {}^9C_1(2)^1\left(-\frac{1}{3}x\right)^8 + {}^9C_2(2)^2\left(-\frac{1}{3}x\right)^7 + {}^9C_3(2)^3\left(-\frac{1}{3}x\right)^6$ $= -\frac{1}{19683}x^9 + (9)(2)\left(\frac{1}{6561}x^8\right) + (36)(4)\left(-\frac{1}{2187}x^7\right) + (84)(8)\left(\frac{1}{729}x^6\right)$ $= -\frac{1}{19683}x^9 + \frac{2}{729}x^8 - \frac{16}{243}x^7 + \frac{224}{243}x^6$ then they can gain SC: B1 M1 A1 A0 A0
		B1 — For a simplified $-\frac{1}{19683}x^9$
		M1: — $({}^9C_1)(...)(x^8)$ or $({}^9C_2)(...)(x^7)$ or $({}^9C_3)(...)(x^6)$ or $({}^9C_8)(...)(x^8)$ or $({}^9C_7)(...)(x^7)$ or $({}^9C_6)(...)(x^6)$
		1st A1: — At least two correct terms from ${}^9C_1(2)^1\left(-\frac{1}{3}x\right)^8 + {}^9C_2(2)^2\left(-\frac{1}{3}x\right)^7 + {}^9C_3(2)^3\left(-\frac{1}{3}x\right)^6$ which can be un-simplified or simplified.
10. (b)	**Note**	Give 1st M0 (unless recovered) for any extra x terms in their expansion of $\mathrm{f}(x)$ or for any additional x terms in $\pm 3('768')x \pm '512'ax$ or for any additional terms in $\pm 3('768') \pm '512'a$.
	Note	Give M1 dM1 for $\pm 3('768')x \pm '512'ax \Rightarrow a = ...$ or for $\pm 3('768') \pm '512'a = 0 \Rightarrow a = ...$
	Note	Valid solutions include $2^9 a - 9(2^8) = 0$ or $\frac{36(2^7)}{9}a - \frac{(3)(9)(2^8)}{3} = 0 \Rightarrow a = \frac{9}{2}$
	Note	Allow 1st M1 for $3(-768x) + \frac{a}{x}(512x^2) = 0$ or $0x$
	Note	M1 dM1 A1 can be given for $K\left(1 + {}^9C_1\left(-\frac{1}{6}x\right) + {}^9C_2\left(-\frac{1}{6}x\right)^2 + ...\right)$ where $K \neq 2^9$ or $K \neq 512$ leading to $a = \frac{9}{2}$ in Q10(b). E.g. $K = \frac{1}{512}$ gives $\frac{a}{512} - \frac{3(3)}{1024} = 0 \Rightarrow a = \frac{9}{2}$

Question Number	Scheme	Notes	Marks
11.	$f(x) = 13 + 3x + (x+2)(x+k)^2$; given $(x+3)$ is a factor of $f(x)$		
(a)(i),(ii)	$f(-3) = 13 + 3(-3) + (-3+2)(-3+k)^2 = 0$	Applies $f(\pm 3)$ to obtain an expression in k only and sets their expression equal to 0	M1
	$4 - (-3+k)^2 = 0$ **(See note)** $(-3+k)^2 = 4$ $-3+k = \pm 2$ $k = 5, 1$ or $4 - (k^2 - 6k + 9) = 0$ $k^2 - 6k + 5 = 0$ $(k-5)(k-1) = 0$ $k = 5, 1$	**dependent on the previous M mark** Correct valid method for solving their quadratic in k to give at least one value of $k = ...$	dM1
		Correct method for finding $k = 5$ (***answer is given***) **and** finds $k = 1$	A1
			(3)
(a) (i) **Alt**	$\{x = -3, k = 5 \Rightarrow\}$	**Use this Alt method for 1st M1 only**	
	$f(-3) = 13 + 3(-3) + (-3+2)(-3+5)^2$ $\{= 13 - 9 - 4\} = 0 \quad \Rightarrow k = 5$	Uses $x = -3, k = 5$ to correctly show that $f(-3) = 0$ **and** concludes that $k = 5$	M1
			(1)
(b) (i)	$f(x) = 13 + 3x + (x+2)(x+5)^2$ $= 13 + 3x + (x+2)(x^2 + 10x + 25)$ $= 13 + 3x + x^3 + 10x^2 + 25x + 2x^2 + 20x + 50$	Attempts to multiply out $f(x)$ with $k = 5$ to give a 4-term cubic of the form $\pm Ax^3 \pm Bx^2 \pm Cx \pm D$; $A, B, C, D \neq 0$	M1
	$= x^3 + 12x^2 + 48x + 63$	$x^3 + 12x^2 + 48x + 63$	A1
	Hence $f(x) = (x+3)(x^2 + 9x + 21)$	Uses their simplified cubic and $(x+3)$ in an attempt to find the quadratic factor. e.g. Attempts to divide by $(x+3)$ using long division to give $x^2 \pm kx + ...,\ k = \text{value} \neq 0$ e.g. factorising/equating coefficients to obtain $(x+3)(x^2 \pm kx \pm c),\ k = \text{value} \neq 0$, c can be 0	M1
		$(x+3)(x^2 + 9x + 21)$ seen on one line	A1
	Note: Give final M0 for attempting to divide by $(x-3)$ **Note:** Give final M0 for factorising/equating coefficients to give $(x-3)(x^2 \pm kx \pm c)$ **Note:** You can recover work for (b)(i) in (b)(ii)		**(4)**
(b)(ii) **Way 1**	$\{b^2 - 4ac =\}\ 9^2 - 4(1)(21)$	Applies $b^2 - 4ac$ on their "$x^2 + 9x + 21$" where $a, b, c \neq 0$. This could be part of the quadratic formula (i.e. the $b^2 - 4ac$ part) or embedded in $b^2 < 4ac$.	M1
	e.g. $b^2 - 4ac = -3 < 0 \Rightarrow$ no solution and so $x = -3$ **e.g.** $b^2 - 4ac = -3 < 0 \Rightarrow$ no solution and the only solution comes from $x + 3 = 0$	Finds $b^2 - 4ac = -3$, states $-3 < 0 \Rightarrow$ no solution **and either** $x = -3$ **or** only solution comes from $x + 3 = 0$	A1 **cso**
	Note: Give A0 for stating '$(x+3)$ is the only solution'. **Note:** If they refer to the solution of $x = -3$ it must be correct (not e.g. $x = 3$) for A1 **cso** **Note:** Give A0 for $b^2 - 4ac = -3 < 0 \Rightarrow$ no solution and $x^2 + 9x + 21 < 0 \Rightarrow x = -3$ **Note:** $x = -3$ must clearly be a part of their solution for A1 **Note:** The solution $x = -3$ must be referred to in (b)(ii)		**(2)**
			9

Question Number	Scheme	Notes	Marks
11. (ii)(b) **Way 2**	$\{(x^2+9x+21)=0 \Rightarrow\}$ $\left(x+\frac{9}{2}\right)^2 - \frac{81}{4} + 21 = 0$ $\left(x+\frac{9}{2}\right)^2 = -\frac{3}{4}$ or $x+\frac{9}{2} = \pm\sqrt{-\frac{3}{4}}$	Completes the square on their $"x^2+bx+c"$ where $b,\, c \neq 0$ to make $\left(x+\frac{b}{2}\right)^2$ or $\left(x+\frac{b}{2}\right)$ the subject.	M1
	e.g. {Quadratic} has no solutions and so $x=-3$	$\left(x+\frac{9}{2}\right)^2 = -\frac{3}{4}$ or $x+\frac{9}{2} = \pm\sqrt{-\frac{3}{4}}$ or $x+\frac{9}{2} = \sqrt{-\frac{3}{4}}$, $\Rightarrow$ no solution (or maths error) **and either** $x=-3$ **or** only solution comes from $x+3=0$	A1 **cso**
	e.g. {Quadratic} has no solutions and so the only solution comes from $x+3=0$		
			(2)
11. (ii)(b) **Way 3**	$\{(x^2+9x+21)=0 \Rightarrow\}$ $x = \frac{-9 \pm \sqrt{81-4(1)(21)}}{2}$	Applies b^2-4ac on their $"x^2+9x+21"$ where $a,\, b,\, c \neq 0$. **Note:** This must be seen as part of the quadratic formula.	M1
	e.g. $x = \frac{-9 \pm \sqrt{-3}}{2} \Rightarrow$ {Quadratic} has no solutions and so $x=-3$.	$x = \frac{-9 \pm \sqrt{-3}}{2}$ $\Rightarrow$ no solution (or maths error) **and either** $x=-3$ **or** only solution comes from $x+3=0$	A1 **cso**
	e.g. $x = \frac{-9 \pm \sqrt{-3}}{2} \Rightarrow$ {Quadratic} has no solutions and so the only solution comes from $x+3=0$		
			(2)

	Question 11 Notes	
11. (a)	**Note**	'= 0' can be implied in their working for A1
	Note	1st M can be given for applying $f(\pm 3)$ to their manipulated $f(x) = \dots$
	Note	ALT: $f(-3) = 13 + 3(-3) + (-3+2)(-3+5)^2 = 0 \Rightarrow k = 5$ is sufficient for 1st M1
	Note	Give dM0 for simplifying $13+3(-3)+(-3+2)(-3+k)^2 = 0$ to give $13-9+(-1)(-3+k)^2 = 0 \Rightarrow 3(-3+k)^2 = 0$
	Note	Give dM0 for simplifying $13+3(-3)+(-3+2)(-3+k)^2 = 0$ to give • $4-(-3+k)^2 = 0 \Rightarrow 4-9-k^2 = 0$ or $4-(9-6+k^2) = 0 \Rightarrow k = \dots$
	Note	Condone writing $-k^2+6k+5=0 \Rightarrow (k-5)(k-1)=0 \Rightarrow k=5, 1$ for A1
	Note	Give final A1 for $-k^2+6k-5=0$ or $k^2-6k+5=0 \Rightarrow k=5, 1$ with no intermediate working.

		Question 11 Notes Continued
11. (b)(i)	**Note**	Condone $(x+5)^2 \to x^2+25$ as part of their working for the 1st M mark.
	Note	Condone 2nd M1 e.g. for $x^3+12x^2+48x+63 \to (x+3)(x^2+12x+48)$
(b)(ii)	**Note**	When a student refers to 'solution' it is assumed that they mean a 'real solution'.
	Note	'<0' or 'it is negative' must also be stated in a discriminant method for A1
	Note	A correct discriminant calculation, e.g. $9^2-4(1)(21)$, $81-84$ or -3 is sufficient as part of their working for A1. E.g. Give M1 A1 for $b^2-4ac=81-84<0$, so no solution $\Rightarrow x=-3$
	Note	Give A0 for incorrect working, e.g. $9^2-4(1)(21)=-5<0$
	Note	Give M1 A1 cso for $x=-\frac{9}{2}\pm\frac{\sqrt{3}}{2}\mathrm{i}\,,-3$
	Note	Allow the statement 'as $y=\mathrm{f}(x)$ is a cubic {function}, and cubic functions have at least one solution, $\mathrm{f}(x)\{=0\}$ has one solution' written in place of either '**either** $x=-3$ **or** only solution comes from $x+3=0$' for the A1 mark

Question Number	Scheme	Notes	Marks
12.	$y = \tan x,\ y = 5\cos x\ ;\ 0 < x \le 2\pi$		
(a)	$5\cos x = \tan x$	Sets $5\cos x = \tan x$	B1
	$5\cos x = \dfrac{\sin x}{\cos x}\quad \{\Rightarrow 5\cos^2 x = \sin x\}$	Applies $\tan x = \dfrac{\sin x}{\cos x}$ to their equation or correctly multiplies both sides by $\cos x$	M1
	$5(1 - \sin^2 x) = \sin x$	Uses $\cos^2 x = 1 - \sin^2 x$ to form an equation in just $\sin x$	M1
	$5\sin^2 x + \sin x - 5 = 0$ *	Correct proof with no notational errors	A1 * **cso**
			(4)
(b)	• $\sin x = \dfrac{-1 \pm \sqrt{1 - 4(5)(-5)}}{10}$ $\left\{ = \dfrac{-1 \pm \sqrt{101}}{10} = 0.9049...,\ -1.1049... \right\}$ • $5\left(\sin x + \frac{1}{10}\right)^2 - \frac{1}{20} - 5 = 0 \Rightarrow \sin x = ...$ $\left(\sin x + \frac{1}{10}\right)^2 - \frac{1}{100} - 1 = 0 \Rightarrow \sin x = ...$	Attempts to solve the quadratic = 0 by correct quadratic formula or by completing the square to give $\sin x = ...$, (but condone just $x = ...$ instead of $\sin x = ...$). **Note:** Factorisation attempts score M0. **Note:** The negative square root can be omitted in their working.	M1
	$x = 1.13135...,\ 2.01024...$ $\{\Rightarrow x_A = 1.13,\ x_B = 2.01\ (2\text{ dp})\}$	**dependent on the previous M mark** Uses 'arcsin' to obtain at least one value of x (in radians or in degrees) written down to at least one decimal place. Accept dM1 for any of $x =$ awrt 1.1, awrt 2.0, awrt 64.8, awrt 115.2, awrt 3.6, awrt 5.9, awrt 204.6 or awrt 335.4	dM1
		At least one of either $x =$ awrt 1.13, awrt 2.01, awrt 64.82 or awrt 115.18	A1
		Both $x =$ awrt 1.13 **and** $x =$ awrt 2.01 and no extra solutions in the range $(0, 2\pi]$ **or** for $x_A =$ awrt 1.13 **and** $x_B =$ awrt 2.01	A1
	Note: Work for part (b) cannot be recovered in part (c).		**(4)**
(c) (i)	22	22	B1
	• 2 solutions every 2π (or $360°$) plus 2 solutions in the final π (or $180°$) **or** states $2(10) + 2$ • 20 solutions in 20π (or $1800°$) plus two solutions in the final π (or $180°$) **or** states $20 + 2$ • 20 solutions for $0 < x < 20\pi$ so 22 solutions for $0 < x \le 21\pi$ • each solution is repeated another 10 more times	**dependent on the previous B mark** Acceptable reason or acceptable calculation.	dB1
(ii)	40	40	B1
	• 2 solutions every π (or $180°$) **or** states $2(20)$ • 4 solutions every 2π (or $360°$) **or** states $4(10)$	**dependent on the previous B mark** Acceptable reason or acceptable calculation.	dB1
			(4)
			12

		Question 12 Notes
12. (b)	**Note**	**Completing the square:** Give M1 for either $5\left(\sin x \pm \frac{1}{10}\right)^2 \pm q \pm 5 = 0 \Rightarrow \sin x = ...$ or for $\left(\sin x \pm \frac{1}{10}\right)^2 \pm q \pm 1 = 0 \Rightarrow \sin x = ...;\ q \neq 0$
	Note	Give M0 dM0 A0 A0 for writing down $x = 1.13,\ 2.01$ from no working.
	Note	Give M0 dM0 A0 A0 for writing down $x =$ awrt 1.13, awrt 2.01, awrt 64.82 or awrt 115.18 from no working.
	Note	Condone 1st M1 for writing down (from their graphical calculator) $\sin x =$ awrt 0.9
	Note	Give M1 dM1 A1 A0 for '$\sin x = 0.9 \Rightarrow x = 1.13$'
	Note	Give M1 dM1 A1 A1 for '$\sin x = 0.9 \Rightarrow x = 1.13,\ 2.01$'
	Note	Give 2nd A0 for incorrectly deducing $x_A =$ awrt 2.01 **and** $x_B =$ awrt 1.13

Question Number	Scheme	Notes	Marks
13. (a)	$\frac{1}{2}r^2\theta = 200 \quad \left(\text{or } \frac{\theta}{2\pi} = \frac{200}{\pi r^2}\right)$	States or uses $\frac{1}{2}r^2\theta = 200$, o.e.	B1
	$P = r + r + r\theta$	States or uses $\{P =\} = 2r + r\theta$ o.e. Allow B1 for $\{P =\}2r + l,\ l = r\theta$	B1
	$\frac{1}{2}r^2\theta = 200 \Rightarrow$ • $r\theta = \frac{400}{r} \Rightarrow P = 2r + \frac{400}{r}$ * • $\theta = \frac{400}{r^2} \Rightarrow P = 2r + r\left(\frac{400}{r^2}\right) \Rightarrow P = 2r + \frac{400}{r}$ *	Applies a complete process of substituting $r\theta = ...$ or $\theta = ...$, where '...' $= \mathrm{f}(r)$ into an expression for the perimeter which is of the form $P = \lambda r + \mu\theta;\ \lambda, \mu \neq 0$	M1
		Correct proof with some reference to $P =$, $P \rightarrow$ or P: as part of their proof. **Note:** 'Perimeter' can be written in place of P.	A1 *
			(4)
(b)	$\frac{\mathrm{d}P}{\mathrm{d}r} = 2 - 400r^{-2}$	Differentiates $Cr + \frac{D}{r}$ to give $P + Qr^{-2}$; $C, D, P, Q \neq 0$	M1
		$\left\{\frac{\mathrm{d}P}{\mathrm{d}r} =\right\}\ 2 - 400r^{-2}$, o.e.	A1
	$\left\{\frac{\mathrm{d}P}{\mathrm{d}r} = 0 \Rightarrow\right\}\ 2 - \frac{400}{r^2} = 0$ $\Rightarrow 2r^2 - 400 = 0 \Rightarrow r^2 = ...\ \{= 200\}$	Sets their $\frac{\mathrm{d}P}{\mathrm{d}r} = 0$ and rearranges to give $r^{\pm n} = k$, $k > 0$, $n=2$ or 3	M1
	$\{r = 10\sqrt{2} \Rightarrow\}$ $P = 2(10\sqrt{2}) + \frac{400}{10\sqrt{2}} = 40\sqrt{2}$	**dependent on the previous mark** Substitutes their r (where $r > 0$), which has been found by solving $\frac{\mathrm{d}P}{\mathrm{d}r} = 0$, into $P = 2r + \frac{400}{r}$	dM1
		$P = 40\sqrt{2}$ or $\sqrt{1600}$ or $20\sqrt{8}$ or $\frac{80}{\sqrt{2}}$ or any exact equivalent in the form $a\sqrt{b}$ or $\frac{a}{\sqrt{b}}$	A1
			(5)
(c) **Way 1**	$\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = 800r^{-3} > 0 \Rightarrow$ Minimum {value of P}	Differentiates to give $\left\{\frac{\mathrm{d}^2P}{\mathrm{d}r^2} =\right\}\ \pm Kr^{-3}$, $K \neq 0$	M1
		$800r^{-3}$, > 0 **and** minimum **Note:** ft is only allowed on their '$r = \sqrt{200}$' value from (b), where $r > 0$	A1 ft **cso**
	NB: A1 is **cso**, so calculations for P'' using their '$r = \sqrt{200}$' must be correct to at least 2 sf		**(2)**
(c) **Way 2**	$\{r = 10\sqrt{2} = 14.142... \Rightarrow\}$ $r = 14.1 \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}r} = -0.01197... < 0$ $r = 14.2 \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}r} = 0.01626... > 0$ $\Rightarrow$ Minimum {value of P}	Applies a value on each side of their $r = 10\sqrt{2}$ (where $r > 0$) to an expression of the form $P + Qr^{-2}$; $P, Q \neq 0$	M1
		Correct evaluations to at least 1 sf, $< 0, > 0$ **and** minimum	A1 ft **cso**
			(2)
			11

	Question 13 Notes	
13. (b)	**Note**	The 2nd M mark can be implied. Give 2nd M for $2-\dfrac{400}{r^2}=0 \rightarrow r=\sqrt{200}$ or $r=10\sqrt{2}$ or $r=$ awrt 14.1
	Note	Give final dM1 A0 for $r=14.14... \Rightarrow P=$ awrt 56.6 without reference to a correct exact value for P.
	Note	Give 2nd M0 for $2-\dfrac{400}{r^2}<0 \Rightarrow r<10\sqrt{2}$ but give 2nd M1 dM1 2nd A1 for $2-\dfrac{400}{r^2}<0 \Rightarrow r<10\sqrt{2} \Rightarrow P=2(10\sqrt{2})+\dfrac{400}{10\sqrt{2}}=40\sqrt{2}$
	Note	Give 2nd M0 for $2-\dfrac{400}{r^2}>0 \Rightarrow r>10\sqrt{2}$ but give 2nd M1 dM1 2nd A1 for $2-\dfrac{400}{r^2}>0 \Rightarrow r>10\sqrt{2} \Rightarrow P=2(10\sqrt{2})+\dfrac{400}{10\sqrt{2}}=40\sqrt{2}$
(c)	**Note**	Ignore poor differentiation notation or the lack of differentiation notation in part (c).
	Note	Condone '$\dfrac{d^2P}{dr^2}=800r^{-3}>0 \Rightarrow$ Minimum value of r' for final A1
	Note	Using their $r=10\sqrt{2}$ from (b), give M1 A1 for any of • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=\dfrac{800}{(10\sqrt{2})^3}>0 \Rightarrow$ Minimum • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=0.2828...>0 \Rightarrow$ Minimum • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=0.2828...>0 \Rightarrow P_{\min}$ • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=\dfrac{\sqrt{2}}{5}...>0 \Rightarrow$ Minimum
	Note	Using their $r=10\sqrt{2}$ from (b), give M1 A0 for any of • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=\dfrac{800}{10\sqrt{2}^{\,3}}>0 \Rightarrow$ Minimum {poor bracketing} • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=\dfrac{800}{(40\sqrt{2})^3}=0.0044...>0 \Rightarrow$ Minimum • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=0.282... \Rightarrow$ Minimum {No reference to >0} • $\dfrac{d^2P}{dr^2}=800r^{-3} \Rightarrow \dfrac{d^2P}{dr^2}=\dfrac{800}{(10\sqrt{2})^3}=8>0 \Rightarrow$ Minimum {incorrect evaluation}

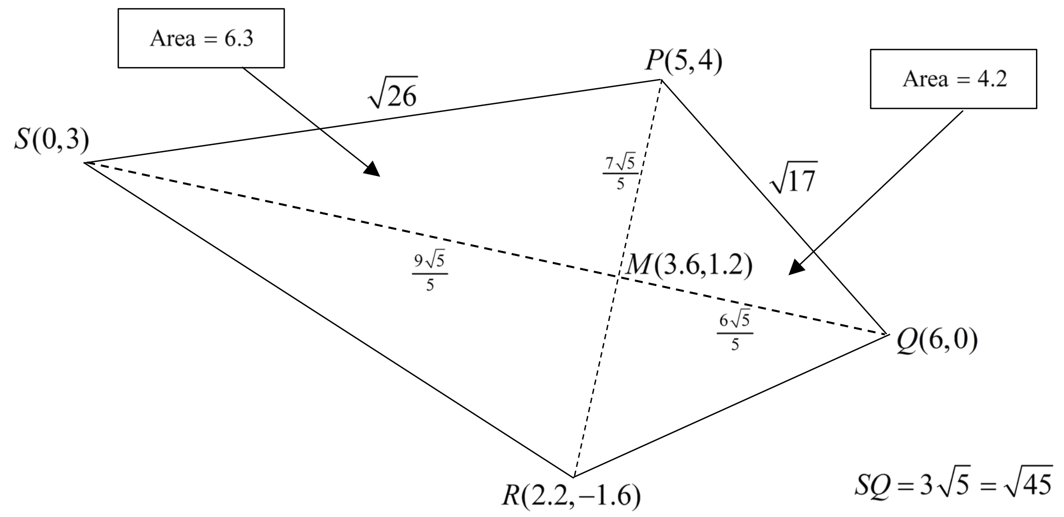
Question Number	Scheme	Notes	Marks
14.	(i) $G_1 = 22$, $G_5 = 130$; $G_1, G_2, G_3, \ldots$ is a geometric sequence (ii) $T_1 = 208$, $T_2 = 207.2$; $T_1, T_2, T_3, \ldots$ is a arithmetic sequence		
(i)	$a = 22,\ ar^4 = 130$ or $22r^4 = 130$	Writes down $a = 22$ **and** $ar^4 = 130$ **or** writes down a correct equation in r only.	M1
	$r = \sqrt[4]{\dfrac{130}{22}}$ $\{= 1.559122245...\}$	$r = \sqrt[4]{\dfrac{130}{22}}$ **or** $\sqrt[4]{\dfrac{65}{11}}$ **or** awrt 1.56	A1
	$\{G_2 = ar \Rightarrow\}$ $G_2 = 22('1.5591...')$	**dependent on the previous M mark** Obtains r from $r^4 = \dfrac{130}{22}$ o.e. and applies $22(\text{their } r)$	dM1
	$= 34.3$ (km h^{-1}) **cao**	34.3 **cao** **Note:** Ignore the units	A1 **cao**
	Note: Condone a copying error (or slip) on one of either '22' or '130' for the M marks.		**(4)**
(ii)	$\{T_n = 0 \Rightarrow a + (n-1)d = 0 \Rightarrow\}$		
(a) **Way 1**	e.g. • $208 + (n-1)(-0.8) = 0 \Rightarrow n = 261$ • $n = \dfrac{208}{0.8} \Rightarrow n = 260$	**Either** applies $a + (n-1)d = 0$ with $a = 208,\ d = -0.8$ to find $n = \ldots$ **or** deduces $n = \dfrac{208}{0.8}$	M1
		Finds $n = 261$ or $n = 260$	A1
	• $S_{261} = \dfrac{261}{2}(2(208) + (260)(-0.8))$ $\left\{= \dfrac{261}{2}(208)\right\}$ • $S_{260} = \dfrac{260}{2}(2(208) + (259)(-0.8))$ $\{= 130(208.8)\}$ • $S_{261} = \dfrac{261}{2}(208 + 0)$ $\left\{= \dfrac{261}{2}(208)\right\}$ • $S_{260} = \dfrac{260}{2}(208 + 0.8))$ $\{= 130(208.8)\}$	**dependent on the previous M mark** **Either** applies $S_n = \dfrac{n}{2}(2a + (n-1)d)$ with $a = 208,\ d = -0.8,\ n = "261"$ or with $a = 208,\ d = -0.8,\ n = "260"$ **or** applies $S_n = \dfrac{n}{2}(a + l)$ with $a = 208,\ n = "261",\ l = 0$ or with $a = 208,\ n = "260",\ l = 0.8$	dM1
	$\{$Maximum value of $S_n\}$ $= 27144$ **cao**	27144	A1 **cao**
			(4)
(a) **Way 2**	$S_n = \dfrac{n}{2}(2(208) + (n-1)(-0.8)) = \dfrac{n}{2}(416 - 0.8n + 0.8)$ $= \dfrac{n}{2}(416.8 - 0.8n) = 208.4n - 0.4n^2$ • $\dfrac{dS_n}{dn} = 208.4 - 0.8n = 0 \Rightarrow n = \dfrac{208.4}{0.8}$ $S_n = -0.4(n^2 - 521n) = -0.4((n - 260.5)^2 - (260.5)^2)$	Applies $S_n = \dfrac{n}{2}(2a + (n-1)d)$ (with $a = 208,\ d = -0.8$) and **either** a valid attempt (i.e. $n^k \to n^{k-1}$) to differentiate with respect to n, sets the result equal to 0 (condone > 0 or < 0) to find $n = \ldots$ **or** a valid attempt to complete the square	M1
	$n = 260.5$ or $S_n = -0.4((n - 260.5)^2 - (260.5)^2)$	Uses a correct algebra to find or deduce $n = 260.5$ Also allow $S_n = -0.4(n - 260.5)^2 + 27144.1$	A1
	• $S_{260} = 208.4(260) - 0.4(260)^2$ • $S_{261} = 208.4(261) - 0.4(261)^2$	**dependent on the previous M mark** Applies an integer value for n which either side of their $n = "260.5"$ to their $S_n = 208.4n - 0.4n^2$ or to a valid formula for S_n. **(See notes)**	dM1
	$\{$Maximum value of $S_n\}$ $= 27144$ **cao**	Concludes maximum sum is 27144	A1 **cao**
			(4)
(ii) (b)	522	522	B1 **cao**
			(1)
			9

	Question 14 Notes	
14. (ii)	**Note**	Condone 1st M1 1st A1 for $208 + (n)(-0.8) = 0 \Rightarrow n = 260$
	Note	Give 1st M0 1st A0 for $208 + (n-1)(0.8) = 0 \Rightarrow n = -261$ but allow 1st M1 1st A1 for $208 + (n-1)(0.8) = 0 \Rightarrow n = -261 \rightarrow n = 261$ (recovered)
	Note	**Way 1:** If a valid method gives a decimal value for *n*, then dM1 will then be given for a correct method using $S_n = \frac{n}{2}(2a+(n-1)d)$ or $S_n = \frac{n}{2}(a+l)$ with $\lfloor n \rfloor$ (i.e. where $\lfloor n \rfloor$ the integer part of *n*)
	Note	**Way 2:** If a valid method gives a decimal value for *n*, then dM1 mark will then be given for a correct method of applying S_n with integer *n* which is either side of their decimal value of *n*. E.g. If $n = 260.5$ then either $n = 260$ or $n = 261$ must be applied to an S_n expression for dM1.
	Note	**Way 2:** If a valid method gives an integer value for *n*, then dM1 mark will then be given for a correct method of applying S_n with either n or $n-1$ E.g. If $n = 250$ then either $n = 250$ or $n = 249$ must be applied to an S_n expression for dM1.
	Note	Give final dM0 A0 for finding $S_{260.5} = \frac{260.5}{2}(2(208)+(260.5)(-0.8)) = 27144.1$ or 27144 without reference to either $S_{261} = \frac{261}{2}(2(208)+(260)(-0.8)) = 27144$ or $S_{260} = \frac{260}{2}(2(208)+(259)(-0.8)) = 27144$
	Note	Allow 1st M1 1st A1 for finding $S_n = 208.4n - 0.4n^2$ **and** using their calculator to deduce $n = 260.5$

Question Number	Scheme	Notes	Marks
15.	$C_1: x^2+(y-3)^2=26$, centre S; $C_2:(x-6)^2+y^2=17$, centre Q		
(a)	$\{SQ=\}\ \sqrt{3^2+6^2}\ =3\sqrt{5}$	States or implies that S and Q are distances 3 and 6 from O	M1
		Applies $SQ=\sqrt{3^2+6^2}$ or $SQ^2=3^2+6^2$	dM1
		$3\sqrt{5}$	A1 **cao**
			(3)
(b)(i)	$C_1: x^2+y^2-6y+9=26$ $C_2: x^2-12x+36+y^2=17$ Subtracting gives: $-6y+9-(-12x+36)=9$	Attempts to multiply out both brackets followed by a correct method of eliminating both x^2 and y^2 from their simultaneous equations.	M1
	$-6y+9+12x-36=9$ $12x-36=6y$ $y=2x-6$ *	Correct proof with no errors seen in their working. **Note:** Condone omission of '$=0$' where appropriate.	A1 *
(b)(ii) **Way 1**	$(x-6)^2+(2x-6)^2=17$ $x^2-12x+36+4x^2-24x+36=17$ $5x^2-36x+72=17$	Substitutes $y=2x-6$ into either of their circle equations and proceeds to form a 3TQ in either x or y	M1
	$5x^2-36x+55=0$	$5x^2-36x+55\ \{=0\}$ $\{$**or** $5y^2-12y-32\ \{=0\}\}$	A1
	$(x-5)(5x-11)=0\Rightarrow x=...$	**dependent on the previous M mark** Correct method for solving their 3TQ $=0$ to find $x=...$	dM1
	• $x=5\Rightarrow y=(2)(5)-6=4$ • $x=2.2\Rightarrow y=(2)(2.2)-6=-1.6$	Substitutes at least one $x=...$ back into an original equation to find at least one $y=...$	dM1
	$P(5,4)$ **and** $R(2.2,-1.6)$	$P(5,4)$ **and** $R(2.2,-1.6)$ or $R(\frac{11}{5},-\frac{8}{5})$	A1
	Note: P: $x=5$, $y=4$ and R: $x=2.2$, $y=-1.6$ is fine for A1		**(7)**
(b)(ii) **Way 2**	$y=\sqrt{26-x^2}+3$, $y=\sqrt{17-(x-6)^2}$ $\sqrt{26-x^2}+3=\sqrt{17-(x-6)^2}$ $26-x^2+6\sqrt{26-x^2}+9=17-x^2+12x-36$ $6\sqrt{26-x^2}=12x-54\Rightarrow\sqrt{26-x^2}=2x-9$ $26-x^2=4x^2-36x+81$	Substitutes one circle into the other circle and uses **valid** algebra to form a 3TQ in either x or y.	M1
	$5x^2-36x+55=0$	$5x^2-36x+55\ \{=0\}$	A1
	then continue to apply the scheme for Way 1		
(c) **Way 1**	$PR=\sqrt{(5-2.2)^2+(4--1.6)^2}$	Uses the distance formula to find the length PR	M1
	$\left\{=\sqrt{\frac{196}{5}}\text{ or }\sqrt{39.2}\text{ or }\frac{14}{5}\sqrt{5}\right\}$		
	$\text{Area}(SPQR)=\frac{1}{2}\left(3\sqrt{5}\right)\left(\frac{14}{5}\sqrt{5}\right)$	**dependent on the previous M mark** Complete correct method to find $\text{Area}(SPQR)$	dM1
	$=21\ (\text{units})^2$	21	A1 **cao**
			(3)
			13

Question 15 Notes

15. (b)(i)	**Note**	An alternative method of completing (b)(i) is to substitute $y=2x-6$ into C_1 and $y=2x-6$ into C_2 and verify that both equations can be manipulated to give the same $5x^2-36x+55=0$
	Note	Methods of proof involving a gradient of 2 and a point lying on the line PR will rarely score marks in this part.

Question Number	Scheme	Notes	Marks
15.			
(c) **Way 2**	Let M be the midpoint of PR		
	$M(3.6, 1.2)$ $PM = \sqrt{(5-3.6)^2+(4-1.2)^2}\ \left\{= \frac{7\sqrt{5}}{5}\right\}$ $SM = \sqrt{(0-3.6)^2+(3-1.2)^2}\ \left\{= \frac{9\sqrt{5}}{5}\right\}$ $MQ = \sqrt{(3.6-6)^2+(1.2-0)^2}\ \left\{= \frac{6\sqrt{5}}{5}\right\}$	Finds the midpoint of PR and finds lengths PM, SM, MQ. **Note:** S and Q must be of the form $S(0, \alpha)$ and $Q(\beta, 0)$; $\alpha, \beta \neq 0$	M1
	Area$(SPQR)$ $= 2\left(\frac{1}{2}\left(\frac{9\sqrt{5}}{5}\right)\left(\frac{7\sqrt{5}}{5}\right) + \frac{1}{2}\left(\frac{9\sqrt{5}}{5}\right)\left(\frac{7\sqrt{5}}{5}\right)\right)$	**dependent on the previous M mark** Complete correct method to find Area$(SPQR)$	dM1
	$= 2(6.3+4.2) = 21$ (units)^2	21	A1 **cao**
			(3)
(c) **Way 3**	$\cos(S\hat{P}Q) = \dfrac{(\sqrt{26})^2+(\sqrt{17})^2-(\sqrt{45})^2}{2(\sqrt{26})(\sqrt{17})}$ $\Rightarrow S\hat{P}Q = 92.7263...$ or $1.6183...$	Uses SP, PQ and SQ in a correct method of using the cosine rule to find angle $S\hat{P}Q = ...$ **Note:** S and Q must be of the form $S(0, \alpha)$ and $Q(\beta, 0)$; $\alpha, \beta \neq 0$	M1
	Area$(SPQR) = 2\left(\frac{1}{2}\sqrt{26}\sqrt{17}\sin 92.7263...\right)$	**dependent on the previous M mark** Complete correct method to find Area$(SPQR)$	dM1
	$= 21$ (units)^2	21	A1 **cao**
			(3)

	Question 15 Notes	
15. (b)(ii)	**Note**	In the following solution the student obtains a 3TQ in y, but this has come from an incorrect method of undoing the square root to incorrectly obtain the line $(26-(y-3)^2+36)+y^2=17$ $x = \sqrt{26-(y-3)^2}$, $(x-6)^2+y^2=17 \Rightarrow \left(\sqrt{26-(y-3)^2}-6\right)^2+y^2=17$ $\Rightarrow (26-(y-3)^2+36)+y^2=17 \Rightarrow -2y^2+6y+36=0$ Therefore this solution gets M0 A0 dM0 dM0 A0

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