Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level
In Core Mathematics C12 (WMA01) Paper 01

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PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- └ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\,|pq|=|c|,\ \text{leading to } x=\dots$$

$$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\,|pq|=|c| \text{ and } |mn|=|a|\ ,\ \text{leading to } x=\dots$$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to $x=\dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1.(a)	$\frac{6}{\sqrt{5}-\sqrt{2}}=\frac{6}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$	M1
	$=\frac{6\left(\sqrt{5}+\sqrt{2}\right)}{5-2}=2\sqrt{5}+2\sqrt{2}$	A1
		(2)
(b)	$\sqrt{5}x=\sqrt{2}x+18\sqrt{5}\Rightarrow\left(\sqrt{5}-\sqrt{2}\right)x=18\sqrt{5}$	M1
	$\Rightarrow x=\frac{18\sqrt{5}}{\left(\sqrt{5}-\sqrt{2}\right)}=3\sqrt{5}\times(\text{a})$	dM1
	$\Rightarrow x=3\sqrt{5}\times2\left(\sqrt{5}+\sqrt{2}\right)=30+6\sqrt{10}$	A1
		(3) **(5 marks)**

(a)

M1 For multiplying the numerator and denominator by $\sqrt{5}+\sqrt{2}$ or $-\sqrt{5}-\sqrt{2}$

A1 For achieving $2\sqrt{5}+2\sqrt{2}$ following one correct intermediate line after $\frac{6}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

(b) Hence

M1 For attempting to collect and combine the terms in x on one side of the equation. There must be some attempt to combine the terms which may be implied by a subsequent line. Note $\sqrt{3}x=18\sqrt{5}$

dM1 For using part (a) and attempting to find $3\sqrt{5}\times(\text{a})$

A1 $30+6\sqrt{10}$. ISW after a correct answer. Do not allow $30+6\sqrt{5}\sqrt{2}$

Or otherwise (i)

M1 For attempting to collect and combine the terms in x on one side of the equation. This may be implied by a subsequent line, e.g. $\frac{18\sqrt{5}}{\left(\sqrt{5}-\sqrt{2}\right)}$ Note $\sqrt{3}x=18\sqrt{5}$

dM1 For starting again and multiplying the numerator and denominator by $\sqrt{5}+\sqrt{2}$

For this to be awarded the candidate cannot just write down the answer.

Look for, for example, $\frac{18\sqrt{5}}{\left(\sqrt{5}-\sqrt{2}\right)}\times\frac{\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)}=\frac{18\sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)}{5-2}$

A1 $30+6\sqrt{10}$ following at least one correct line between $\frac{18\sqrt{5}}{\left(\sqrt{5}-\sqrt{2}\right)}\times\frac{\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)}$ and the answer

Or otherwise (ii)- squaring approach

M1 Attempts to square both sides (not each term) producing an equation of the form

$...x^2=...x^2+...+...\sqrt{10}x$ FYI the correct equation is $5x^2=2x^2+1620+36\sqrt{10}x$

dM1 Collects terms to form 3TQ and solves.

FYI $x^2-12\sqrt{10}x-540=0\Rightarrow x=6\sqrt{10}\pm30$

A1 $30+6\sqrt{10}$ ONLY

Question Number	Scheme	Marks
2.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\times 2x--\frac{1}{4}x^{-2}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=4x+\frac{1}{4}x^{-2}$ oe	A1
		(3)
(b)	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{1}{2}}=4\times\frac{1}{2}+\frac{1}{4\times\left(\frac{1}{2}\right)^2}=(3)$	M1
	$y+3=3\left(x-\frac{1}{2}\right)\Rightarrow y=3x-\frac{9}{2}$	dM1 A1
		(3) **(6 marks)**

(a)

M1 For reducing a correct power by one on either x term.

The indices must be processed and not left as, for example, x^{2-1}

Look for either x or x^1 or x^{-2} or $\frac{1}{x^2}$

A1 Correct (but may be un simplified) See line 1 scheme for possible expression

Allow here a correct simplified / unsimplified expression with an additional '+ c'.

A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=4x+\frac{1}{4}x^{-2}$ or exact simplified equivalent. Allow $4x\leftrightarrow 4x^1$

ISW after a correct answer. They may attempt to write as a single fraction or write e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=4x+\frac{1}{4\sqrt{x}}$

(b)

M1 For substituting $x=\frac{1}{2}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finding a numerical answer. Unlikley to be scored if there is a '+ c'

dM1 For correct method of finding the equation of the tangent. Eg $y+3=\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\frac{1}{2}}\left(x-\frac{1}{2}\right)$

Condone one error on the sign of the $\frac{1}{2}$ or the -3.

If the form $y=mx+c$ is used they must proceed to $c=\ldots$

A1 $y=3x-\frac{9}{2}$ or $y=3x-4.5$. ISW after the correct answer.

It must be written in this form and not left $m=3,\ c=-\frac{9}{2}$

SC. If a calculator is used to find $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\frac{1}{2}}=3$ **without sight of** $\frac{\mathrm{d}y}{\mathrm{d}x}=4x+\frac{1}{4}x^{-2}$ then you may allow the final two marks in (b) for correct method to find a correct tangent.

Question Number	Scheme	Marks
3.(a)	$4(x-3)<2x-7 \Rightarrow 4x-12<2x-7 \Rightarrow 2x<...$	M1
	$\Rightarrow x<\frac{5}{2}$	A1
		(2)
(b)	$2x^2-5x-63=0 \Rightarrow (2x+9)(x-7)=0$	
	Critical values $x=-\frac{9}{2},7$	B1
	Chooses inside region	M1
	$-\frac{9}{2}\leqslant x\leqslant 7$	A1
		(3)
(c)	An attempt to combine (a) and (b)	M1
	$-\frac{9}{2}\leqslant x<\frac{5}{2}$	A1
		(2) **(7 marks)**

(a)

M1 Attempts to expand the bracket, collect terms and proceed to $2x<...$

Don't be too concerned by the mechanics but look for $4x-12<2x-7$ and $2x<...$

A1 $x<\frac{5}{2}$ or $x<2.5$

(b)

B1 Critical values of $x=-\frac{9}{2},7$. May be implied from their solution set

M1 Chooses the inside region for their critical values. Condone $-\frac{9}{2}<x<7$ for this mark.

Ignore any diagrams or graphs. Score the text only and it must be seen in part (b).

A1 $-\frac{9}{2}\leqslant x\leqslant 7$ which must be seen in (b)

Allow to be written separately as $x\geqslant-\frac{9}{2}, x\leqslant 7$ (with or without a comma), as $x\geqslant-\frac{9}{2}$ and $x\leqslant 7$ with the '**and**' but **not** with or. Note that this may be written down (from a calculator) for all three marks. There are other correct versions using set notation.

(c)

M1 For a correct attempt to combine their (a) and (b).

For example if they write (a) $x<\frac{5}{2}$ (b) $x\leqslant-\frac{9}{2}, x\leqslant 7$ then (c) $x\leqslant-\frac{9}{2}$ scores M1 A0

if they write (a) $x<\frac{5}{2}$ (b) $x\leqslant-\frac{9}{2}, x\geqslant 7$ then (c) $x\leqslant-\frac{9}{2}$ scores M1 A0

Condone slips on the inequalities. Eg $-\frac{9}{2}\leqslant x\leqslant\frac{5}{2}$

A1 $-\frac{9}{2}\leqslant x<\frac{5}{2}$ Allow written as two separate inequalities. See part (b) for how to apply.

Question Number	Scheme	Marks
4.	$\dfrac{4\sqrt{x}-3}{2x^{2}}=\dfrac{4\sqrt{x}}{2x^{2}}-\dfrac{3}{2x^{2}}=2x^{-\frac{3}{2}}-\dfrac{3}{2}x^{-2}$	M1 A1
	$\displaystyle\int\frac{4\sqrt{x}-3}{2x^{2}}\,\mathrm{d}x=-4x^{-\frac{1}{2}}+\frac{3}{2}x^{-1}+c$	dM1 A1 A1
		(5 marks)

M1 Attempts to write as a sum of two terms. Award if any index is correct and processed

Look for $Ax^{m}+Bx^{n}$ where $m=-\dfrac{3}{2}$ or $n=-2$.

A1 $2x^{-\frac{3}{2}}-\dfrac{3}{2}x^{-2}$ oe such as $\dfrac{1}{2}\left(4x^{-\frac{3}{2}}-3x^{-2}\right)$ on one line with the indices processed

dM1 Raises the power by one. One index must have been correct and processed

Look for one of the following $\rightarrow ...x^{-\frac{1}{2}}+...x^{-1}$

A1 For one correct term either $-4x^{-\frac{1}{2}}$ or $+\dfrac{3}{2}x^{-1}$ which must be simplified.

If candidate attempts to integrate $\dfrac{1}{2}\left(4x^{-\frac{3}{2}}-3x^{-2}\right)$ you may award for one of $\dfrac{1}{2}\left(-8x^{-\frac{1}{2}}+3x^{-1}\right)$

A1 Fully correct including an arbitrary constant Eg. $-4x^{-\frac{1}{2}}+\dfrac{3}{2}x^{-1}+c$.

Accept exact equivalent answers such as $-\dfrac{4}{\sqrt{x}}+\dfrac{3}{2x}+c$ or $-\dfrac{4}{\sqrt{x}}+\dfrac{1.5}{x}+c$

Also accept the factorised form $\dfrac{-8x^{-\frac{1}{2}}+3x^{-1}}{2}$ or equivalent

(i)

Question Number	Scheme	Marks
5 (i)	$4^y = 10^{3000} \Rightarrow y\log 4 = 3000\log 10$	M1
	$y = \frac{3000}{\log 4} = 4983$	A1
		(2)
(ii)	$\log_4 \frac{(x+4)}{(2-x)^2} = \frac{1}{2}$ $\log_4 (x+4) = \log_4 2(2-x)^2$	M1 dM1
	$\frac{x+4}{(2-x)^2} = 2$ oe	A1
	Attempts to solve the **correct** equation $2x^2 - 9x + 4 = 0 \Rightarrow x = ...$	dM1
	$x = \frac{1}{2}$ only	A1
		(5)
		(7 marks)

M1 Takes logs of both sides and uses power law. Score for $y\log 4 = 3000\log 10$ or $y\log 4 = 3000$

Note that stating that $y = \log_4 10^{3000}$ is not worthy of any marks unless the 3000 is subsequently brought down as a coefficient. This is because 10^{3000} is too large to be found via a calculator.

A1 AWRT 4983

If there is no working then award AWRT 4983 both marks

(ii)

M1 For one correct log law. Award for either $2\log_4(2-x) \rightarrow \log_4(2-x)^2$ or $\frac{1}{2} = \log_4 2$

Note that these are the only two you should accept for this mark

dM1 Correctly combines two of the terms using log laws. See scheme

Award for $\log_4(x+4) - \log_4(2-x)^2 = \log_4 \frac{(x+4)}{(2-x)^2}$ $\frac{1}{2} + \log_4(2-x)^2 = \log_4 2(2-x)^2$ OR

$\log_4(x+4) - \frac{1}{2} = \log_4 \frac{(x+4)}{2}$. Note that M1 dM1 can be scored at the same time.

A1 Correct equation in x (not involving lns).

dM1 Attempts at solving the correct equation leading to two solutions (or the correct solution). Dependent upon the first M1 only.

A1 $x = \frac{1}{2}$ only

SC: Note that a candidate who writes

$\frac{\log_4(x+4)}{\log_4(2-x)^2} = \frac{1}{2}$ AND goes on to get a correct equation and solution should be awarded 10110

Question Number	Scheme	Marks
6 (a) (i)	$P(0,4)$	B1
(ii)	$Q(6,13)$	B1
		(2)
(b)	The gradient of line $l_1 = \frac{3}{2}$ oe	B1
	Perpendicular gradient $"-\frac{2}{3}"$	M1
	$y-"13"=-\frac{2}{3}(x-6)$	M1
	$2x+3y-51=0$	A1
		(4)
(c)	Method of finding R $y=0\Rightarrow 2x-51=0\Rightarrow x=...\left(\frac{51}{2}\right)$	M1
	Full method of finding area = trapezium + triangle (oe)	
	$=\frac{6}{2}("4"+"13")+\frac{1}{2}\times"13"\times\left("\frac{51}{2}"-6\right)$	M1
	$=177.75$	A1
		(3)
		(9 marks)

(a) (i)

B1 Allow the coordinates to be written separately. Do not allow $P=4$

Condone $x=0\Rightarrow 2y=8\Rightarrow y=4$ which implies the coordinate pair

(ii)

B1 Allow the coordinates to be written separately.

(b)

B1 The gradient of line $l_1 = \frac{3}{2}$ oe. This may be implied by subsequent work.

M1 For knowing that the gradient of l_2 is the negative reciprocal of l_1

This requires evidence for the l_1

M1 For the equation of a line through their $Q(6,13)$ with a changed $\frac{3}{2}$

A1 $2x+3y-51=0$ or any multiple thereof

(c)

M1 For the method of finding the x coordinate of R. $y=0\Rightarrow 2x-51=0\Rightarrow x=...\left(\frac{51}{2}\right)$

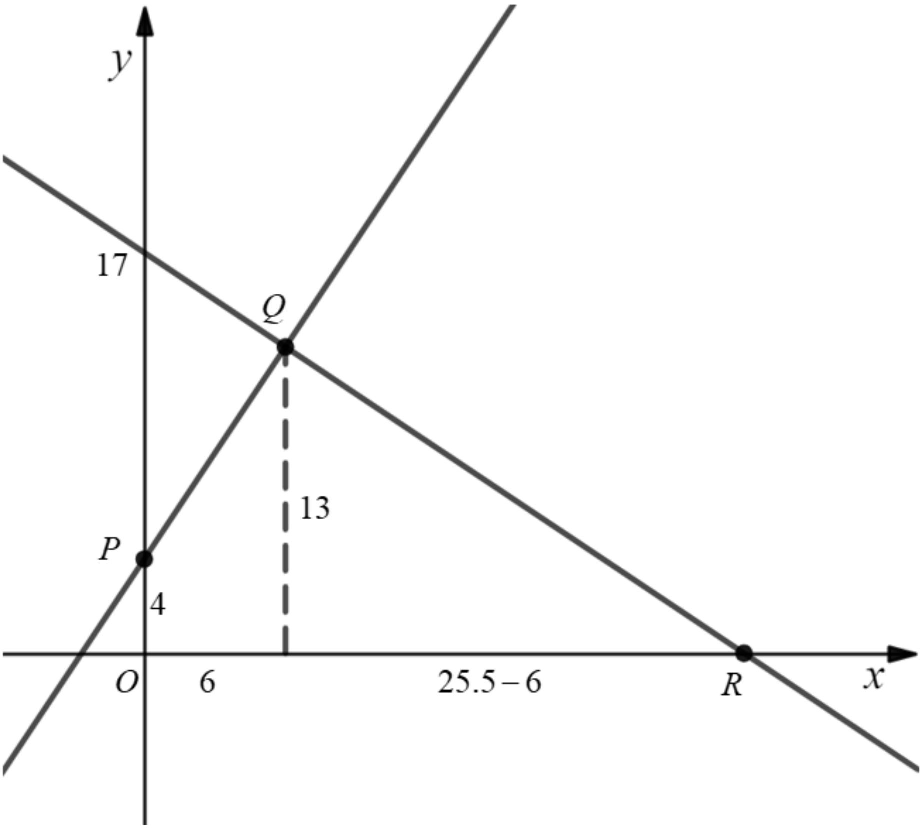
M1 For a full method of finding the area of shape *OPQR*. There are many versions. See diagram

Allow trapezium + triangle. (See scheme)

Allow rectangle + triangle + triangle $"4"\times 6+\frac{6}{2}("13"-"4")+\frac{1}{2}\times"13"\times\left("\frac{51}{2}"-6\right)$

A1 177.75 oe such as $\frac{711}{4}$. Remember to isw after a correct answer.

Handy Diagram



There are lots of different methods to find the total area, Look at each one carefully.

Allow triangle −triangle $"\frac{1}{2}\times 17\times 25.5-\frac{1}{2}(17-4)\times 6'$

A rather unusual one is triangle *OPQ* + triangle *OQR* = $\frac{1}{2}\times 4\times 6+\frac{1}{2}\times 25.5\times 13$

Allow Integration attempts here.

So $\int_{0}^{"6"}\left(\frac{3}{2}x+4\right)\mathrm{d}x+\int_{"6"}^{"\frac{51}{2}"}\left("17-\frac{2}{3}x"\right)\mathrm{d}x=$ is fine for the method

Allow answer to be written down as the method has been shown.

Any unusual methods that you feel deserve credit and you don’t know how to mark please send to review.

Question Number	Scheme	Marks
7. (a)	Attempts $12^2+7.6^2-2\times12\times7.6\cos80$ $BD=13.04$ m CAO	M1 A1 **(2)**
(b)	Finds area $ABD=\frac{1}{2}\times12\times7.6\sin80=(44.9)$ Attempts angle ABD using sin/cosine rules Eg $\frac{\sin ABD}{7.6}=\frac{\sin80}{"13.04"}$ or $\cos ABD=\frac{12^2+"13.04"^2-7.6^2}{2\times12\times"13.04"}$ Angle ABD = awrt 35.0° Attempts to find the area of triangle $BCD=\frac{1}{2}\times13.04\times10\times\sin(73-35.03)$ Adds together the area of triangle ABD and triangle BCD Total Area = awrt 85 m^2	M1 M1 A1 dM1 dddM1 A1 **(6)** **(8 marks)**

There is no SC for candidates who bisect angle *ABC* or who incorrectly assume *ABCD* is a cyclic quadrilateral

(a)

M1 Attempts to find BD or BD^2 using the cosine rule.

Allow an attempt at "$12^2+7.6^2-2\times12\times7.6\cos80$ " condoning slips

A1 $BD=13.04$ m (Note that this is NOT awrt)

(b)

M1 Attempts to find the area of $ABD=\frac{1}{2}\times12\times7.6\sin80$ (This is implied by awrt 44.9)

M1 A full attempt to find angle *ABD*. The most common approaches are using the sine or cosine rules as shown in scheme. Follow through on the value of their *BD*. A valid alternative is to find angle *BDA* and use "angles in a triangle = 180°."

A1 Angle ABD = awrt 35.0° Allow 35 degrees following correct method

dM1 An attempt to find the area of triangle *BCD* using $\frac{1}{2}\times"13.04"\times10\times\sin(73-"35.03")=(40.1)$.

This is dependent upon the previous M only.

dddM1 Dependent upon all three method marks in (b). It is for adding together the area of triangle *ABD* and triangle *BCD*

A1 awrt 85

……

Watch for alternative methods in (a), and (b). For instance (a) could be done by Pythagoras' Theorem
Part (b) could be found by adding the areas Triangle + Trapezium + Triangle. See diagram 2

M1: Attempts any relevant area

$$\frac{1}{2}\times7.6\sin80°\times7.6\cos80° \text{ or } \frac{(12-7.6\cos80°-10\cos73°)}{2}\times\{7.6\sin80°+10\sin73°\}$$

$$\frac{1}{2}\times10\sin73°\times10\cos73°$$

M1 Attempts any two relevant areas (See above)

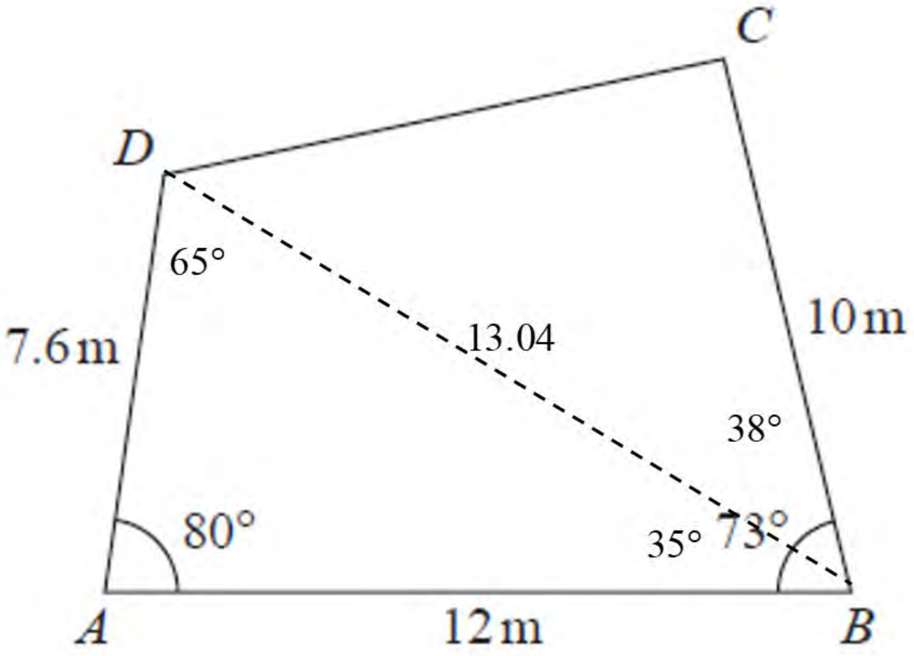
A1 Correct value or expression for two area's

dM1 Attempts the three relevant areas

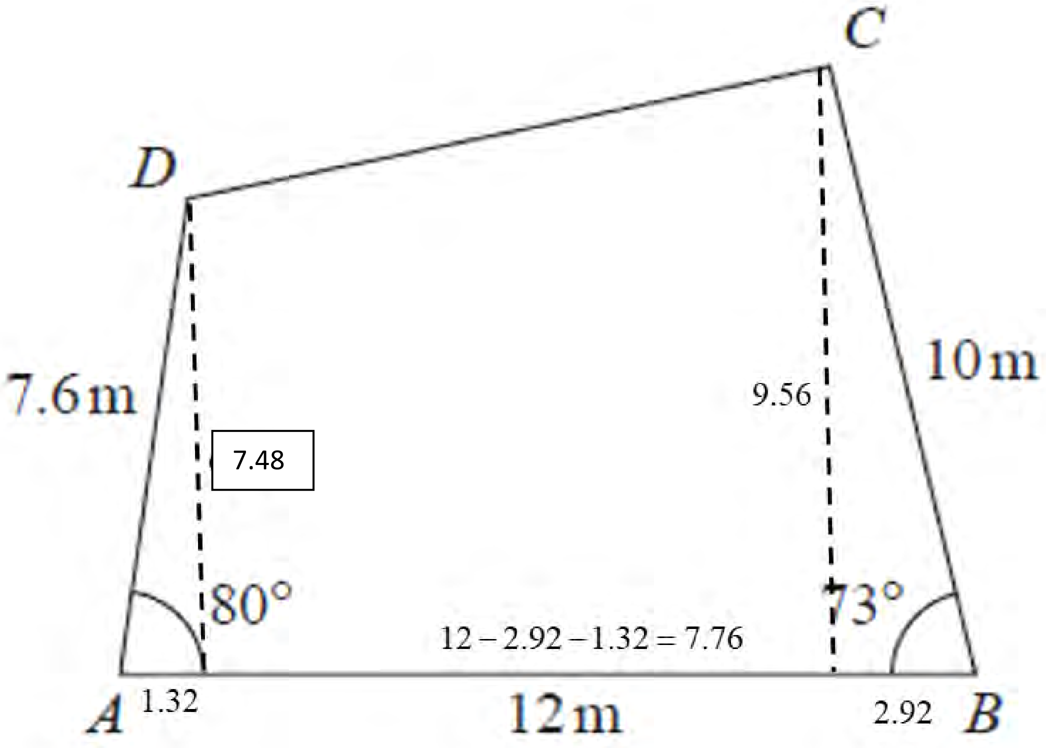
dddM1 For adding together the three relevant areas

A1 awrt 85

Useful diagram One:



Useful diagram Two:



Question Number	Scheme	Marks
8. (a)	Substitutes $y=2x+k$ into $2x^2+y^2+4x-2y=3$ to form an equation in *x*.	M1
	$2x^2+\underline{\underline{4x^2+4kx+k^2}}+4x-4x-2k=3$	A1
	$6x^2+4kx+k^2-2k-3=0$ *	A1*
		(3)
(b)	Attempts to use $b^2-4ac=0$ oe with $a=6, b=4k, c=k^2-2k-3$	M1
	$k^2-6k-9=0$ oe	A1
	$(k-3)^2=18 \Rightarrow k=3\pm3\sqrt{2}$ CSO	dM1 A1
		(4) **(7 marks)**

(a)

M1 Attempts to fully substitute $y=2x+k$ into $2x^2+y^2+4x-2y(=3)$ to form an equation in only x

A1 For $(2x+k)^2=4x^2+4kx+k^2$ which may be unsimplified. The M must have been scored

A1* Expands to show the $-4x-2k$ before proceeding without any errors to given answer.

(b)

M1 Attempts to use $b^2-4ac...0$ oe with $a=6, b=4k, c=k^2-2k-3$

A1 Correct 3TQ expression E.g. $k^2-6k-9...0$ oe such as $8k^2-48k-72...0$

dM1 Candidates now must be solving $b^2-4ac=0$ and making a correct attempt to find at least one value for *k*. Allow decimal answers here

A1 $(k=)3\pm3\sqrt{2}$ Allow exact equivalents such as $k=3\pm\sqrt{18}$. They do not have to state *k* =... but don't accept *x* = …. Award this mark where they follow $3\pm3\sqrt{2}$ with decimal equiavlents.
We will not be applying the ISW principle for students who find $k=3\pm3\sqrt{2}$ and then say as $b^2-4ac>0$, $k>3+3\sqrt{2}, k<3-3\sqrt{2}$. This is marked the same way as those who start with $b^2-4ac>0$. Score M1 A1 M0 A0

It is possible to solve this via differentiation

………

Way One: Differentiates the equation from (a)

E.g. Differentiates $6x^2+4kx+k^2-2k-3=0 \Rightarrow 12x+4k=0 \Rightarrow x=-\frac{k}{3}$ and then substitutes this into $6x^2+4kx+k^2-2k-3=0 \Rightarrow k^2-6k-9=0$. Score M1 for the complete attempt, the A1 for the correct result.

……

Way Two: Via implicit differentiation

Differentiates $2x^2+y^2+4x-2y=3 \Rightarrow 4x+2y\frac{\mathrm{d}y}{\mathrm{d}x}+4-2\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and sets $\frac{\mathrm{d}y}{\mathrm{d}x}=2$

Condone slips on coefficients. Candidates then need to solve their $x+y=0$ simultaneously with $2x^2+y^2+4x-2y=3$ to find (x,y)

M1: For the complete attempt

A1: For $x=-1\pm\sqrt{2}, y=1\mp\sqrt{2}$,

dM1: For using $y=2x+k$ with found (x,y) to find at least one value for k.

A1: As main scheme

……

Question Number	Scheme	Marks
9. (a)	Attempts either $S_{10}=\frac{10}{2}\{2\times50+9\times12\}$ or $S_9=\frac{9}{2}\{2\times50+8\times12\}$	M1
	Calculates $S_{10}=1040$ or $S_9=882$	A1
	Attempts both $S_{10}=\frac{10}{2}\{2\times50+9\times12\}$ and $S_9=\frac{9}{2}\{2\times50+8\times12\}$	dM1

Question Number	Scheme	Marks
	Calculates $1000 - S_9 = 118$ *	A1*
		(4)
(b)	Sets $S_8 = \dfrac{d\left(1.02^8 - 1\right)}{1.02 - 1} = 1000$	B1
	$d = \dfrac{1000 \times 0.02}{\left(1.02^8 - 1\right)} = 116.51 \text{ km}$	M1 A1
		(3)
		(7 marks)

(a) Method One: Using S_9 and S_{10}

M1 Attempts either $S_{10} = \frac{10}{2}\{2 \times 50 + 9 \times 12\}$ or $S_9 = \frac{9}{2}\{2 \times 50 + 8 \times 12\}$ oe

A1 Calculates $S_{10} = 1040$ or $S_9 = 882$

dM1 Attempts both $S_{10} = \frac{10}{2}\{2 \times 50 + 9 \times 12\}$ and $S_9 = \frac{9}{2}\{2 \times 50 + 8 \times 12\}$

A1* Calculates $1000 - S_9 = 118$ *

See over page for alternative methods

(b)

B1 Sets $S_8 = \dfrac{d\left(1.02^8 - 1\right)}{1.02 - 1} = 1000$. Condone $\dfrac{d\left(1.02^8 - 1\right)}{1.02 - 1} ...1000$ where … could be an inequality.

M1 Attempts to solve an equation of the form $\dfrac{d\left(1.02^8 - 1\right)}{1.02 - 1} = 1000$ condoning errors on the 1.02 or 1000 but not the '8'. Do not be concerned by the mechanics of this but it must lead to a value for *d*.

A1 awrt 116.51km. Answers such as $d \geqslant 116.51$ will likely score B1 M1 A0

Alternative methods seen in part (a)

………

Method Two: Using S_9 or S_{10} with u_{10}

M1 Attempts either $S_{10} = \frac{10}{2}\{2 \times 50 + 9 \times 12\}$ or $S_9 = \frac{9}{2}\{2 \times 50 + 8 \times 12\}$ oe

A1 Calculates $S_{10} = 1040$ or $S_9 = 882$

dM1 Attempts $S_9 = \frac{9}{2}\{2 \times 50 + 8 \times 12\}$ and $u_{10} = 50 + 9 \times 12$

or $S_{10}=\frac{10}{2}\{2\times 50+9\times 12\}$ and $u_{10}=50+9\times 12$

A1* Shows calculations that prove student *A* finishes the ride in 118km

Using $S_9=882$ and $u_{10}=158$ proves that will finish on day 10 riding 118 km

Using $S_{10}=1040$ and $u_{10}=158$ proves that will finish on day 10 riding (158-40) = 118 km

……

Method Two: Using S_9 and n when $S=1000$

M1: Attempts either $S_9=\frac{9}{2}\{2\times 50+8\times 12\}$ or attempts to solve $1000=\frac{n}{2}\left(100+12(n-1)\right)\Rightarrow n=...$

via a correct attempt at solving a 3TQ in *x* (usual rules)

A1: Either $S_9=882$ or $1000=\frac{n}{2}\left(100+12(n-1)\right)\Rightarrow n=9.75$ via $6n^2+44n-1000=0\Rightarrow$

dM1 For both of the above

A1: For a correct explanation that proves student *A* finishes the ride in 118km on 10th day

……

Method Three: Students may attempt the question term by listing terms and totals

M1: Attempt at finding S_9 (see table). Expect to see an attempt where they are adding on the daily totals

A1: 882

dM1: Can be scored for the 158

A1: For an explanation

Day	Day Totals	Cumulative
1	50	50
2	62	112
3	74	186
4	86	272
5	98	370
	110	480
7	122	602
8	134	736
9	146	882
10	158	1040

Question Number	Scheme	Marks
10. (a)	Attempts $f(-2) = -2\times(-2)^3 + 7\times(-2)^2 + 10\times(-2) - 24$	M1
	Achieves $f(-2) = 0$ and states $(x+2)$ is a factor, hence divisible	A1
		(2)
(b)	$-2x^3 + 7x^2 + 10x - 24 = (x+2)(-2x^2 + 11x - 12)$	M1
	$= (x+2)(-2x+3)(x-4)$	dM1 A1
		(3)
(c) (i)	$x^3 - 2x^2 - 8x = x(x^2 - 2x - 8) = x(x-4)(x+2)$	M1 A1
(ii)	Hence $\dfrac{-2x^3 + 7x^2 + 10x - 24}{x^3 - 2x^2 - 8x} = \dfrac{-2x+3}{x} = -2 + \dfrac{3}{x}$	M1 A1
		(4) **(9 marks)**

(a) Division in part (a) is M0

M1 Attempts $f(-2) = -2\times(-2)^3 + 7\times(-2)^2 + 10\times(-2) - 24$ condoning sign slips etc.

You must see an attempt to substitute so expect to see embedded -2's or some correctly calculated values. Look for two of "16" , "28" or "-20".

Merely writing $f(-2) = 0$ without sight of embedded terms or correctly calculated values is M0

A1 Achieves $f(-2) = 0$ following embedded -2's or correctly calculated values and states $(x+2)$ is a factor, hence divisible

All three elements are expected

- the correct calculation showing $f(-2) = 0$. Condone $f(x) = 0$ as long as you see $x = -2$ stated or embedded
- a statement that $(x+2)$ is a factor or that there is no remainder
- a minimal conclusion "hence divisible". Allow hence true, QED, ✓

(b)

M1 Attempts to divide $\mathrm{f}(x)$ by $(x+2)$ achieving a quadratic factor. This may be awarded from part (a) if division was attempted.

If division is used look for the first two terms $\begin{array}{r} -2x^2+11x+.... \\ x+2\overline{\big)-2x^3+7x^2+10x-24} \\ -2x^3-4x^2 \qquad\qquad \end{array}$

If inspection is used look for first and last terms

$$-2x^3+7x^2+10x-24=(x+2)\left(-2x^2\pm..x\pm12\right)$$

dM1 Attempts to factorise the quadratic factor. Usual rules apply

A1 $(x+2)(-2x+3)(x-4)$ or equivalent versions $(x+2)(2x-3)(4-x)$ and $-(x+2)(2x-3)(x-4)$

You should ignore any subsequent working such as finding roots. Ignore = 0 on the rhs of f (x)

……

Additional notes to part (b):

Part (b) is Hence but allow all marks for candidates who have divided.

Some candidates who merely use their calculators and write down the roots $-2,\frac{3}{2},4$ are unlikely to factorise correctly with most getting $(x+2)\left(x-\frac{3}{2}\right)(x-4)$ or $(x+2)(2x-3)(x-4)$

Candidates who do get the **correct factorisation** can score SC 100

……

Mark parts (c) together as one

(c)(i)

M1 Attempts to factorise x^3-2x^2-8x by taking out a factor of x followed by attempt at quadratic (usual rules). May be awarded in (c)(ii)

A1 $x^3-2x^2-8x=x(x-4)(x+2)$ May be awarded in (c)(ii)

Ignore = 0 on the rhs of expression

Both marks should be awarded for candidates who write the roots following correct factorisation.

(c)(ii)

M1 Cancels out factors and attempts to divide $\frac{Ax+B}{x}\to A+\frac{B}{x}$

An alternative is to attempt $\frac{-2x^3+7x^2+10x-24}{x^3-2x^2-8x}=\frac{-2x^3+4x^2+16x+3x^2-6x-24}{x^3-2x^2-8x}$

$$=-2+\frac{3\cancel{\left(x^2-2x-8\right)}}{x\cancel{\left(x^2-2x-8\right)}}=A+\frac{B}{x}$$

A1 $-2+\frac{3}{x}$

Question Number	Scheme	Marks
11. (a)	Attempts $(x\pm3)^2+\left(y\pm\frac{9}{2}\right)^2 ....=0$	M1
(i)	Centre $\left(3,-\frac{9}{2}\right)$	A1
(ii)	Radius = $\frac{3\sqrt{5}}{2}$ oe	B1
		(3)
(b)	Attempts $\left(\frac{3\sqrt{5}}{2}\right)^2=h^2+\left(\frac{5}{2}\right)^2 \Rightarrow h=\sqrt{5}$	M1 A1
	$x=3+\sqrt{5}, x=3-\sqrt{5}$	dM1 A1
		(4) **(7 marks)**

(a)

M1 Attempts to complete the square. E.g. $(x\pm3)^2+\left(y\pm\frac{9}{2}\right)^2 ....=0$.

Imply if centre is given as $\left(\pm3,\pm\frac{9}{2}\right)$

A1 Centre $\left(3,-\frac{9}{2}\right)$

B1 Radius = $\frac{3\sqrt{5}}{2}$ oe such as $\frac{\sqrt{45}}{2}, \sqrt{11.25}$ ISW after a correct answer

(b)

M1 Attempt to find the distance of the chord from the mid-point using Pythagoras. $(r)^2=h^2+\left(\frac{5}{2}\right)^2 \Rightarrow h=...$

A1 $h=\sqrt{5}$

dM1 Finds the equation of at least one of the chords using a correct method.
Allow $x="3"\pm"h"$. It must be $x=...$ and not $y=...$

A1 For $x=3+\sqrt{5}, x=3-\sqrt{5}$

There are lots of incorrect methods that involve substituting $x=0\Rightarrow y=...$ or alternatively substituting $y=0\Rightarrow x=...$ These attempts score M0 A0 dM0 A0

……

Alt in (b)

M1 Substitutes $y = "-\frac{9}{2}" \pm \frac{5}{2}$ into $x^2 + y^2 - 6x + 9y + 18 = 0$ to produce an equation in x

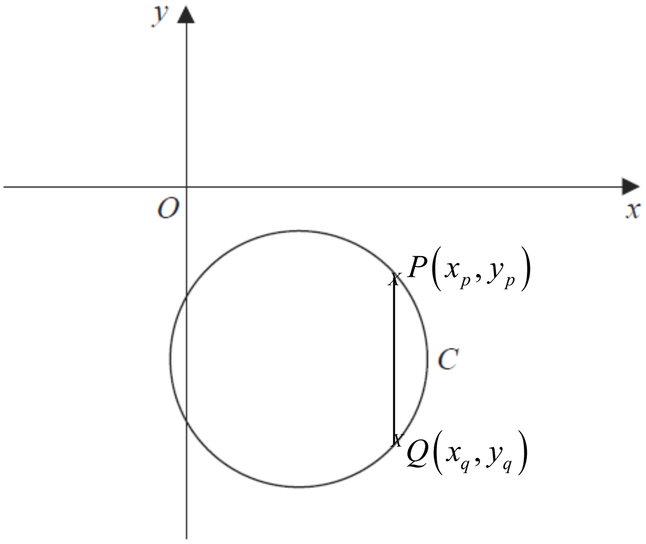
A1 Correct equation for x $x^2 - 6x + 4 = 0$

dM1 Solves their $x^2 - 6x + 4 = 0$ to find one value of x. . It must be $x = ...$ and not $y = ...$

A1 For $x = 3 + \sqrt{5}, x = 3 - \sqrt{5}$

……

A version of the Alt above that does not score the M1 until a great deal of work has been done



Both P and Q lie on the circle so

$x_p^2 + y_p^2 - 6x_p + 9y_p + 18 = 0$ and $x_q^2 + y_q^2 - 6x_q + 9y_q + 18 = 0$

Subtracting and using $x_p = x_q \Rightarrow y_p^2 - y_q^2 + 9y_p - 9y_q = 0$

Attempting to use the above with $y_p - y_q = 5$ to form an equation in just y_p or y_q

and A1 for $25 - 10y_q + y_q^2 - y_q^2 + 9(y_q - 5) - 9y_q = 0 \Rightarrow y_q = -7$ and therefore $y_p = -2$

Only now can you apply the alt scheme.

M1 Substitutes $y = "-7"$ or $y = "-2"$ into $x^2 + y^2 - 6x + 9y + 18 = 0$ to produce an equation in x

……

Question Number	Scheme	Marks
12. (a)	2^7 OR 128 seen as the constant term	B1
	$\left(2-\frac{x}{8}\right)^7 = ...+7\times2^6\left(-\frac{x}{8}\right)^1+21\times2^5\left(-\frac{x}{8}\right)^2+...$	M1A1
	$=128-56x+\frac{21}{2}x^2+...$	A1
		(4)
	$\left(2-\frac{x}{8}\right)^{10}(a+bx)=\left(128-56x+\frac{21}{2}x^2\right)(a+bx)$	
(b)	$128a=16\Rightarrow a=\frac{1}{8}$ oe	M1A1
		(2)
(c)	$128b-56a=249\Rightarrow b=2$	M1A1
		(2) **(8 marks)**

(a)

B1 2^7 OR 128 seen as the constant term. Even award for ${}^7C_0\times2^7$

M1 For a correct attempt at term 2 **or** term 3 of the binomial expansion for $(a+b)^n$ with a =2, $b=\pm\frac{x}{8}$ and n=7 Allow coefficients to be written ${}^7C_1$ Condone missing brackets.

A1 A completely correct unsimplified 2nd **and** 3rd term but with ${}^7C_1$, ${}^7C_2$ processed.

Accept $...+7\times2^6\left(-\frac{x}{8}\right)^1+\frac{7\times6}{2}\times2^5\left(-\frac{x}{8}\right)^2+$

A1 Fully correct and simplified $128-56x+\frac{21}{2}x^2$. Accept written separately $128, -56x, \frac{21}{2}x^2$

ISW after a correct answer and ignore any additional terms.

(b) **Mark (b) and (c) together**

M1 For setting either $2^7\times a=16$ or setting their $'128'\times a=16$ leading to a value for a

Their 128 cannot be 1

A1 $a=\frac{1}{8}$. Accept equivalents such as 0.125

(c)

M1 Sets their $'128'$, $\times b\pm$ their $'56'a=249$ with their value of a to find b

or sets $"128"\times bx\pm$ their $'56'ax=249x$ with their value of a to find b

A1 $b=2$

How to mark when candidates attempt $\mathbf{2}^7\left(1\pm\frac{x}{16}\right)^7$

Question Number	Scheme	Marks
12. (a)	2^7 OR 128 seen as the constant term in an expression. E.g. $128+...x+...x^2$	B1
	$\left(1-\frac{x}{16}\right)^7 = ...+7\times\left(-\frac{x}{16}\right)^1+21\times\left(-\frac{x}{16}\right)^2+...$	M1A1
	$=128-56x+\frac{21}{2}x^2+...$	A1
		(4)

B1 This B mark is not awarded until the constant has been isolated with the bracket multiplied out.

So you may see $\mathbf{2}^7+\left(1\pm\frac{x}{16}\right)^7=129+...x+....$ is B0

M1 For a power series expansion on $\left(1\pm\frac{x}{16}\right)^7$ but NOT $\left(1\pm\frac{x}{8}\right)^7$

Look for a correct attempt at term 2 **or** term 3 of the binomial expansion.

So for example award for $\pm\frac{7x}{16}$ or ${}^7C_2\times\left(\frac{x}{16}\right)^2$

A1 A completely correct unsimplified 2nd **and** 3rd term but with ${}^7C_1$, ${}^7C_2$ processed

Look for ... $\left(....+7\times\left(-\frac{x}{16}\right)+\frac{7\times 6\left(-\frac{x}{16}\right)^2}{2!}+..\right)$ FYI $1-\frac{7}{16}x+\frac{21}{256}x^2$

A1 Fully correct and simplified $128-56x+\frac{21}{2}x^2$. Accept written separately $128,-56x,\frac{21}{2}x^2$

ISW after a correct answer and ignore any additional terms.

Question Number	Scheme	Marks
13. (i)	$\sin\left(2\theta+\frac{3\pi}{8}\right)=\frac{1}{2}\Rightarrow\left(2\theta+\frac{3\pi}{8}\right)=\frac{5\pi}{6}$ or $\frac{13\pi}{6}$	M1
	$\Rightarrow 2\theta+\frac{3\pi}{8}=\frac{5\pi}{6}\Rightarrow\theta=...$	M1
	$\Rightarrow\theta=\frac{11\pi}{48},\frac{43\pi}{48}$	A1, A1
		(4)
(ii)	$5\sin x=4\tan x\Rightarrow 5\sin x=4\times\frac{\sin x}{\cos x}$	
	$\Rightarrow\cos x=\frac{4}{5}$ (or $\sin x=0$)	M1 A1
	$x=36.9°$ and $323.1°$	dM1A1
	$x=180°$ and $360°$	B1
		(5) **(9 marks)**

(i)

M1 For $\arcsin\frac{1}{2}$ Implied by $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$ or 30, 150 or 390

M1 Correct order of operations to find one value of θ.

Do not allow degrees and radians to be mixed here.

They can work entirely in degrees with $\frac{3\pi}{8}=67.5°$

Allow for $\left(2\theta+\frac{3\pi}{8}\right)=\frac{\pi}{6}\Rightarrow-\frac{5\pi}{48}$ Expect to see the correct order of operations

Sight of the decimal answers awrt 0.72, awrt 2.81 may imply this but this should be following a correct equation.

A1 One correct value either $\frac{11\pi}{48}$ or $\frac{43\pi}{48}$. Exact solutions are required

A1 Two correct values and no others in the range $\frac{11\pi}{48}$ and $\frac{43\pi}{48}$. Exact solutions are required

(ii)

M1 Uses $\tan x=\frac{\sin x}{\cos x}$ or $\frac{\sin x}{\tan x}=\cos x$ to form an equation $\cos x=k$

A1 $\cos x=\frac{4}{5}$

dM1 Takes arccos to find at least one value for x. For degrees accept awrt $x=37°$ or $323°$

It may be implied by a correct radian value to 1dp, eg awrt 0.6 radians

A1 $x=$ awrt $36.9°$ and awrt $323.1°$ given as the only answers for $\cos x=\frac{4}{5},\ 0<x\leqslant 360°$

B1 $x=180°$ and $360°$ and no others for $0<x\leqslant 360°$. Note that $x=0,180°$ and $360°$ is B1

Question Number	Scheme	Marks
14. (a)	$Q=(4,0)$	B1 **(1)**
(b)	$\mathrm{f}(x)=(x+2)^2(4-x)=\left(x^2+4x+4\right)(4-x)$	M1
	$=16+12x-x^3$	A1
	Area $=\left[16x+6x^2-\frac{1}{4}x^4\right]_{(-2)}^{(4)}$	dM1 A1 ft
	$=[64+96-64]-[-32+24-4]=108$ CSO	M1 A1 **(6)**
(c)	For 486 of ft on 4.5 × their 108 States 4.5 times bigger	B1 ft B1 **(2)** **(9 marks)**

(a)

B1 $Q=(4,0)$ Accept coordinates written separately or without brackets

Do not accept $(4,0)$ and $(-2,0)$ unless it has been made clear that $Q=(4,0)$

(b)

M1 Attempts to multiply out. This may be seen in part (a)

Look for $\mathrm{f}(x)=(x+2)^2(4-x)=\left(x^2+...+4\right)(4-x)=....$

Or $\mathrm{f}(x)=(x+2)(x+2)(4-x)=(x+2)\left(8.....-x^2\right)=....$

Or if attempted in head $\mathrm{f}(x)=-x^3+........16$

A1 $16+12x-x^3$

dM1 Raises the power by one in at least **two** (different) terms. It is dependent upon the first M1

A1ft Follow through on their cubic.

Do not award this A mark if they adapt their cubic. E.g. $16+12x-x^3 \leftrightarrow x^3-12x-16$

M1 Uses the limits −2 and 4 on a changed function.

Also allow both final marks for $=\left[16x+6x^2-\frac{1}{4}x^4\right]_{-2}^{4}=108$

A1 CSO For reaching 108 and showing all relevant steps (See line above)

(c)

B1ft For 486 or follow through on their answer to $(b)\times 4.5$. **May be awarded without method**

B1 States 4.5 times bigger

……

Answers with limited or no working.

Case 1: No working

Writes $\int_{-2}^{4}(x+2)^2(4-x)\,\mathrm{d}x=108$ scores no marks

Case 2: Some working

Writes $\int_{-2}^{4}(x+2)^2(4-x)\,\mathrm{d}x=\int_{-2}^{4}16+12x-x^2\mathrm{d}x=108$ scores M1 A1 dM0 A0 M0 A0

Case 3: Shows all calculus, appreciates the two limits and proceeds to the correct answer which was not given

$\int_{-2}^{4}(x+2)^2(4-x)\,\mathrm{d}x=\int_{-2}^{4}16+12x-x^3\mathrm{d}x=\left[16x+6x^2-\frac{1}{4}x^4\right]_{-2}^{4}=108$ scores M1 A1 M1 A1 M1 A1

……

Question	Scheme	Marks

15. (a)	A correct formula for P or A which may be unsimplified Either $P=6r+4r\theta$ **or** $A=2r^2+4r^2\theta$	B1
	Attempts to use area = 30. Look for $30=Mr^2+Nr^2\theta$	M1
	Obtains $4r\theta=\frac{30-2r^2}{r}$, $\theta=\frac{30-2r^2}{4r^2}$ or $r\theta=\frac{30-2r^2}{4r}$ oe	A1
	Substitutes their $4r\theta$, θ, or $r\theta$ into $P=Rr+Sr\theta\rightarrow P=\mathrm{f}(r)$	dM1
	$P=6r+\frac{30-2r^2}{r}=4r+\frac{30}{r}$ *	A1*
		(5)
(b)	Differentiates with $r^{-1}\rightarrow r^{-2}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}r}=4-\frac{30}{r^2}$	A1
	Solves $\frac{\mathrm{d}P}{\mathrm{d}r}=0\rightarrow\left(r=\sqrt{\frac{15}{2}}\right)$ and substitutes into P	M1
	$P=4\sqrt{\frac{15}{2}}+\frac{30\sqrt{2}}{\sqrt{15}}=2\sqrt{30}+2\sqrt{30}=4\sqrt{30}$ oe	A1
(c)		**(4)**
	Finds $\frac{\mathrm{d}^2P}{\mathrm{d}r^2}=\frac{60}{r^3}$ at their r	M1
	States that $\frac{\mathrm{d}^2P}{\mathrm{d}r^2}>0$ hence minimum	A1
		(2) **11 marks**

(a)

B1 A correct formula for P or A.

Accept unsimplified version such as $P=2r+2r+r+r+2r\theta+2r\theta$

$$\text{or } A=2r\times r+\frac{1}{2}(2r)^2\theta+\frac{1}{2}(2r)^2\theta$$

It must be on one line and the bracketing, if required, must be correct.

If it is only given in terms of one variable it must be correct.

M1 Attempts to use the area of the stand is 30 m^2

Condone slips/incorrect formulae but score for a form $30=Mr^2+Nr^2\theta$ (terms may not be collected)

A1 Obtains $4r\theta=\frac{30-2r^2}{r}$, $\theta=\frac{30-2r^2}{4r^2}$ or $r\theta=\frac{30-2r^2}{4r}$ oe such as $2\theta=\frac{30}{2r^2}-1$

dM1 This is for a valid method of using their $30=Mr^2+Nr^2\theta$ in a valid formula for P to get a formula for P in terms of r.

For example substitutes their $4r\theta$, θ, or $r\theta$ (oe) into $P=Rr+Sr\theta\rightarrow P=\mathrm{f}(r)$

It is dependent upon the previous M

A1* Obtains the given answer with no incorrect work. P should appear in the solution in the correct place somewhere.

Allow calculations in (b) and (c) which have incorrect LH sides E.g. $P', P'', \frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

Parts (b) and (c) may be marked as one.

(b)

M1 Differentiates $r^{-1} \to r^{-2}$

A1 $\frac{\mathrm{d}P}{\mathrm{d}r} = 4 - \frac{30}{r^2}$

M1 Solves $\frac{\mathrm{d}P}{\mathrm{d}r} = 0 \to \left(r = \sqrt{\frac{15}{2}}\right)$ and uses the model with r to find P. Also common is $r = \frac{\sqrt{30}}{2}$

This cannot be scored from unsolvable equations such as $4 + \frac{30}{r^2} = 0$

A1 Correct working with surds and no incorrect working seen to arrive at given answer.

Allow $r = \sqrt{\frac{15}{2}} \Rightarrow P = 4\sqrt{\frac{15}{2}} + \frac{30\sqrt{2}}{\sqrt{15}} = 4\sqrt{30}$ o.e. Allow $2\sqrt{120}$ or $\sqrt{480}$

Processing may come from calculator

(c)

M1 Finds $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{k}{r^3}$ and calculates its value at their $r = \sqrt{\frac{15}{2}}$ or its sign (at their $r = \sqrt{\frac{15}{2}}$)

Only award for when $r > 0$….and not made positive from a negative answer

E.g Score M1 for $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{k}{r^3} > 0$ if $k > 0$ or $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{k}{r^3} < 0$ if $k < 0$

A1 Scored for a correct second derivative, a correct r (2sf), a correct calculation or statement +correct conclusion. See below for two examples

- Either stating that $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{60}{r^3}$, $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} =$ awrt $3 > 0$ at $r = 2.7$, P is a minimum.
- Or stating that as $r > 0$ $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = \frac{60}{r^3} > 0$, P is a minimum. (Their r in (c) must be correct to 2sf)

……

Alt solutions to part (c) using $\frac{\mathrm{d}P}{\mathrm{d}r}$ and P.

M1: Finds the value of $\frac{\mathrm{d}P}{\mathrm{d}r}$ either side of (and close to) their $r = \sqrt{\frac{15}{2}}$ Suggest 2.6 and 2.8

Alternatively finds the value of P either side of (and close to) their $r = \sqrt{\frac{15}{2}}$

A1: All calculations correct with statement (could be in graph form) and correct conclusion

……

Question Number	Scheme	Marks
16.(a)	Uses common ratios $\Rightarrow \frac{1+\cos\theta}{2\sin\theta}=\frac{4\sin\theta}{1+\cos\theta}$	M1
	$\Rightarrow 1+2\cos\theta+\cos^2\theta=8\sin^2\theta$	A1
	Uses $\sin^2\theta=1-\cos^2\theta \Rightarrow 1+2\cos\theta+\cos^2\theta=8\left(1-\cos^2\theta\right)$	dM1
	$9\cos^2\theta+2\cos\theta-7=0$ * cso	A1*
		(4)
(b)	Attempts to solve $9\cos^2\theta+2\cos\theta-7=0\Rightarrow(9\cos\theta-7)(\cos\theta+1)=0$	M1
	$\cos\theta=\frac{7}{9}$	A1
		(2)
(c)	(i) $a=2\sin\theta=2\sqrt{1-\cos^2\theta}=2\sqrt{1-\frac{49}{81}}=\frac{8}{9}\sqrt{2}$	M1 M1 A1
	(ii) $r=\frac{1+\cos\theta}{2\sin\theta}=\frac{1+\frac{7}{9}}{\frac{8}{9}\sqrt{2}}=\sqrt{2}$	M1 A1
		(5) **(11 marks)**

(a)

M1 Attempts to use common ratios to produce a correct equation usually $\frac{1+\cos\theta}{2\sin\theta}=\frac{4\sin\theta}{1+\cos\theta}$ or $(1+\cos\theta)\times\frac{1+\cos\theta}{2\sin\theta}=4\sin\theta$

A1 Correct equation: It must not be fractional and brackets must be expanded

Look for $1+2\cos\theta+\cos^2\theta=8\sin^2\theta$ or its equivalent

dM1 Uses $\sin^2\theta=1-\cos^2\theta \Rightarrow 1+2\cos\theta+\cos^2\theta=8\left(1-\cos^2\theta\right)$ to get an equation in $\cos\theta$.

It may be implied by a following line (as long as it is not the given answer)

A1* cso $9\cos^2\theta+2\cos\theta-7=0$ with all relevant stages shown.
Withhold this mark for incorrect notation etc within the body of their answer, not for part of their thought process. Eg withhold $\cos^2\theta \leftrightarrow \cos\theta^2$ without a bracket.

(b)

M1 Attempts to solve $9\cos^2\theta+2\cos\theta-7=0$ with usual rules for factorisation, formula but allow calculator use to give a value for $\cos\theta=..$ or θ. Allow with $y=\cos\theta\Rightarrow y=...$

A1 Chooses $\cos\theta=\frac{7}{9}$. Alternatively rejects $\cos\theta=-1$ Condone $\theta=\arccos\left(\frac{7}{9}\right)$

They cannot just leave both roots.

If they state $\theta=38.9°$ following $\cos\theta=\frac{7}{9}$ then the A1 mark may be awarded

This must be seen in part (b) and not part (c).

In part (c) the candidate must not choose solutions from $\cos\theta = \pm 1 \Rightarrow$ e.g. $\theta = 180°$ as these trivialise the solutions.

(c)(i)

M1 Attempts to find the value of $\sin\theta$ or $2\sin\theta$ for their value for $\cos\theta$ or θ

For example when $\cos\theta = \frac{7}{9}$ this is implied by awrt a = 1.26, $\sin\theta =$ awrt 0.63 or $\sin 38.9°$

Even allow for $\sin\arccos\left(\frac{7}{9}\right)$ or $a = 2\sin\arccos\left(\frac{7}{9}\right)$

Do not allow an attempt from incorrect statements such as $\sin\theta = 1-\cos\theta$ unless preceded by the correct $\sin^2\theta = 1-\cos^2\theta$

An alternative is via $\frac{1+\cos\theta}{r}$ where $r^2 = \frac{4\sin\theta}{2\sin\theta} = 2$

M1 Attempts to find the exact value of $\sin\theta$ using $\sin\theta = \sqrt{1-\cos^2\theta}$ with their exact value for $\cos\theta$ An equally valid attempt is to draw a triangle with the adjacent = 7 and hypotenuse = 9 and using Pythagoras' theorem to find the opposite side

A1 Finds the exact value of $a = 2\sin\theta = \frac{8}{9}\sqrt{2}$

(c)(ii)

M1 Attempts $r = \frac{1+\cos\theta}{2\sin\theta}$ with their value for $\cos\theta$ or θ For correct $\cos\theta = \frac{7}{9}$ this is implied by 1.4

You can follow through on an incorrectly found value of $\sin\theta$ in (c)(i)

Condone $r = \frac{1+\frac{7}{9}}{2\sin\left(\arccos\frac{7}{9}\right)}$

A1 $\sqrt{2}$

Note that a calculator will give 1.41 This scores 1 0

……

Note that part (c) can be done without part (b).

(c)(ii)

M1: Using the first and third terms we get $\frac{a_3}{a_1} = r^2 = \frac{4\sin\theta}{2\sin\theta} = 2$

A1: Common ratio = $\sqrt{2}$

(c)(i)

M1M1: Uses $a_1 = \frac{1+\cos\theta}{r} = \frac{1+\frac{7}{9}}{\sqrt{2}}$

A1: $a = \frac{8}{9}\sqrt{2}$ or exact simplified equivalent.

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