Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel International Advanced Level in Core Mathematics C12 (WMA01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$125\times\left(\frac{2}{5}\right)^3=\ldots$ or $8\times\left(\frac{2}{5}\right)^{-3}=\ldots$ or $r^3=\frac{8}{125}\Rightarrow r=\ldots$ oe	M1
	$125\times\left(\frac{2}{5}\right)^3=8\Rightarrow r=\frac{2}{5}*$ or $r=\sqrt[3]{\frac{8}{125}}=\frac{2}{5}*$	A1*
		(2)
(b)	$a=\frac{125}{\left(\frac{2}{5}\right)^3}$ or $a=\frac{8}{\left(\frac{2}{5}\right)^6}$ $\left(=\frac{15625}{8}=1953.125\right)$	M1
	$S_\infty=\frac{"\frac{15625}{8}"}{1-\frac{2}{5}}$ or $S_{10}=\frac{"\frac{15625}{8}"\left(1-\left(\frac{2}{5}\right)^{10}\right)}{1-\frac{2}{5}}$	M1
	$\pm\left(S_\infty-S_{10}\right)=\pm\left(\frac{78125}{24}-3254.867\right)=\pm0.341$	dM1
	$\pm\left(S_\infty-S_{10}\right)=\pm0.341$	A1
		(4)
		(6 marks)

(a)

M1 Attempts to verify that $r=\frac{2}{5}$, using the fourth term and r to reach the seventh term or vice versa. Alternatively obtains a correct equation for $r^{\pm3}$ and proceeds to find a value for r. Accept $(r=)\ \sqrt[3]{\frac{8}{125}}$ oe

A1* Concludes that $r=\frac{2}{5}$ (requires a tick, $\therefore$,$\Rightarrow$, $r=\frac{2}{5}$) or solves to obtain $r=\frac{2}{5}$.

Minimum acceptable for both marks is $r=\sqrt[3]{\frac{8}{125}}=\frac{2}{5}$ or $r^3=\frac{8}{125}\Rightarrow r=\frac{2}{5}$. Ignore dubious working.

(b)

M1 Correct method to find the first term using $r=\frac{2}{5}$. May be implied by $\frac{15625}{8}$ or 1953.125

M1 Correct method to find S_∞ or S_{10} using their "a" and $r=\frac{2}{5}$. If they attempt to find S_{10} then they must use $n=10$.

dM1 Both correct formulae for S_∞ and S_{10} and attempts $\pm\left(S_\infty-S_{10}\right)$. It is dependent on the previous method mark.

A1 awrt ±0.341

Question Number	Scheme	Marks
2(a)	$8^x = 2^{3x}$	B1
	$\dfrac{8^x}{2^{x-1}} = 2^{3x-(x-1)}$	M1
	$2^{2x+1} \Rightarrow a=2,\ b=1$	A1
		(3)
(b)	$2\sqrt{2} = 2^{1.5}$	B1
	$"1.5" = "2x+1" \Rightarrow x = ...$	M1
	$x = \dfrac{1}{4}$	A1cao
		(3)
Alt(a)	$8^x = 2^{3x}$	B1
	$\left(2^{3x} =\right)\ 2^{x-1} \times 2^{ax+b} = 2^{x-1+ax+b}$	M1
	$a=2,\ b=1$	A1
Alt(a)	$\ln 8^x - \ln 2^{x-1} = \ln 2^{ax+b}$	
	$3x\ln 2 - (x-1)\ln 2 = \ln 2^{ax+b}$	B1
	$(2x+1)\ln 2 = (ax+b)\ln 2$	M1
	$2x+1 = ax+b$	A1
	$a=2,\ b=1$	
Alt(b)	$("2x+1")\ln 2 = \ln 2\sqrt{2}$	B1
	$("2x+1")\ln 2 = \ln 2\sqrt{2} \Rightarrow x = ...$	M1
	$x = \dfrac{1}{4}$	A1
		(6 marks)

(a)

B1 Sight of 8^x being written as 2^{3x}, $\left(2^x\right)^3$ or $\left(2^3\right)^x$. This does not have to be written as part of the equation and my be implied. In the alternative method you may see sight of one of the given expressions or it may be written as $3\ln 2$

M1 Subtracts their powers of 2. (may be implied eg. $2^{2x\pm1}$). Alternatively, they may multiply both sides by 2^{x-1} and add their powers of 2. They do not need to simplify for this mark and it is possible in the alternative method to achieve B0M1A0. **Do not award** for incorrect application of laws of indices which may lead to a correct power (i.e. $\frac{2^{3x}}{2^{x-1}} = 1^{3x-(x-1)}$)

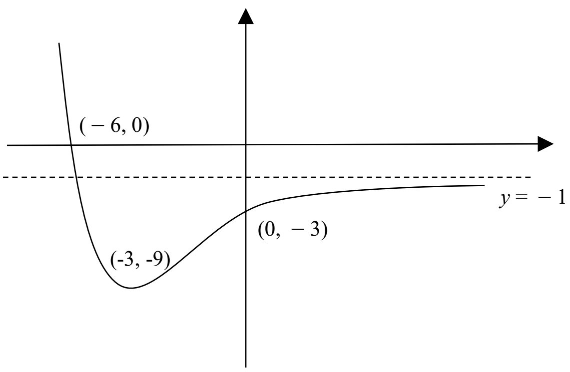
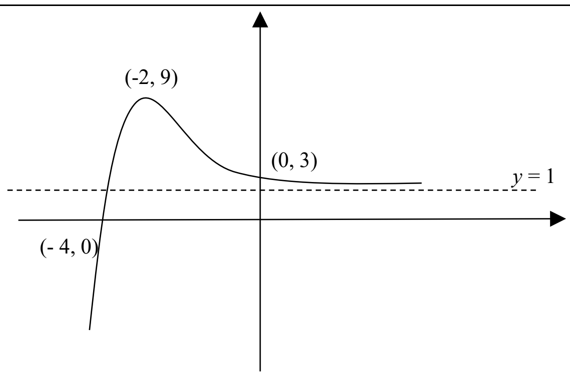
A1 $a = 2,\ b = 1$ cao. Condone 2^{2x+1}. Do not award from incorrect work.

(b)

B1 Sight or implied use of $2^{1.5}$ or $2^{\frac{3}{2}}$. Alternatively they take lns of both sides (or log of any base) and correctly apply the power rule to achieve $("2x+1")\ln 2 = \ln 2\sqrt{2}$ oe

M1 Sets their powers of 2 equal and solves for *x*. In the alternative method it is for rearranging to find a value for *x*. Condone sign slips only which may also be in their copying of their answer from part (a).

A1 $x = \frac{1}{4}$ cao

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3(a)</td><td>
</td><td>B1B1B1</td></tr>
<tr><td></td><td></td><td>(3)</td></tr>
<tr><td>(b)</td><td>
</td><td>B1B1B1</td></tr>
<tr><td></td><td></td><td>(3)</td></tr>
<tr><td></td><td></td><td>(6 marks)</td></tr>
</table>

(a)

B1 Correct shape and position that is in all of quadrants 2,3 and 4 only (must not appear in quadrant 1) and has a minimum point in quadrant 2. Allow slips of the pen towards the asymptote but as a rule of thumb the graph should be higher than $(0,-3)$ (or their y-intercept)

B1 All three coordinates $(-3,-9)$, $(-6,0)$ and $(0,-3)$ clearly labelled on the sketch in the correct places (eg. $(0,-3)$ must be a negative y-intercept, $(-3,-9)$ must be a turning point in quadrant 3, $(-6,0)$ must be a negative x-intercept).
Condone the coordinates the wrong way round if the point of intersection is in the correct place. Allow -6 or -3 being labelled as intersection points on the axes. Condone any extra points of intersection and condone the graph "stopping" at $("-6",0)$ for this mark.

B1 Asymptote given as $y=-1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic but this must be the only horizontal asymptote offered by the candidate. A graph is not needed for this mark.

(b)

B1 A graph with correct shape and position that must be in all of quadrants 1, 2 and 3 only (must not appear in quadrant 4) with a maximum in quadrant 2. Allow slips of the pen towards the asymptote but as a rule of thumb the graph should be lower than $(0,3)$ (or their y-intercept).
At least one coordinate or the equation of the asymptote must be adapted to those given in the original graph for f(x).

B1 All three coordinates $(-2,9)$, $(-4,0)$ and $(0,3)$ clearly labelled on the sketch in the correct places. Condone the coordinates the wrong way round if the point of intersection is in the correct place. Allow -4 or 3 being labelled as intersection points on the axes. Condone any extra points of intersection and condone the graph "stopping" at $(-4,0)$ for this mark.

B1 Asymptote given as $y=1$. This could appear on the diagram or within the text. Note that the curve does not need to be asymptotic but this must be the only horizontal asymptote offered by the candidate. A graph is not needed for this mark.

Question Number	Scheme	Marks
4	$\dfrac{2x^4-8}{5\sqrt{x}}=\dfrac{2x^4}{5\sqrt{x}}-\dfrac{8}{5\sqrt{x}}=\dfrac{2}{5}x^{\frac{7}{2}}-\dfrac{8}{5}x^{-\frac{1}{2}}$	M1
	$\left(\dfrac{dy}{dx}=\right)\ ...x^1+...x^{-2}+...x^{"\frac{5}{2}"}+...x^{"-\frac{3}{2}"}$	M1
	$\left(\dfrac{dy}{dx}=\right)\ 10x-\dfrac{1}{2}x^{-2}+\dfrac{7}{5}x^{\frac{5}{2}}+\dfrac{4}{5}x^{-\frac{3}{2}}$	A1A1A1A1
		(6 marks)

Notes

M1 Attempts to split $\dfrac{2x^4-8}{5\sqrt{x}}$ to obtain two terms where one is of the form $px^{\frac{7}{2}}$ or $qx^{-\frac{1}{2}}$. They may do this by writing as two separate fractions or eg. $(2x^4-8)\times\dfrac{1}{5}x^{-\frac{1}{2}}$ and multiplying out the bracket. May be implied by their differentiated terms.

M1 Reduces a correct power by 1 on any term $x^n \to x^{n-1}$ (i.e. $...x^2\to...x^1$, $...x^{-1}\to...x^{-2}$, $...x^{\frac{7}{2}}\to...x^{\frac{5}{2}}$, $...x^{-\frac{1}{2}}\to...x^{-\frac{3}{2}}$._ The indices must have been processed for this mark. Follow through on their $px^{\frac{7}{2}}$ or $qx^{-\frac{1}{2}}$ but do not allow for incorrect differentiation (i.e. $\left(\dfrac{2x^4-8}{5}\right)x^{-\frac{1}{2}}\to...\left(\dfrac{2x^4-8}{5}\right)x^{-\frac{3}{2}}$. They would need to fully attempt the product or quotient rule on that final term if they have not split into separate terms initially)

A1 Any one term correct and simplified (see below)

A1 Any two terms correct and simplified (see below)

A1 Any three terms correct and simplified (see below)

A1 $\left(\dfrac{dy}{dx}=\right)\ 10x-\dfrac{1}{2}x^{-2}+\dfrac{7}{5}x^{\frac{5}{2}}+\dfrac{4}{5}x^{-\frac{3}{2}}$ appearing all on one line and fully simplified. Isw after this.

Note allow exact simplified equivalents eg. $-\dfrac{1}{2x^2}$, $+\dfrac{7\sqrt{x^5}}{5}$, $+\dfrac{4}{5x^{\frac{3}{2}}}$, $0.8x^{-\frac{3}{2}}$ **but NOT eg** $10x^1$ $-\dfrac{2}{4x^2}$

Beware of $\dfrac{1}{2x}\to 2x^{-1}\to -2x^{-2}\to -\dfrac{1}{2x^2}$ is M1A0

Question Number	Scheme	Marks
5(a)	$u_2 = k - \frac{8}{1}$ or $u_3 = k - \frac{8}{"k-8"}$	M1
	$u_2 = k - \frac{8}{1}$ and $u_3 = k - \frac{8}{k-8}$ oe	A1
		(2)
(b)	$u_3 = 6 \Rightarrow k - \frac{8}{k-8} = 6$	M1
	$"k^2 - 14k + 40" \; (=0)$ oe	M1
	$"(k-4)(k-10)" = 0 \Rightarrow k = ...$ or $k = \frac{"14" \pm \sqrt{"14"^2 - 4 \times "1" \times "40"}}{2 \times "1"}$	dM1
	$(k =)\ 4,\ 10$	A1
		(4)
		(6 marks)

We are now marking this on Epen2 as M1A1 M1M1M1A1

(a)

M1 Either $(u_2 =)\ k - \frac{8}{1}$ or $(u_3 =)\ k - \frac{8}{"k-8"}$.

A1 $(u_2 =)\ k - \frac{8}{1}$ and $(u_3 =)\ k - \frac{8}{k-8}$ isw after a correct unsimplified expression for both.

(b)

M1 Sets their $"u_3" = 6$ to form an equation in k

M1 Rearranges their equation (which could be from eg $u_2 = u_3$) to form a 3TQ =0 or equivalent. (Eg $"k^2 = 14k - 40"$) Do not be concerned with the mechanics of their rearrangement and if they only have an expression for their 3TQ equation this may be implied by further work (i.e. attempting to solve their equation)

dM1 Attempts to solve their 3TQ. Apply general marking principles for solving a quadratic. If they use their calculator you may need to check. It is dependent on the previous method mark.

A1 $(k =)\ 4,\ 10$. Do not allow $x = ...$

Question Number	Scheme	Marks
6(a)	$\left(1+\frac{1}{4}x\right)^{12}=1+\binom{12}{1}\left(\frac{1}{4}x\right)+\binom{12}{2}\left(\frac{1}{4}x\right)^2+\binom{12}{3}\left(\frac{1}{4}x\right)^3+\ldots$	M1
	$=1+3x,+\frac{33}{8}x^2,+\frac{55}{16}x^3+\ldots$	B1A1A1
		(4)
(b)	$\left(3+\frac{2}{x}\right)^2=9+\frac{12}{x}+\frac{4}{x^2}$ oe	B1
	$"\left(9+\frac{12}{x}+\frac{4}{x^2}\right)""\left(1+3x+\frac{33}{8}x^2+\frac{55}{16}x^3+\ldots\right)"=\ldots$ At least two of: $"9\times 3x"$, $"\frac{12}{x}\times\frac{33}{8}x^2"$, $"\frac{4}{x^2}\times\frac{55}{16}x^3"$	M1
	$"9\times 3x"+"\frac{12}{x}\times\frac{33}{8}x^2"+"\frac{4}{x^2}\times\frac{55}{16}x^3"=\ldots x$	dM1
	$=\frac{361}{4}$	A1
		(4)
		(8 marks)

(a)

M1 An attempt at the binomial expansion to get the third **and/or** fourth term. The **correct** binomial coefficient needs to be combined with the correct power of x. Ignore bracket errors and omission of or incorrect powers of $\frac{1}{4}$. Accept any notation for ${}^{12}C_2$ or ${}^{12}C_3$, e.g. $\binom{12}{2}$ or $\binom{12}{3}$ or 66 or 220 from Pascal's triangle or $\frac{12\times 11}{2!}$ or $\frac{12\times 11\times 10}{3!}$ which may be written as e.g. $\frac{12\times(12-1)\times(12-2)}{3!}$

B1 $1+3x$ only. Accept unsimplified equivalent expressions such as $1+3x^1$ or $1+\frac{3x^1}{1}$ or $1,3x$ or $1+12\times\frac{1}{4}x^1$ but the binomial coefficient must be numerical (i.e. NOT $1+\binom{12}{1}\frac{1}{4}x^1$)

A1 $+\frac{33}{8}x^2$ or $+\frac{55}{16}x^3$. Fractions must be simplified and accept $4.125x^2$ or $3.4375x^3$

A1 $1+3x+\frac{33}{8}x^2+\frac{55}{16}x^3+\ldots$ Ignore any terms after this and accept written as a list. Fractions must be simplified and accept the coefficients as decimals.

(b)

B1 $9+\frac{12}{x}+\frac{4}{x^2}$ or unsimplified equivalent for the expansion of $\left(3+\frac{2}{x}\right)^2$. The two $\frac{6}{x}$ terms do not need to have been combined for this mark but do not allow additional terms

M1 Attempt to multiply their expansion of $\left(3+\frac{2}{x}\right)^2$ by their answer to (a) to correctly find at least two terms in x.. Their expansions must be of the form:

$$\left(a+\frac{b}{x}+\frac{c}{x^2}\right)\left(P+Qx+Rx^2+Sx^3\right) \quad \text{where } a,c\neq 0\text{, possibly } S=0 \text{ or } P=0$$

Do not be concerned with the multiplication of other terms that do not produce an x term.

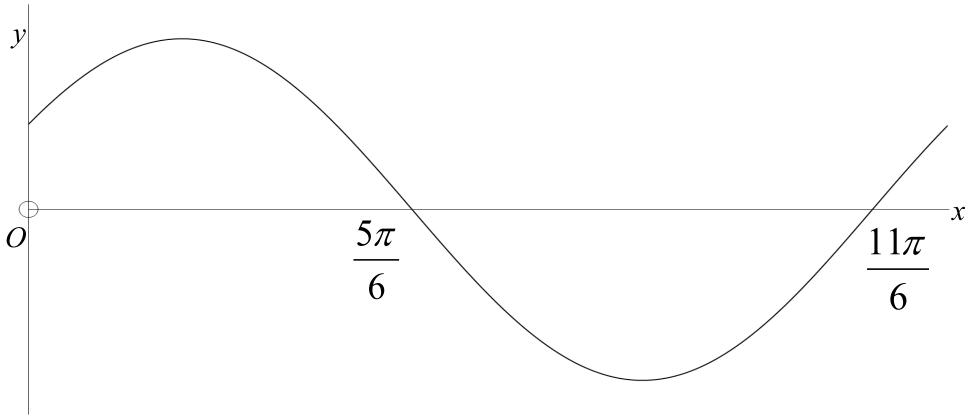
dM1 Attempts to add all necessary x terms (or just the coefficients) to get a single term in x or a single coefficient of x.

This will arise from calculating

$$\underline{aQ}x+\underline{bR}x+\underline{cS}x=\ldots x \qquad a,Q,b,R,c,S \text{ all non-zero}$$

A1 $\frac{361}{4}$ or exact equivalent (including 90.25). Do not accept $\frac{361}{4}x$

Note an expansion of $\left(9+\frac{4}{x^2}\right)$ in part (b) can score a maximum of B0M1M0A0

Question Number	Scheme	Marks
7(a)		B1B1B1
		(3)
(b)	States $h = \frac{\pi}{8}$ or uses in the trapezium rule	B1
	$\underline{\{0.5 + 0.866 + 2(0.793 + 0.966 + 0.991)\}}$	M1A1
	$\frac{1}{2} \times \frac{\pi}{8}\underline{\{6.866\}} = 1.35$	A1
		(4)
		(7 marks)

(a)

B1 Correct shape and position for sin x that has been translated horizontally only. Look for one complete cycle between 0 and 2π that has a positive y-intercept and finishes above the x-axis at 2π . The graph should be above and below the x-axis. Ignore any graph to the left of the y-axis. If there is more than one cycle then $x = 2\pi$ must be labelled/marked in the correct place. Ignore any graph afterwards. Ignore any scale on the y-axis.

B1 Either $\left(\frac{5\pi}{6}, 0\right)$ **or** $\left(\frac{11\pi}{6}, 0\right)$. Allow in degrees for this mark $(150°, 0)$ or $(330°, 0)$. May be indicated as just $\frac{5\pi}{6}$ or $\frac{11\pi}{6}$ on the x-axis which is acceptable. Condone awrt 2.62 or awrt 5.76 radians for this mark.

B1 $\left(\frac{5\pi}{6}, 0\right)$ **and** $\left(\frac{11\pi}{6}, 0\right)$ and no others between 0 and 2π Must be in radians. $\left(1\frac{5\pi}{6}, 0\right)$ is acceptable. May be indicated as just $\frac{5\pi}{6}$ or $\frac{11\pi}{6}$ on the x-axis which is acceptable and a graph is not required for either B marks. No decimals accepted.

(b)

B1 States $h = \frac{\pi}{8}$ or for using a strip width of $\frac{\pi}{8}$ or awrt 0.39

M1 Scored for the correct $\{......\}$ outer bracket structure. It needs to contain first y value plus last y value and the inner bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values.

If the only mistake is a copying error or is to omit one value from inner bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values. Allow recovery of invisible brackets.

A1 For the correct bracket $\{......\}$

Note $\frac{1}{2}\times\frac{\pi}{8}\times 0.5+0.866+2\left(0.793+0.966+0.991\right)=$ awrt 1.35 B1M1A1A1

$\frac{1}{2}\times\frac{\pi}{8}\times 0.5+0.866+2\left(0.793+0.966+0.991\right)=$ awrt 6.46 B1M1A0A0

$\frac{1}{2}\times\frac{\pi}{8}\times(0.5+0.866)+2\left(0.793+0.966+0.991\right)=$ awrt 5.77 B1M1A0A0

A1 1.35 only

NB: Separate trapezia may be used: B1 for $h = \frac{\pi}{8}$, M1 for 1/2 $h(a + b)$ used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Beware if they evaluate the integral between 0 and $\frac{\pi}{2}$ the answer is 1.37 which would be A0 (and possibly 0 marks if no application of the trapezium rule is shown).

Question Number	Scheme	Marks
8(a)	Arc length $BD = r\theta = 6\times 3.5 = 21$ (cm)	M1A1
		(2)
(b)	$\angle AEB = \frac{3}{2}\pi - 3.5 \quad (= \text{awrt } 1.2)$	B1
	$AB^2 = 9^2 + 6^2 - 2\times 9\times 6\cos(\text{"}\angle AEB\text{"}) \quad (=79.1...)$	M1
	$P = 6 + 9 + \text{"21"} + \text{"}AB\text{"}$	dM1
	$P =$ awrt 44.9 (cm)	A1
		(4)
(c)	Area triangle $ABE = \frac{1}{2}\times 6\times 9\sin\left(\text{"}\frac{3}{2}\pi - 3.5\text{"}\right)$ or $\frac{1}{2}\times 9\times 6\cos(\text{"}3.5-\pi\text{"})$ (≈ 25.3)	M1
	Area sector $BDE = \frac{1}{2}\times 6^2\times 3.5$ (= awrt 63)	B1
	$\frac{1}{2}\times 6\times 9\sin\left(\text{"}\frac{3}{2}\pi - 3.5\text{"}\right) + \frac{1}{2}\times 6^2\times 3.5 = \text{"25.3"} + \text{"63"} = ...$ (dep B1M1)	dM1
	Area = awrt 88.3 (cm^2)	A1
		(4)
		(10 marks)

Notes

(a)

M1 A correct method to find the length of the arc BD. They may work in degrees (i.e. $\frac{\theta}{360}\times 2\pi\times 6$ where θ is between 200 and 201 degrees.

A1 21 (cm)

(b)

B1 Finds angle $\angle AEB$. Accept awrt 1.2 radians or awrt $69.5°$. May appear in any part or appear on the diagram.

M1 Correctly uses the cosine rule to find an expression for the length AB $(=8.89...)$ or $AB^2\ (=79.1...)$ using lengths 6, 9 and their $\angle AEB$ $(\neq 3.5)$

dM1 Adds the lengths 6, 9, their (a) and their AB to achieve a value for the perimeter. You may need to check this with your calculator. Do not allow their AB to be 9 unless it has been rounded from 8.89.... (The triangle should not be assumed to be isosceles). This mark is dependent on the previous method mark.

A1 awrt 44.9 (cm)

(c) Equivalent working in degrees is acceptable

M1 Attempts to find the area of triangle ABE with correct lengths 6 and 9 and their $\angle AEB$ (FYI the area $ABE \approx 25.3$)

B1 $\frac{1}{2}\times 6^2 \times 3.5$ (or may be implied by awrt 63) for sector BDE

dM1 Attempts to add their area of the sector to the area of the triangle. It is dependent on the previous B and M marks.

A1 awrt 88.3 (cm^2)

Question Number	Scheme	Marks
9(a)	32	B1
		(1)
(b)	$\mathrm{f}(-1)=(-1+k)\left(3(-1)^2+4(-1)-16\right)+32$	M1
	$(-1+k)\left(3(-1)^2+4(-1)-16\right)+32=15\Rightarrow k=...$	dM1
	$k=2$ *	A1*
		(3)
(c)	$(x+2)(3x^2+4x-16)=...$	M1
	$\mathrm{f}(x)=3x^3+10x^2-8x$	A1
	$\mathrm{f}(x)=x(3x^2+10x-8)=x(...)(...)$	dM1
	$\{\mathrm{f}(x)\}=x(3x-2)(x+4)$	A1
		(4)
Alt(b)	Algebraic division $$\begin{array}{r} 3x^2+(1+3k)x+k-17 \\ (x+1)\overline{)3x^3+(4+3k)x^2+(4k-16)x+32-16k} \\ \underline{3x^3+3x^2} \\ (1+3k)x^2+(4k-16)x \\ .... \\ .... \\ \overline{-17k+49} \end{array}$$	M1
	$"-17k+49"=15\Rightarrow k=...$	dM1
	$k=2$ *	A1*
		(8 marks)

(a)

B1 32 (If algebraic division is used then they must clearly state the remainder is 32)

(b)

M1 Attempts $\mathrm{f}(\pm 1)$. It is sufficient to see the value 1 or -1 substituted into the expression for $\mathrm{f}(x)$. They may even expand the expression for $\mathrm{f}(x)$ so condone algebraic errors before they substitute 1 or -1.
In the alternative method they algebraically divide by $(x+1)$ to achieve a quotient of $3x^2$ +…

dM1 Sets their expression for $\mathrm{f}(\pm 1) = 15$ and solves to find *k*. Condone errors in rearrangement and arithmetical slips for this mark
In the alternative method they set their remainder equal to 15 and attempt to find *k*.

A1* Achieves $k = 2$ with no errors in working including brackets. Allow invisible brackets if the intention is clear, however. As a minimum final intermediate step before proceeding to $k = 2$ expect to see $\pm 17k = ...$

(c)

M1 Attempts to multiply out $(x+2)(3x^2+4x-16)$ to reach an expression of the form $Ax^3 + Bx^2 + Cx \ (+D)$ with $A, B, C \neq 0$ with no terms in *k*. This can be awarded from earlier work if $k = 2$ is substituted into an expansion in part (b).

A1 $\mathrm{f}(x) = 3x^3 + 10x^2 - 8x$

dM1 Attempts to factorise their cubic, which must be of the form $Ax^3 + Bx^2 + Cx$ with $A, B, C \neq 0$, to achieve

$$(\mathrm{f}(x) =) \text{ LINEAR} \times \text{QUADRATIC (eg } x(Ax^2 + Bx + C)\text{)}.$$

They may factorise out a different linear factor, it may be seen within an algebraic division method or they may have found all three linear factors. It is dependent on the previous method mark.

A1 $\{\mathrm{f}(x) =\}\ x(3x-2)(x+4)$ on one line. ISW after this.

Question Number	Scheme	Marks
10(a)		
(i)	$(1)^2+(16)^2+4(1)+16p+123=0 \Rightarrow p=...$	M1
	$16p=-384 \Rightarrow (p=)\ -24$	A1
(ii)	Centre is $(-2,\ 12)$	B1ft
(iii)	$r^2="2"^2+"12"^2-123 \Rightarrow r=...$ or $r^2=(1-("-2"))^2+(16-("12"))^2 \Rightarrow r=...$	M1
	$r=5$	A1cao
		(5)
(b)	$m_N=\dfrac{16-"12"}{1-"-2"}\left(=\dfrac{4}{3}\right)$	M1
	$m_T="-\dfrac{3}{4}"$	M1
	$y-16="-\dfrac{3}{4}"(x-1)$	M1
	$3x+4y-67=0$	A1
		(4)
Alt(b)	$2x+2y\dfrac{dy}{dx}+4-24\dfrac{dy}{dx}=0$	M1
	$"2"\times1+"2"\times16\dfrac{dy}{dx}+"4"-"24"\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=...\left(=-\dfrac{3}{4}\right)$	M1
	$y-16="-\dfrac{3}{4}"(x-1)$	M1
	$3x+4y-67=0$	A1
		(9 marks)

Notes

(a)
(i)

M1 Attempts to substitute the coordinates $x = 1, y = 16$ into the equation of the circle and proceeds to find a value for p.

A1 $(p =) - 24$

(ii)

B1ft $(-2, 12)$. Follow through on their value of p so look for coordinates of the form $(-2, -\frac{p}{2})$

(iii)
M1 Uses the equation of the circle for their centre and proceeds to find a value for r or a fully correct attempt to find the length between their centre and $(1,16)$ using Pythagoras (where $r > 0$ and $r^2 \neq 123$). They may have already done this to find the centre of the circle so check earlier work.

A1 $r = 5$ cao (can only be awarded from a correct $p = -24$)

(b)

M1 For an attempt at the gradient between their centre of C and $(1,16)$
Look for an attempt at $\frac{\Delta y}{\Delta x}$ There must be an attempt to subtract on both the numerator and the denominator. It can be implied by their attempt to find the equation of the tangent to C. In the alternative method they implicity differentiate to achieve a form of
$...x + ...y\frac{\mathrm{d}y}{\mathrm{d}x} + ... + ...\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

M1 For an attempt at using $m_T = -\frac{1}{m_N}$ or equivalent to find the gradient of the perpendicular m_2
In the alternative method they substitute in the point $(1,16)$ and proceed to finding a value for their $\frac{\mathrm{d}y}{\mathrm{d}x}$. There is no requirement for a specific differentiated form for this mark.

M1 It is for the method of finding a line passing though $(1,16)$ with a changed gradient.
Eg $\frac{8}{3} \rightarrow \frac{3}{8}$
Look for $(y - 16) =$ changed $m_N(x - 1)$ or equivalent. Both brackets must be correct.
Alternatively uses the form $y = mx + c$ AND proceeds as far as $c = \ldots$
In the alternative method, uses their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find the equation of a straight line passing through $(1,16)$

A1 $3x + 4y - 67 = 0$ Accept $\pm A(3x + 4y - 67 = 0)$ where $A \in \mathbb{N}$

Question Number	Scheme	Marks
11(a)	$2x^2+x+6=mx-2 \Rightarrow 2x^2+x-mx+8\ (=0)$	M1
	$b^2-4ac<0 \Rightarrow (1-m)^2-4(2)(8)<0$	M1
	$m^2-2m-63<0$*	A1*
		(3)
(b)	$m^2-2m-63=0 \Rightarrow (m-9)(m+7)=0 \Rightarrow m=...$	M1
	$(m=)\ \ -7,9$	A1
	Attempt at inside region	M1
	$-7<m<9$	A1
		(4)
		(7 marks)

(a)

M1 Sets $2x^2+x+6=mx-2$ and rearranges the equation $(=0)$. Condone sign slips but all terms must be on one side. The two terms in x do not have to be collected together for this mark and the "=0" may be implied by further work.

M1 Attempts $b^2-4ac...0$ using values of $a=\pm2,\ b=\pm1\pm m,\ c=\pm8$ or ±4 where an inequality or equals sign is used. You may see $b^2...4ac$. It is sufficient to see the values substituted in correctly for this mark and you can condone invisible brackets. They must have achieved a quadratic in x to calculate the discriminant.

A1* Proceeds to given answer with no errors. The correct inequality for the discriminant must appear before the final line.

(b)

M1 Attempts to solve the given quadratic (which may be in terms of another variable) to find at least one of the critical values for m. Apply general marking principles for solving a quadratic.

A1 $(m=)-7,9$ only

M1 Finds inside region for their critical values. They may draw a diagram but they must proceed to an $...<m<...$ (allow use of $\leq$ for one or both inequalities for this mark and they may even be separate statements). May be in terms of x or any other variable for this mark.

A1 $-7<m<9$ **Must be in terms of m**

Accept others such as , $m>-7$ AND $m<9$ $\ m>-7,\ \text{m}<9$, $9>m>-7$,$\{m:-7<m<9\}$

and also accept $(-7,9)$

Do NOT accept $m>-7$ OR $\text{m}<9$ and do NOT accept inequalities shown on a number line.

Correct answers with no working is 4/4 in (b)

Question Number	Scheme	Marks
12(a)	$$14+7\cos x=4\sin^2 x+12$$ $$14+7\cos x=4\left(1-\cos^2 x\right)+12$$ $$4\cos^2 x+7\cos x-2=0$$	M1 M1 A1
		(3)
(b)	$$\left(4\cos x-1\right)\left(\cos x+2\right)=0\Rightarrow\cos x=...$$ $$\cos x=\frac{1}{4}$$ $$x=\cos^{-1}\left("\frac{1}{4}"\right)=\ldots$$ $x =$ awrt 1.32, awrt 4.96/4.97	M1 A1 dM1 A1A1
		(5)
		(8 marks)

(a)

M1 Multiplies both sides by $(3+\sin^2 x)$ to create an equation with only numerators containing trigonometric functions. Typically you may see $7(2+\cos x)=4(3+\sin^2 x)$ which is sufficient and condone errors including only multiplying one of the terms when expanding any brackets. Condone invisible brackets for this mark. You may see eg. $7(2+\cos x)=4(4-\cos^2 x)$ oe if they have used the identity first.

M1 Attempts to use $\sin^2 x = \pm 1 \pm \cos^2 x$ to achieve an equation in $\cos x$ only. They may do this before multiplying both sides by the denominators.

A1 $4\cos^2 x + 7\cos x - 2 = 0$ or exact equivalent eg. $-4\cos^2 x - 7\cos x + 2 = 0$

Accept fractions eg $\dfrac{-4}{7}\cos^2 x - \cos x + \dfrac{2}{7} = 0$

Withhold the final mark for any errors including missing brackets in their solution.

(b) **If the quadratic reached in (a) is incorrect then only the method marks can be scored in (b)**

M1 Solves their quadratic to achieve $\cos x = \alpha$. Apply general marking principles for solving a quadratic but their value, α, must be $|\alpha| \leq 1$. Ignore any other solutions. **Values just stated from a calculator with no working MUST be checked.**

A1 $\cos x = \dfrac{1}{4}$ (ignore the other solution $\cos x = -2$). **Must be achieved from a correct equation.**

dM1 Finds arcos("α") to achieve a value for x. You may need to check this on your calculator (check correct to 2sf). It is dependent on the previous method mark.

A1 One of awrt 1.32 or awrt $4.96/4.97$ (accept awrt $75.5°$ or $284.5°$ for this mark only)

A1 Both awrt 1.32 and awrt $4.96/4.97$ only. Withhold if any additional angles inside the range.

Answers with no working scores 0 marks.

Minimum acceptable for full marks in (b) is $\cos x = \dfrac{1}{4} \Rightarrow x = \text{awrt } 1.32 \text{ and awrt } 4.97$

Question Number	Scheme	Marks
13(a)	Eg $\log_a 900 = \log_a 9 + \log_a 100$ or $\log_a 100 = 2\log_a 10$	M1
	$\Rightarrow \log_a 9 + 2\log_a 10$ or $\Rightarrow \log_a 9 + \log_a 10 + \log_a 10$	dM1
	$p + 2q$ or $p + q + q$	A1
		(3)
(b)	$\log_a 0.3 = \log_a \frac{3}{10} = \log_a 3 - \log_a 10$	M1
	$\log_a 3 = \log_a 9^{\frac{1}{2}}$ or $\log_a 3 = \frac{1}{2}\log_a 9$	B1
	$\frac{1}{2}p - q$ oe (eg. $p - \frac{p}{2} - q$)	A1
		(3)
		(6 marks)

Do not penalise the lack of a base letter *a* in this question.

(a)

M1 Correct use of the addition rule or power law. Look for either $\log_a xy = \log_a x + \log_a y$ or the $\log_a x^n = n\log_a x$ at least once which may be after incorrect working.

Condone $\log_a 900 = \log_a (9\times 10) = \log_a 9 + \log_a 10$ for M1

Do not allow $\log_a 900 = \log_a 9 \times \log_a 100 = \log_a 9 + \log_a 100$

dM1 A correct method to achieve an allowable form of either $\log_a 9 + 2\log_a 10$ or $\log_a 9 + \log_a 10 + \log_a 10$ which is dependent on the previous mark.

A1 $p + 2q$ or $p + q + q$

(b)

M1 Correct use of the subtraction rule. Look for $\left(\log_a \frac{x}{y} = \log_a x - \log_a y\right)$ used at least once which may be after incorrect working. Eg $\log_a 0.3$ as $\log_a 3 - \log_a 10$

B1 Sight or implied use of $\log_a 3 = \log_a 9^{\frac{1}{2}}$ or $\log_a 3 = \frac{1}{2}\log_a 9$. CANNOT be awarded for $\log_a 3^2 = \log_a 9$

A1 $\frac{1}{2}p - q$ oe (eg. $p - \frac{p}{2} - q$)

Question Number	Scheme	Marks
14(a) **(i)&(ii)**	$a+4d=4k$ oe	B1
	$S_8=\frac{1}{2}(8)\left[2a+(8-1)\times d\right]=20k+16$ oe	B1
	…………………………………………………………………………..	
	Eg $4\left[8k-8d+7d\right]=20k+16\Rightarrow d=...$ or $a+4\left(\frac{5k+4-2a}{7}\right)=4k\Rightarrow a=...$	M1
	$(d=)\ 3k-4$ oe (see notes) or $(a=)\ 16-8k$ *	A1
	Eg $a=4k-4d=4k-4(3k-4)\Rightarrow a=...$ or $16-8k+8d=24\Rightarrow d=...$	M1
	$(a=)\ 16-8k$ and $(d=)\ 3k-4$ oe (see notes) *	A1*
		(6)
(b)	$16-8k+8(\text{"}3k-4\text{"})=24\Rightarrow k=...$	M1
	$(k=)\ 2.5$ oe	A1
		(2)
(c)	$a=16-8\times\text{"}2.5\text{"}(=-4),\ d=\text{"}3\text{"}\times\text{"}2.5\text{"}-\text{"}4\text{"}(=3.5)$	M1
	$S_{20}=\frac{1}{2}(20)\left[2\times(\text{"}-4\text{"})+(20-1)\times\text{"}3.5\text{"}\right]$	dM1
	$(S_{20}=)\ 585$	A1
		(3)
Alt(b)	$4d=24-4k\Rightarrow 4(\text{"}3k-4\text{"})=24-4k\Rightarrow k=...$	M1
	$(k=)\ 2.5$ oe	A1
Alt(c)	$S_{20}=\frac{1}{2}(20)\left[2\times(16-8k)+(20-1)\times(\text{"}3k-4\text{"})\right]=\frac{1}{2}(20)\left[2\times\text{"}-4\text{"}+(20-1)\times(\text{"}3.5\text{"})\right]$	M1
		dM1
	$S_{20}=\frac{1}{2}(20)\left[2\times\text{"}-4\text{"}+(20-1)\times(\text{"}3.5\text{"})\right]$	A1
	$(S_{20}=)\ 585$	

		(11 marks)

(a) **Mark parts (i) and (ii) together and allow *a* and *d* to be other variables such as y and x. Do not allow work seen in other parts to earn marks in (a)**

(i)&(ii)

B1 Sight of $a+4d=4k$ oe

B1 Sight of an expression for the sum of the first 8 terms set $=20k+16$ which is usually $\frac{1}{2}(8)\left[2a+(8-1)\times d\right]=20k+16$ oe. Condone invisible brackets. They may also write $a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d+a+7d=20k+16$ which is correct.

M1 Attempts to solve simultaneously to find *a* or *d* in terms of *k* only. Do not be concerned with the mechanics and condone invisible brackets. There are other ways of doing this but the requirement is to end up with an equation in just *k* leading to *a*= or *d* =

Note that if they use the given answer $a=16-8k$, without proof, then the maximum that they can score in (a) is B1B1M0A0M1A0.

A1 Either $(d=)\ 3k-4$ (oe of the form $\frac{Ak+B}{C}$ eg. $\frac{12k-16}{4}$) or $(a=)\ 16-8k$ * which must be from correct work.

Note A marks cannot be scored using $a=16-8k$, without proof, in S_8 to find *d*

M1 Attempts to solve simultaneously to find the other variable of *a* and *d* in terms of *k* only. Do not be concerned with the mechanics and condone invisible brackets. There are other ways of doing this but the requirement is to end up with an equation in just *k* leading to *a*= or *d*=

A1* Both $(d=)\ 3k-4$ (oe of the form $\frac{Ak+B}{C}$ eg. $\frac{12k-16}{4}$) and $(a=)16-8k$ with no obvious errors. **Note A marks cannot be scored using $a=16-8k$, without proof, in S_8 to find *d***

(b)

M1 Uses the given *a,* and their *d* in terms of *k,* in a correct term formula for u_9 to find *k.* Alternatively they may set the difference between the 5th and 9th terms equal to $24-4k$ and proceeds to find *k*. Condone bracketing errors.

A1 $(k=)\ 2.5$ oe

(c)

M1 Substitutes their value for *k* to find numerical expressions for *a* **and** *d*. If *a* and *d* are stated without working then at least one of them must be correct for their *k, d* and the given *a.*

dM1 Substitutes their numerical expressions for *a*, *d* and $n=20$ into a correct formula for. It is sufficient to see these embedded within the formula to score this mark. It is dependent on the first method mark.

A1 $(S_{20}=)\ 585$ cao

Alt method: If they write their sum in terms of *k* to give:
$(S_{20}=)\ \frac{1}{2}(20)\left[2\times(16-8k)+(20-1)\times(\text{"}3k-4\text{"})\right]$ they **MUST** then substitute their value of *k*

for $16-8k$ and their $"3k-4"$. This will score the first method mark. If the formula is correct the dependent second method mark is also scored.

Question Number	Scheme	Marks
15(a)	Area $= xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ or $P = 2y + x + \frac{1}{2}\pi x$ oe	B1
	Area $= xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ and $P = 2y + x + \frac{1}{2}\pi x$ oe	B1
	$y = \frac{100}{x} - \frac{\pi x}{8} \Rightarrow P = 2\left(\frac{100}{x} - \frac{\pi x}{8}\right) + x + \frac{\pi x}{2}$	M1
	$P = \frac{1}{4}x(4+\pi) + \frac{200}{x}$ *	A1*
		(4)
(b)	$\left(\frac{dP}{dx} =\right) 1 + \frac{\pi}{4} - \frac{200}{x^2}$	B1
	$1 + \frac{\pi}{4} - \frac{200}{x^2} = 0 \Rightarrow x^2 = \frac{800}{4+\pi}$	M1
	$x = \sqrt{\frac{800}{4+\pi}}$	A1
		(3)
(c)	$\left(\frac{d^2P}{dx^2} =\right) \frac{400}{x^3} = \frac{400}{"\sqrt{\frac{800}{4+\pi}}"} = \ldots$ or makes reference to the sign of $\frac{d^2P}{dx^2}$	M1
	As $\frac{d^2P}{dx^2} = \frac{400}{x^3} > 0$ when $x > 0$, P is a minimum	A1
		(2)
(d)	$P = \frac{1}{4}x(4+\pi) + \frac{200}{x} = \ldots$	M1
	= awrt 37.8 (m)	A1
		(3)
Alt(c)	Eg Substitutes in $x = 10, x = 11$ into $\left(\frac{dP}{dx} =\right)$ $"1 + \frac{\pi}{4} - \frac{200}{"10"^2}" = -0.2\ldots < 0$ and $\left(\frac{dP}{dx} =\right)$ $"1 + \frac{\pi}{4} - \frac{200}{"11"^2}" = 0.1\ldots > 0$	M1
	As the gradient changes from negative to positive $\Rightarrow$ minimum	A1
		(11 marks)

Condone alternatives such as $f'(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ rather than derivatives in terms of P and x

(a)

B1 Sight of or implied use of $xy+\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ oe OR $2y+x+\frac{1}{2}\pi x$ oe possibly unsimplified

B1 Sight or implied use of both $2y+x+\frac{1}{2}\pi x$ oe AND $xy+\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ oe possibly unsimplified

M1 Sets their expression for the area equal to 100, rearranges to make y the subject and substitutes this into their expression for the perimeter. Don't be concerned with the mechanics of rearranging the subject for this mark.

A1* cso Achieves the given answer with no errors including brackets.

(b)

B1 $\left(\frac{dP}{dx}=\right)\ 1+\frac{\pi}{4}-\frac{200}{x^2}$ oe (unsimplified although the power must be processed)

M1 Sets their $\frac{dP}{dx}=0$, which must be of the form $\frac{dP}{dx}=A+Bx^{-2}$ $A\neq 0,\ B<0$, and proceeds to $x^{\pm 2}=C$ where $C>0$. (awrt 112 or awrt 0.00893 is sufficient evidence for this mark)

A1 $x=\sqrt{\frac{800}{4+\pi}}$ or exact equivalent eg $\frac{20\sqrt{2}}{\sqrt{4+\pi}}$. Do not accept decimals (eg most likely to be 10.58…) and there should not be fractions on the numerator or denominator so do not accept eg $\frac{10\sqrt{2}}{\sqrt{1+\frac{\pi}{4}}}$ or $\sqrt{\frac{200}{1+\frac{\pi}{4}}}$ $\pm\sqrt{\frac{800}{4+\pi}}$ is A0

If $x=10.6$ or better in part (b), they have access to A marks in (c) and (d)

(c)

M1 Finds an expression for $\frac{d^2P}{dx^2}$ which must be of the form Ax^{-3}, $A>0$ and **either** substitutes their x value from (b) into their $\frac{d^2P}{dx^2}$ to find a value for $\frac{d^2P}{dx^2}$, **or** refers to the sign of the second derivative. Alternatively they may substitute x values either side of their value found in (b) into their $\frac{dP}{dx}=A+Bx^{-2}$ to find two values for $\frac{dP}{dx}$

A1 cso Correct second derivative $\frac{d^2P}{dx^2}=\frac{400}{x^3}$ (on both sides), a value of $x=10.6$ or better in (b) and proceeds to justify with a minimum conclusion of $\frac{400}{x^3}>0$ when $x>0$, P is a minimum. Where they calculate the value of the second derivative it must be correct (0.3 (1sf)) followed by a conclusion..

In the alternative method the answers to the two values substituted in must be either side of $x=\sqrt{\frac{800}{4+\pi}}$, the values of $\frac{dP}{dx}$ calculated must be correct (1sf) and they must conclude that the gradient changes from negative to positive implying a minimum point (or equivalent).

(d)

M1 Substitutes their value from (b) into the given equation for P to find a value for their minimum perimeter of the garden. It is sufficient to see the value substituted into the given expression followed by an answer, although you may need to check if no working is shown.

A1 awrt 37.8

Question Number	Scheme	Marks
16(a)	$A(0,12)$	B1
	$2x^2-11x+12=0\Rightarrow x=...$	M1
	$\left(\frac{3}{2},\ 0\right),(4,\ 0)$	A1
		(3)
(b)	**The general approach to marking part (b) should be** **B1 Correct x-coordinate at D** **M1 Integrating the curve or curve-line** **A1 Correct integration** **dM1 Substituting in their limits to find an area that is completely above the x-axis** **A1 Correct value for area for their integration** **ddM1 Correct method to find the shaded area** **A1 34.125 cso**	
Eg WAY1	At D, $x=\frac{11}{2}$	B1
	……………………………………………………………………………… Integration: $\int 2x^2-11x+12\ \mathrm{d}x=\frac{2x^3}{3}-\frac{11x^2}{2}+12x(+c)$ or $\int 12-2x^2+11x-12\ \mathrm{d}x=12x-\frac{2x^3}{3}+\frac{11x^2}{2}-12x(+c)$	M1A1
	……………………………………………………………………………….. Substitution of limits: $(18)-\left[\frac{2x^3}{3}-\frac{11x^2}{2}+12x\right]_{"4"}^{"5.5"}$ $=(18)-\left(\left(\frac{2("5.5")^3}{3}-\frac{11("5.5")^2}{2}+12("5.5")\right)-\left(\frac{2("4")^3}{3}-\frac{11("4")^2}{2}+12("4")\right)\right)$ $\left[-\frac{2x^3}{3}+\frac{11x^2}{2}\right]_4^{5.5}=\left(-\frac{2("5.5")^3}{3}+\frac{11("5.5")^2}{2}\right)-\left(-\frac{2("4")^3}{3}+\frac{11("4")^2}{2}\right)=...$	dM1
	$=10.125$	A1

	Finding the area of the triangle AOC with base vertices $x=0$ **and** $x="4"$ Area of triangle = $\frac{1}{2}\times"4"\times"12"="24"$ Area of $R = 24+10.125=...$ ……………………………………………………………………….. $=34.125$ oe $\left(=\frac{273}{8}\right)$	 ddM1 A1
		(7)
Alt(b) **WAY2**	At D, $x=\frac{11}{2}$ Integration: $\int 2x^2-11x+12\ \mathrm{d}x=\frac{2x^3}{3}-\frac{11x^2}{2}+12x(+c)$ ……………………………………………………………………….. Substitution of limits: $\left[\frac{2x^3}{3}-\frac{11x^2}{2}+12x\right]_{"4"}^{"5.5"}$ $=\left(\frac{2("5.5")^3}{3}-\frac{11("5.5")^2}{2}+12("5.5")\right)-\left(\frac{2("4")^3}{3}-\frac{11("4")^2}{2}+12("4")\right)=...$ $=7.875$ ……………………………………………………………………….. Finding the area of the trapezium with vertices AD, (“5.5”,0), (“4”,0) $=\frac{1}{2}\times"12"\times\left("\frac{11}{2}"+\left("\frac{11}{2}"-"4"\right)\right)=42$ Area of $R=42-7.875=...$ $=34.125$ oe $\left(=\frac{273}{8}\right)$	B1 M1A1 dM1 A1 ddM1 A1cso

Alt(b) **WAY3**	At D, $x=\frac{11}{2}$ Integration: $\int 2x^2-11x+12\ \mathrm{d}x=\frac{2x^3}{3}-\frac{11x^2}{2}+12x(+c)$ …………………………………………………………………………….. Substitution of limits: $\left[\frac{2x^3}{3}-\frac{11x^2}{2}+12x\right]_{"4"}^{"5.5"}$ $=\left(\frac{2("5.5")^3}{3}-\frac{11("5.5)^2}{2}+12("5.5")\right)-\left(\frac{2("4")^3}{3}-\frac{11("4")^2}{2}+12("4")\right)=...$ $=7.875$ ………………………………………………………………………………. Finding the area of the unshaded triangle with vertices, (0,“12”), (“4”,0), (0,0) $=\frac{1}{2}\times"12"\times"4"=24$ Area of $R=12\times 5.5-7.875-24=...$ …………………………………………………………………………….. $=34.125$ oe $\left(=\frac{273}{8}\right)$	B1 M1A1 dM1 A1 ddM1 A1cso
		(10 marks)

We are now marking this on epen2 as B1M1A1 B1M1A1dM1A1ddM1A1

Mark parts (a) and (b) together

(a)

B1 $(0,12)$. May be indicated on the graph. Accept 12 being indicated on the y-axis and accept $x=0, y=12$

M1 Solves the given quadratic. Apply general marking principles for solving a quadratic. Award for $\frac{3}{2}$ or 4 without any working shown

A1 Correct coordinates $\left(\frac{3}{2}, 0\right), (4, 0)$ which may be indicated on the graph. Accept $\frac{3}{2}$ and 4 being indicated on the x-axis. Can also be written as $x=\frac{3}{2}, y=0$ and $x=4, y=0$. Ignore labelling of coordinates as *A,B* and *C*.

(b)

B1 At *D*, $x=\frac{11}{2}$ which may be indicated on the graph

M1 Attempts to integrate the equation of the curve (which may be combined with the line $y=12$)Award for any one term $x^n \to x^{n+1}$ but the powers must have been processed (eg do not award for x^{2+1})

A1 $\frac{2x^3}{3}-\frac{11x^2}{2}+12x(+c)$ or $12x-\frac{2x^3}{3}+\frac{11x^2}{2}-12x(+c)$ With or without the constant of integration. Ignore any spurious notation.

dM1 Substitutes in their limits and subtracts either way round to find an area that is completely above the x-axis. Therefore, they can integrate between $x=\text{"}4\text{"}$ and $x=\text{"}5.5\text{"}$ or $x=0$ and $x=\text{"}\frac{3}{2}\text{"}$. It is dependent on the previous method mark.

A1 10.125 or in the alternative method 7.875 $(=\frac{63}{8})$

ddM1 A complete method for finding the required area. Must have scored all previous method marks in (b).

A1 34.125 or exact equivalent $\frac{273}{8}$

Note the question says using algebraic integration so answers with no integration work shown can only get a maximum of B1M0A0dM0A0ddM0A0 in (b)

Any responses that you think are credit worthy and you are unsure how to mark please send to review (eg. they may translate the graph first or any attempt to use the line

$y = 12 - 3x\ (AC)$)